Precast, Prestress Bridge Girder Design Example

PGSuper Training

WSDOT Bridge and Structures Office
Abstract
This document is a detailed design example of a 165 ft long, WF74G precast-prestressed concrete bridge girder. The example illustrates design, analysis, and load rating of a typical exterior girder in a horizontally curved structure. The primary design elements include alignment and bridge geometrics, loads and load distribution, flexure design for optimized fabrication, shear design, deflections and camber, slab haunch build-up, lateral stability during initial lifting of the girder from the casting bed and hauling the girder from the fabrication facility to the bridge site, treatment of prestress losses, and load rating. This example is intended to illustrate the calculations performed by the BridgeLink::PGSuper computer program.
# Table of Contents

1. Introduction ................................................................................................................................................................................................. 1
   1.1 Sign Convention ................................................................................................................................................................................... 1

2. Bridge Description ............................................................................................................................................................................................ 1
   2.1 Site Conditions .................................................................................................................................................................................... 1
   2.2 Roadway .............................................................................................................................................................................................. 1
   2.3 Bridge Layout ...................................................................................................................................................................................... 2

3. Design Preliminaries .......................................................................................................................................................................................... 4
   3.1 Construction Sequence ........................................................................................................................................................................ 4
   3.2 Girder Length ..................................................................................................................................................................................... 5
   3.3 Section Properties ............................................................................................................................................................................... 6
       3.3.1 Effective Flange Width .......................................................................................................................................................... 6
       3.3.2 Composite Girder Properties .............................................................................................................................................. 6
       3.3.3 First Moment of Area of deck slab, ........................................................................................................................................ 8
       3.3.4 Section Property Summary .................................................................................................................................................. 8

4. Structural Analysis .......................................................................................................................................................................................... 9
   4.1 Step 1 Design for Final Service Conditions ........................................................................................................................................ 9
       4.1.1 Stresses due to loads on non-composite section ..................................................................................................................... 9
       4.1.2 Stresses due to loads on the composite section ..................................................................................................................... 10
       4.1.3 Check Estimate of Final Concrete Strength .......................................................................................................................... 10
   4.2 Step 2 - Design for Lifting without Temporary Top Strands ........................................................................................................... 14
       4.2.1 Proportion Strands ................................................................................................................................................................. 14
       4.2.2 Prestress losses ........................................................................................................................................................................... 14
       4.2.3 Concrete strength for form stripping .................................................................................................................................... 14
       4.2.4 Check girder stability .............................................................................................................................................................. 14
   4.3 Step 3 - Design for Shipping ............................................................................................................................................................... 21
   4.4 Step 4 - Design for Lifting with Temporary Top Strands ............................................................................................................... 21
       4.4.1 Prestress losses ........................................................................................................................................................................... 21
       4.4.2 Check girder stability .............................................................................................................................................................. 21
   4.5 Step 5 – Revise Shipping Design .......................................................................................................................................................... 24
       4.5.1 Estimate Prestress Losses at Shipping .................................................................................................................................... 24

5. Structural Analysis .......................................................................................................................................................................................... 24
   5.1 Analysis Results Summary ................................................................................................................................................................. 24
   5.2 Limit State Responses ....................................................................................................................................................................... 24
   5.3 Live Load Distribution Factors ......................................................................................................................................................... 24

6. Flexure Design ............................................................................................................................................................................................... 24
   6.1 Step 1 Design for Final Service Conditions ........................................................................................................................................ 24
       6.1.1 Stresses due to loads on non-composite section ..................................................................................................................... 24
       6.1.2 Stresses due to loads on the composite section ..................................................................................................................... 24
       6.1.3 Check Estimate of Final Concrete Strength .......................................................................................................................... 24
   6.2 Step 2 - Design for Lifting without Temporary Top Strands ........................................................................................................... 28
       6.2.1 Proportion Strands ................................................................................................................................................................. 28
       6.2.2 Prestress losses ........................................................................................................................................................................... 28
       6.2.3 Concrete strength for form stripping .................................................................................................................................... 28
       6.2.4 Check girder stability .............................................................................................................................................................. 28
   6.3 Step 3 - Design for Shipping ............................................................................................................................................................... 31
   6.4 Step 4 - Design for Lifting with Temporary Top Strands ............................................................................................................... 31
       6.4.1 Prestress losses ........................................................................................................................................................................... 31
       6.4.2 Check girder stability .............................................................................................................................................................. 31
   6.5 Step 5 – Revise Shipping Design .......................................................................................................................................................... 34
       6.5.1 Estimate Prestress Losses at Shipping .................................................................................................................................... 34

7. Structural Analysis .......................................................................................................................................................................................... 34
   7.1 Analysis Results Summary ................................................................................................................................................................. 34
   7.2 Limit State Responses ....................................................................................................................................................................... 34
   7.3 Live Load Distribution Factors ......................................................................................................................................................... 34

8. Flexure Design ............................................................................................................................................................................................... 34
   8.1 Step 1 Design for Final Service Conditions ........................................................................................................................................ 34
       8.1.1 Stresses due to loads on non-composite section ..................................................................................................................... 34
       8.1.2 Stresses due to loads on the composite section ..................................................................................................................... 34
       8.1.3 Check Estimate of Final Concrete Strength .......................................................................................................................... 34
   8.2 Step 2 - Design for Lifting without Temporary Top Strands ........................................................................................................... 38
       8.2.1 Proportion Strands ................................................................................................................................................................. 38
       8.2.2 Prestress losses ........................................................................................................................................................................... 38
       8.2.3 Concrete strength for form stripping .................................................................................................................................... 38
       8.2.4 Check girder stability .............................................................................................................................................................. 38
   8.3 Step 3 - Design for Shipping ............................................................................................................................................................... 41
   8.4 Step 4 - Design for Lifting with Temporary Top Strands ............................................................................................................... 41
       8.4.1 Prestress losses ........................................................................................................................................................................... 41
       8.4.2 Check girder stability .............................................................................................................................................................. 41
   8.5 Step 5 – Revise Shipping Design .......................................................................................................................................................... 44
       8.5.1 Estimate Prestress Losses at Shipping .................................................................................................................................... 44
4.5.2 Check Girder Stability ................................................................. 44
4.5.3 Check concrete strength ............................................................. 54
4.6 Step 6 – Check Erection Stresses .................................................... 55
  4.6.1 Losses between Transfer to Deck Placement ............................ 55
  4.6.2 Stresses ............................................................................. 57
4.7 Step 7 – Check Final Conditions .................................................. 58
  4.7.1 Losses from Deck Placement to Final ..................................... 58
  4.7.2 Stresses ............................................................................. 60
  4.7.3 Moment Capacity ................................................................. 63
4.8 Check Splitting Resistance ........................................................... 68
4.9 Check Confinement Zone Reinforcement ....................................... 69
5 Shear Design .................................................................................. 69
  5.1 Locate Critical Section for Shear ................................................ 69
  5.2 Check Ultimate Shear Capacity .................................................. 70
    5.2.1 Compute Nominal Shear Resistance ..................................... 70
    5.2.2 Check Requirement for Transverse Reinforcement ............... 73
    5.2.3 Check Minimum Transverse Reinforcement ......................... 73
    5.2.4 Check Maximum Spacing of Transverse Reinforcement ........ 73
  5.3 Check Longitudinal Reinforcement for Shear ............................... 74
  5.4 Check Horizontal Interface Shear ............................................... 74
    5.4.1 Check Nominal Capacity .................................................... 75
    5.4.2 Check Minimum Reinforcement ........................................... 76
6 Check the “A” Dimension ................................................................. 76
  6.1 Slab and Fillet ........................................................................... 76
  6.2 Profile Effect ............................................................................ 77
    6.2.1 Vertical Curve .................................................................... 78
    6.2.2 Horizontal Curve ............................................................. 79
    6.2.3 Profile Effect .................................................................... 79
  6.3 Girder Orientation Effect ........................................................... 79
  6.4 Excess Camber ......................................................................... 80
    6.4.1 Compute Creep Coefficients .............................................. 82
    6.4.2 Compute Deflections ......................................................... 82
  6.5 Check Required Haunch ............................................................ 83
  6.6 Compute Lower Bound Camber at 40 days ................................. 83
    6.6.1 Creep Coefficients ......................................................... 83
    6.6.2 Compute Deflections ......................................................... 83
  6.7 Check for Possible Girder Sag .................................................... 83
List of Figures

Figure 2-1: Bridge Plan ............................................................................................................................................................ 2
Figure 2-2: Bridge Section at Station 7+82.50 ...................................................................................................................... 2
Figure 2-3: Girder Dimensions ................................................................................................................................................ 3
Figure 2-4: Slab Detail ............................................................................................................................................................ 3
Figure 3-1 Assumed Construction Sequence .......................................................................................................................... 5
Figure 3-2 Connection Geometry ............................................................................................................................................. 5
Figure 3-3 Effective Flange Width .......................................................................................................................................... 6
Figure 3-4 Centroid of Non-composite and Composite Section .............................................................................................. 8
Figure 3-5 Main slab section for slab loading .......................................................................................................................... 11
Figure 3-5: Slab Haunch ......................................................................................................................................................... 12
Figure 3-6: HL93 Live Load Model .......................................................................................................................................... 14
Figure 3-7: εg Detail ............................................................................................................................................................... 15
Figure 4-1: Optimized Fabrication Girder Design Procedure ................................................................................................. 20
Figure 4-2: Optimum Strand Arrangement ............................................................................................................................ 25
Figure 4-3: Equilibrium of Hanging Girder ........................................................................................................................... 29
Figure 4-4: Girder Self-Weight Deflection during Lifting ...................................................................................................... 29
Figure 4-5: Offset Factor ....................................................................................................................................................... 31
Figure 4-6: Equilibrium during Hauling ................................................................................................................................. 45
Figure 4-7: Prestress induced Deflection based on Storage Datum .......................................................................................... 46
Figure 4-8: Girder Self-Weight Deflection during Storage .................................................................................................. 46
Figure 4-9: Discretized Girder Section for Strain Compatibility Analysis ................................................................................ 65
Figure 5-1: Graphical method to Determine Critical Section Location .................................................................................... 70
Figure 5-2: Flexural Tension Side of Beam with Area of Concrete and Area of Steel Identified ........................................ 72
Figure 6-1: Slab + Fillet Effect .................................................................................................................................................. 77
Figure 6-2: General Method for Profile Effect ........................................................................................................................ 77
Figure 6-3: Vertical Curve Effect ........................................................................................................................................... 78
Figure 6-4: Horizontal Curve Effect ..................................................................................................................................... 79
Figure 6-5: Top Flange Effect ............................................................................................................................................... 80
Figure 6-6: Camber Effect ................................................................................................................................................. 80
Figure 6-7: Camber Diagram ............................................................................................................................................... 81
## Revisions

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<th>Name</th>
<th>Revision</th>
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<td>10/8/2018</td>
<td>1.0</td>
<td>Brice</td>
<td>Original example</td>
</tr>
<tr>
<td>03/09/2022</td>
<td>2.0</td>
<td>Brice</td>
<td>Updated calculations of capacity reduction factor for flexure</td>
</tr>
<tr>
<td>03/18/2022</td>
<td>3.0</td>
<td>Brice</td>
<td>Changed bridge configuration and girder type so design includes temporary top strands for shipping.</td>
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<tr>
<td>06/02/2022</td>
<td>3.1</td>
<td>Brice</td>
<td>Evaluation of total reinforcement strain from moment capacity analysis did not include the initial strain. The calculation is updated to show the total strain is the sum of the net tensile strain and the initial strain.</td>
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<tr>
<td>10/19/2022</td>
<td>3.2</td>
<td>Brice</td>
<td>Editorial and format revisions</td>
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1 Introduction

The purpose of this document is to illustrate how the PGSuper computer program performs its computations. PGSuper is a computer program for the design, analysis, and load rating of precast, prestressed concrete girder bridges.

A design example followed by a load rating analysis illustrates the engineering computations performed by PGSuper. PGSuper uses a state-of-the-art iterative design algorithm and other iterative computational procedures. Only the final iterative steps are of interest. To avoid lengthy iterations in this document, trial variables are “guessed” based on the final iterations produced by the software.

PGSuper uses 16 decimals of precision. There will be minor differences between these “hand” calculations and numbers reported by PGSuper. When noted, these calculations adopt numeric values reported by PGSuper.

1.1 Sign Convention

This document and PGSuper use the following sign convention.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>Tension</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>Upward Deflection</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>Downward Deflection</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>Top Section Modulus</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>Bottom Section Modulus</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>Strand Eccentricity above Centroid</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>Strand Eccentricity below Centroid</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

2 Bridge Description

2.1 Site Conditions

Normal Exposure

Average Ambient Relative Humidity: 75%

2.2 Roadway

Alignment

<table>
<thead>
<tr>
<th>PI Station</th>
<th>Back Tangent</th>
<th>Delta</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>10+00</td>
<td>N 34° 45’ 32” W</td>
<td>12° 34’ 15” R</td>
<td>6000 ft</td>
</tr>
</tbody>
</table>

Profile

<table>
<thead>
<tr>
<th>PVI Station</th>
<th>PVI Elevation</th>
<th>Grade in (g₁)</th>
<th>Grade out (g₂)</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>9+00</td>
<td>100.00</td>
<td>-2%</td>
<td>-1.5%</td>
<td>600 ft</td>
</tr>
</tbody>
</table>

Superelevations

<table>
<thead>
<tr>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04 ft</td>
<td>–0.04 ft</td>
</tr>
</tbody>
</table>

2.3 Bridge Layout

Back of Pavement Seat, Abutment 1, 7+00
Back of Pavement Seat, Abutment 2, 8+65
Abutments are oriented at S 58° 41’ 13.47” W
Abutment 1, Skew Angle 0°
Abutment 2, Skew Angle 1° 34’ 32.28” L

Figure 2-1: Bridge Plan

Figure 2-2: Bridge Section at Station 7+82.50
Girders

6 WF74G @ 7'-0"
measured radial at BPS

A = 923.531 in²
Iₓ = 734356.0 in⁴
Iᵧ = 72018.4 in⁴
Y₁ = 38.343 in
Y₂ = 35.657 in
S₁ = 19152.5 in³
S₂ = 20594.8 in³
Perimeter = 289.284 in

Wₓf = 49.0 in
Wᵧf = 38.375 in
tₓweb = 6.125 in

f'c = to be determined
wₓc = 155 lb/ft³
wᵧc = 165 lb/ft³ (including rebar)

Figure 2-3: Girder Dimensions

Harping points at 0.4L from the end of the girder.

Interior Diaphragms

Rectangular – Between girders only. H = 62.875 in
T = 8.00 in

Located at 0.25L₁, 0.50L₁ and 0.75L₁.

Slab

Gross Depth = 7.5 in
Overhang Edge Depth = 7.0 in
Overhang = Varies
Edge Offset = 21'-0"
Overall width = 42'-0"
Slab Offset ("A" Dimension) = To be determined
Fillet = ¾"
Sacrificial Depth = ½"
f'c = 4 ksi
wₓc = 150 lb/ft³
wᵧc = 155 lb/ft³ (including rebar)
Future Wearing Surface, 0.035 k/ft²

Figure 2-4: Slab Detail

Strands
0.6" Diameter \( f_{pu} = 270.0 \text{ ksi} \)
Grade 270 \( f_{py} = 243.0 \text{ ksi} \)
Low Relaxation \( E_{ps} = 28500 \text{ ksi} \)
\( a_{ps} = 0.217 \text{ in}^2/\text{per strand} \)

**Traffic Barrier**

42" Single Slope

Design weight = 0.690 kip/ft/barrier

Load is distributed to 3 exterior girders

**Load Modifiers**

<table>
<thead>
<tr>
<th>Ductility</th>
<th>Redundancy</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_D = 1.0 )</td>
<td>( \eta_R = 1.0 )</td>
<td>( \eta_I = 1.0 )</td>
</tr>
</tbody>
</table>

**Criteria**


WSDOT policy is to design using gross section properties (BDM 5.6.2.I) using refined estimate of prestress losses (BDM 5.4.1.C) and elastic gains. PGSuper supports stress analysis with transformed section properties, the LRFD approximate method for estimating prestress losses, and a non-linear time-step analysis.

## 3 Design Preliminaries

The framing of this bridge results in girders that are all the same length. The left exterior girder (Girder A) has the largest overhang at mid-span. Design and load rate Girder A.

### 3.1 Construction Sequence

Figure 3-1 shows the assumed construction sequence. PGSuper models the various construction stages with Construction Events.
3.2 Girder Length

For a typical stub abutment with a Type A connection, the centerline of bearing is located 2’-8.5” from, and measured normal to, the back of pavement seat (BPS). The distance from the centerline bearing to the end of the girder is 1’-8.5” measured normal to the CL Bearing, which is parallel to the back of pavement seat.

Figure 3-1 Assumed Construction Sequence

Figure 3-2 Connection Geometry
The BPS-to-BPS distance of 165 ft is measured along the arc of the horizontal curve. The angle subtended by the girder is \( \delta = \frac{L_c}{R} \cdot \frac{180}{\pi} = 1.575634^\circ \). The chord length of the girder is \( 2R \sin \frac{\delta}{2} = 2(6000) \sin \frac{1.575634}{2} = 164.995 \text{ ft} \).

The bearing-to-bearing span length is \( L_s = 164.995\text{ft} - 2.7083\text{ft} - (2.7083\text{ft})/\cos (1.575634) = 159.578\text{ft} \).

The overall girder length is \( L_g = 159.577\text{ft} + 1.7083\text{ft} + (1.7083\text{ft})/\cos (1.575634) = 162.995\text{ft} \).

### 3.3 Section Properties

Compute the composite section properties. The basic (non-composite) girder section properties are given in the bridge description.

#### 3.3.1 Effective Flange Width

The effective flange width of a composite concrete deck slab is the tributary width of the member (LRFD 4.6.2.6.1). LRFD C4.6.2.6.1 states “These provisions are considered applicable for skew angles less than or equal to 75 degrees, L/S greater than or equal to 2.0 and overhang widths less than or equal to 0.5S”. The maximum overhang is approximately 4 ft which exceeds S/2 = 3.5 ft. For this reason, the overhang portion of the effective flange width is taken to be equal to S/2.

![Figure 3-3 Effective Flange Width](image)

The effective flange width is measured perpendicular to the centerline of the girder.

\[
w_{eff} = (3.5\text{ft} + 3.5\text{ft}) \left( \frac{12\text{in}}{1\text{ft}} \right) \cos 1.575634 = 83.97\text{in}
\]

#### 3.3.2 Composite Girder Properties

Transform the slab to equivalent girder material and use the parallel axis theorem to compute the composite girder properties. At mid-span the bottom of the slab is above the top of the girder by the fillet amount (\( \frac{3}{8} \text{"} \)). If the actual camber exceeds the predicted camber, the \( \frac{3}{8} \text{"} \) fillet can be easily lost. Assume the bottom of the slab is directly on top of the girder. This provides the least stiff section where the maximum demand occurs. For simplicity, use this section model at all locations (BDM 5.6.2.B.1).

PGSuper has options to include the haunch depth in the section properties calculations. Each section can use the minimum haunch depth (fillet dimension) or the actual haunch depth. Using the actual
haunch depth means there is a different set of section properties at every cross section. Using more precise section properties may be desirable for load rating.

Modulus of elasticity of slab concrete

\[ E_c = 120,000K_1w_c^2f_c^{0.33} = (120,000)(1.0)(0.150)^2(4.0)^{0.33} = 4266.223 \text{ ksi} \]

Modulus of elasticity of girder concrete assuming a concrete strength of \( f'_c = 8.7 \text{ ksi} \)

\[ E_c = 120,000K_1w_c^2f_c^{0.33} = (120,000)(1.0)(0.155)^2(8.7)^{0.33} = 5886.891 \text{ ksi} \]

The overall controlling 28-day design concrete strength is 8.7 ksi after multiple design iterations and refinements. See Section 4.6.2.

\[ n = \frac{E_c \text{ slab}}{E_c \text{ girder}} = \frac{4266.223 \text{ ksi}}{5886.891 \text{ ksi}} = 0.725 \]

The sacrificial wearing surface is not part of the structural section. Use the structural slab depth for computing section properties.

\[ t_{\text{slab}} = t_{\text{gross slab depth}} - t_{\text{sacrificial depth}} = 7.5\text{in} - 0.5\text{in} = 7.0\text{in} \]

<table>
<thead>
<tr>
<th>Area</th>
<th>( (\text{Area})(Y_b) )</th>
</tr>
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<tbody>
<tr>
<td>Slab</td>
<td>((0.725)(83.9\text{in})(7.0\text{in}) = 425.793\text{in}^2) = 32998.958in^3</td>
</tr>
<tr>
<td>Girder</td>
<td>923.531in^2</td>
</tr>
<tr>
<td>Total</td>
<td>(A_c = 1349.324\text{in}^2)</td>
</tr>
</tbody>
</table>

\( Y_{bc} = \frac{\sum(\text{Area})(Y_b)}{\sum(\text{Area})} = \frac{65929.303\text{in}^3}{1349.324\text{in}^2} = 48.861\text{in} \)

\( Y_{tc \text{ girder}} = H_g - Y_{bc} = 74.0\text{in} - 48.861\text{in} = 25.139\text{in} \)

<table>
<thead>
<tr>
<th>Area</th>
<th>( d )</th>
<th>( (\text{Area})(d^2) )</th>
<th>( I_o )</th>
<th>( I_o + (\text{Area})(d^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab</td>
<td>425.793in^2</td>
<td>(74.0\text{in} + \frac{7.0\text{in}}{2} - 48.861\text{in} = 28.639\text{in} )</td>
<td>349207.761in^4</td>
<td>(\frac{1}{12}(0.725)(83.9\text{in})(7.0\text{in}) = 1738.653\text{in}^4 )</td>
</tr>
<tr>
<td>Girder</td>
<td>923.531in^2</td>
<td>35.657in - 48.861in = -13.204in</td>
<td>161013.58in^4</td>
<td>734356.0in^4</td>
</tr>
</tbody>
</table>

\[ S_{bc} = -\frac{I_x}{Y_{bc}} = -\frac{1246315.994\text{in}^4}{48.861\text{in}} = -25507.378\text{in}^3 \]
\[ S_{tc\,girder} = \frac{I_x}{Y_{tc\,girder}} = \frac{1246315.994\text{in}^4}{25.113\text{in}} = 49628.319\text{in}^3 \]

### 3.3.3 First Moment of Area of deck slab,

\[ Q_{slab} = A_{slab} \left( Y_{tc\,girder} + \frac{t_{slab}}{2} \right) = 425.793\text{in}^2 \left( 25.113\text{in} + \frac{7\text{in}}{2} \right) = 12183.2125\text{in}^3 \]

### 3.3.4 Section Property Summary

Below are the section properties from PGSuper. They are slightly different than the properties computed above. Use the section properties reported by PGSuper for better agreement between these calculations and the software.

![Figure 3-4 Centroid of Non-composte and Composite Section](image)

**Table 3-1: Section Properties from PGSuper**

<table>
<thead>
<tr>
<th></th>
<th>Girder</th>
<th>Composite Girder</th>
</tr>
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<tbody>
<tr>
<td><strong>Area, A</strong></td>
<td>923.531 in²</td>
<td>1349.614 in²</td>
</tr>
<tr>
<td><strong>I_x</strong></td>
<td>734356.0 in⁴</td>
<td>1246570.6 in⁴</td>
</tr>
<tr>
<td><strong>I_y</strong></td>
<td>72018.4 in⁴</td>
<td>-</td>
</tr>
<tr>
<td><strong>Y_{top,girder}</strong></td>
<td>38.343 in</td>
<td>25.113 in</td>
</tr>
<tr>
<td><strong>Y_{top,slab}</strong></td>
<td>-</td>
<td>32.116 in</td>
</tr>
<tr>
<td><strong>Y_b</strong></td>
<td>35.657 in</td>
<td>48.867 in</td>
</tr>
<tr>
<td><strong>S_{top,girder}</strong></td>
<td>-19152.5 in³</td>
<td>-49599.7 in³</td>
</tr>
<tr>
<td><strong>S_{top,slab}</strong></td>
<td>-</td>
<td>-53531.9 in³</td>
</tr>
<tr>
<td><strong>S_b</strong></td>
<td>20594.8 in³</td>
<td>25509.3 in³</td>
</tr>
<tr>
<td><strong>Q_{slab}</strong></td>
<td>-</td>
<td>12199.9 in³</td>
</tr>
<tr>
<td><strong>Effective Flange Width, W_{eff}</strong></td>
<td>-</td>
<td>83.992 in</td>
</tr>
<tr>
<td><strong>Perimeter</strong></td>
<td>289.284 in</td>
<td>-</td>
</tr>
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</table>
3.4 **Structural Analysis**

There are several significant stages during the life of a prestressed girder. PGSuper automatically models these stages as Construction Events. The events, and the related construction activities, are:

1) **Fabricate girders**
   a) Tension strands, place reinforcement, form girders, cast concrete, concrete curing. Initial relaxation of the prestressing strand occurs.
   b) Strip forms and impart the precompression force into the girder (aka Release)
   c) Move girders into storage area (Initial lifting)
   d) Elapsed time during storage (creep, shrinkage, and relaxation losses occur)

2) **Erect girders**
   a) Prior to erection, the girders must be transported from the fabrication facility to the bridge site
   b) Erect and brace girders
   c) De-tension temporary strands (if applicable)

3) **Cast diaphragms and deck** (dead load is applied to non-composite girder section)

4) **Install railing system** (traffic barriers, sidewalks, etc.) (dead load is applied to composite section)

5) **Final without Live Load** (includes future overlay if applicable)

6) **Final with Live Load**

PGSuper models the individual steps within a Construction Event with Analysis Intervals. For example, Event 1 – Construct Girders, models five analysis intervals: Tension Strands and Cast Concrete, Elapsed Time during Curing, Prestress Release, Lifting, Placement into Storage, and Elapsed Time during Storage.

The analysis intervals are a general modelling approach associated with time-step analysis. Precast girder design normally uses a 2-step time-step analysis. However, PGSuper can perform a refined non-linear time-step analysis. PGSplice uses the non-linear time-step analysis as well.

### 3.4.1 Girder Fabrication

Girder fabrication consists of tensioning strands, placing mild reinforcement, installing girder forms, and placing concrete. Stripping of girder forms occurs after the concrete reaches adequate strength to accommodate the stresses and stability of the girder. The strands are then detensioned but because of bond with the girder concrete, the tension force is transferred into the girder concrete, compressing it. If the prestress force is eccentric to the centroid of the girder and it is sufficient to overcome the self-weight of the girder, the girder cambers upwards. In this condition, the girder bears on its ends and bending stresses develop.

\[
w_{girder} = w_c A_g = (0.165\text{kcf})(923.531\text{in}^2)\left(\frac{1\text{ft}^2}{144\text{in}^2}\right) = 1.058 \text{kflf}
\]

where:
- \(A_g\) = Gross cross-sectional area of the girder
- \(\gamma_c\) = Unit weight of concrete

\[
M_g = \frac{wx}{2}(l - x)
\]

Moment at point of prestress transfer

Prestress transfer occurs over 60 strand diameters (LRFD 5.9.4.3.1)

\[
l_t = 60d_b = (60)(0.6in) = 36in = 3\text{ft}
\]

\[
M_g = \frac{(1.058\text{kflf})(3\text{ft})}{2}(162.995\text{ft} - 3\text{ft}) = 253.912 \text{k}\cdot\text{ft}
\]
Moment at harp point (HP)
Harp point is 0.4L from the end of the girder 

\[ M_g = \frac{(1.058 lbf)(65.198 ft)}{2}(162.995 ft - 65.198 ft) = 3372.99 k \cdot ft \]

Moment at mid-span (0.5L)

\[ M_g = \frac{(1.058 lbf)(162.995 ft)}{2} \left(162.995 ft - \frac{162.995 ft}{2}\right) = 3513.534 k \cdot ft \]

### 3.4.2 Erected Girder
Substructure elements support the girder at permanent bearing locations once erected. Bracing stabilizes the girder. Temporary top strands are detensioned, followed by diaphragm and roadway slab casting. Installation of the railing system occurs after the roadway slab gains adequate strength.

#### 3.4.2.1 Diaphragm and Deck Placement
In this stage, the girder supports its self-weight along with the weight of the diaphragms and slab.

##### 3.4.2.1.1 Diaphragm Loads
The diaphragm load for an interior girder is

\[ P = HWc(S - t_{web}) \]

- \( H \) = Height of interior diaphragm
- \( W \) = Width of interior diaphragm
- \( t_{web} \) = Width of girder web
- \( S \) = Spacing of girders

\[ P = HWc(S - t_{web}) = (62.875 in)(8 in)(0.155 k/in) \left(\frac{1728 in^3}{1 ft^3}\right) = 1728 \text{ ksf} \]

Diaphragms are located at \( 39.894 ft (0.25L) \), \( 79.789 ft (0.5L) \) and \( 119.683 ft (0.75L) \) from the left bearing. Slab Loads

The slab load consists of the main slab and the slab haunch.

#### 3.4.2.1.1.1 Main Slab Load
The slab overhang varies along the length of the girder. Also, the thickness of the slab varies from 7” at the edge of deck to 7.5” at the edge of the girder top flange. The basic loading of the main slab is

\[ w_{slab} = w_e W_{trib} t_{slab} \]

where \( w_e \) is the unit weight of concrete, and

\[ w_{slab} = w_e \left[ \frac{t_{slab}}{2} + \left( W_{overhang} - \frac{w_e}{2} \right) \right] \]

for exterior beams.
**Figure 3-5 Main slab section for slab loading**

$W_{\text{overhang}}$ varies along the length of the span.

Assume a slab offset ("A" dimension) of 12.5in. This assumption will be verified in Section 6.5.

The main slab weight computed by PGSuper is:

<table>
<thead>
<tr>
<th>Location (ft)</th>
<th>$W_{\text{overhang}}$ (in)</th>
<th>Main Slab weight (klf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>42.440</td>
<td>0.725</td>
</tr>
<tr>
<td>15.958</td>
<td>44.732</td>
<td>0.747</td>
</tr>
<tr>
<td>31.916</td>
<td>46.518</td>
<td>0.764</td>
</tr>
<tr>
<td>47.873</td>
<td>47.795</td>
<td>0.775</td>
</tr>
<tr>
<td>63.831</td>
<td>48.564</td>
<td>0.782</td>
</tr>
<tr>
<td>79.789</td>
<td>48.826</td>
<td>0.785</td>
</tr>
<tr>
<td>95.747</td>
<td>48.580</td>
<td>0.782</td>
</tr>
<tr>
<td>111.704</td>
<td>47.826</td>
<td>0.776</td>
</tr>
<tr>
<td>127.662</td>
<td>46.564</td>
<td>0.764</td>
</tr>
<tr>
<td>143.620</td>
<td>44.794</td>
<td>0.748</td>
</tr>
<tr>
<td>159.578</td>
<td>42.516</td>
<td>0.726</td>
</tr>
</tbody>
</table>

### 3.4.2.1.1.2 Slab Haunch Load

The slab haunch load accounts for the buildup of concrete between the top of the girder and the bottom of the main slab. This concrete element has a width equal to the top flange width ($W_{\text{tf}}$) and varies in depth along the length of the girder because of camber and variations in the roadway surface.
WSDOT’s design policy is to assume the top of the girder is flat (no camber) for purposes of determining the slab haunch load (BDM 5.6.2.D.3.iv).

The basic haunch dead load at any given section is

\[ w_{haunch} = W_{tf} t_{haunch} w_c \]

Assuming a slab offset (“A” dimension) of 12.5in, the slab haunch load at the start of the span is

\[ t_{haunch} = A - t_{slab} = 12.5in - 7.5in = 5.0in \]

\[ w_{haunch} = (49in)(5in)(0.155ksi) \left( \frac{1ft^2}{144in^2} \right) = 0.264 klf \]

The vertical curve causes the haunch depth to vary along the length of the girder. The table below lists the haunch loading for half the span. This load is modeled as linear load segments.

<table>
<thead>
<tr>
<th>Location</th>
<th>( t_{haunch}(\text{in}) )</th>
<th>( w_{haunch}(\text{klf}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0L</td>
<td>5.0</td>
<td>0.264</td>
</tr>
<tr>
<td>0.1L</td>
<td>4.795</td>
<td>0.253</td>
</tr>
<tr>
<td>0.2L</td>
<td>4.635</td>
<td>0.244</td>
</tr>
</tbody>
</table>
3.4.2.2 **Superimposed Dead Loads**

Application of superimposed dead loads occurs after the deck has reached adequate strength. The superimposed dead loads consist of the traffic barrier and the overlay if present. The composite section is resisting these loads.

3.4.2.2.1 **Traffic Barrier**

The traffic barrier weight is distributed over \( n \) exterior girders, if there are \( 2n \) or more girders, otherwise the weight of the traffic barrier per girder is

\[
W_{tb} = \frac{W_{tb\text{left}} + W_{tb\text{right}}}{n},
\]

where \( N \) is the number of girders in the span. From BDM 5.6.3.2.B.2.d, \( n = 3 \).

\[
w_{tb} = \frac{W_{tb}}{n} = \frac{0.690\text{klf}}{3 \text{ girders}} = 0.230 \frac{\text{klf}}{\text{girder}}
\]

AASHTO permits equal distribution for barrier loads to all girders.

3.4.2.3 **Open to Traffic**

3.4.2.3.1 **Future Overlay**

Evenly distribute the weight of the future wearing surface to all girders. The curb-to-curb width of the deck is 39.833ft.

\[
w_o = \frac{(39.833\text{ft})(0.035\text{ksf})}{6 \text{ girder}} = 0.232 \frac{\text{klf}}{\text{girder}}
\]

Take care when applying the future overlay loading. Certain stress conditions are worse before the overlay is applied and others are worse after it is applied.

3.4.2.3.2 **Live Load**

The design live load is the HL93 notional model defined in the AASHTO LRFD BDS.

The vehicular live loading is the combination of the:

- design truck or design tandem (LRFD 3.6.1.1)
- design lane load (LRFD 3.6.1.2.1)

The design truck consists of three axles. Axle weights and spacing are, 8.0 kip, 14.0 ft, 32.0 kip, 14.0 to 30.0 ft, 32.0 kip. See Figure 3-6 below.

The design tandem consists of a pair of 25.0 kip axles spaced 4.0 ft apart.

The design lane load is 0.640 klf, uniformly distributed along the length of the span.
Apply a dynamic load allowance (impact) of 33% to the design truck and design tandem portions of the live load response.

The fatigue live load is the design truck with the rear axle spacing fixed at 30 ft. The dynamic load allowance for fatigue is 15%.

### 3.4.3 Analysis Results Summary

The following load responses are computed by PGSuper.

#### 3.4.3.1 At Release

<table>
<thead>
<tr>
<th>Loading</th>
<th>Transfer Point</th>
<th>Harp Point</th>
<th>Mid-Span</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>253.96 k·ft</td>
<td>3373.66 k·ft</td>
<td>3514.22 k·ft</td>
</tr>
</tbody>
</table>

#### 3.4.3.2 At Bridge Site

<table>
<thead>
<tr>
<th>Loading</th>
<th>0.5L_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder after erection</td>
<td>3368.43 k·ft</td>
</tr>
<tr>
<td>Diaphragm</td>
<td>140.14 k·ft</td>
</tr>
<tr>
<td>Slab</td>
<td>2465.61 k·ft</td>
</tr>
<tr>
<td>Haunch</td>
<td>760.28 k·ft</td>
</tr>
<tr>
<td>Traffic Barrier</td>
<td>732.12 k·ft</td>
</tr>
<tr>
<td>Future Overlay</td>
<td>739.63 k·ft</td>
</tr>
<tr>
<td>*Design LLIM (HL-93)</td>
<td>5485.09 k·ft</td>
</tr>
<tr>
<td>*Fatigue LLIM</td>
<td>2686.86 k·ft</td>
</tr>
</tbody>
</table>

*Live load moments are per lane
3.4.4 Limit State Responses

Group the structural responses into load cases and compute limit state responses. The total factored load, or limit state response, is \( Q = \sum \eta_i q_i \). (LRFD Eqn. 3.4.1-1)

LRFD Table 3.4.1-1 gives the load factors. The limit states of importance are:

- Service I, \( Q = 1.0DC + 1.0DW + 1.0(LL+IM) \)
- Service III, \( Q = 1.0DC + 1.0DW + 0.8(LL+IM) \)
- Strength I, \( Q = 1.25DC + 1.50DW + 1.75(LL+IM) \)
- Fatigue I, \( Q = 0.5DC + 0.5DW + 1.5(LL+IM) \)

WSDOT specifies the live load factor for Service III is 0.8 for design and 1.0 for load rating. See BDM 3.5.2

3.4.5 Live Load Distribution Factors

Compute the live load distribution factors. Select the appropriate cross section type from LRFD Table 4.6.2.1.1-1. A precast I-beam with cast-in-place concrete deck corresponds to cross section k.

WSDOT deviates from the AASHTO LRFD BDS for exterior girders in type k sections as described in BDM 3.9.3.A.

Compute the longitudinal stiffness parameter \( K_g \).

\[
K_g = n \left( I + A e_g^2 \right)
\]

where:

\( n \) = modular ratio between beam and deck material \( n = \frac{E_{beam}}{E_{slab}} \)

\( I \) = moment of inertia of the beam (in\(^4\))

\( A \) = area of beam (in\(^2\))

\( e_g \) = distance between the centers of gravity of the basic beam and deck (in)

\[
n = \frac{5886.9991si}{4266.223ksi} = 1.380
\]

\[
e_g = Y_l + \frac{t_s}{2} = 38.343\text{in} + \frac{7.0\text{in}}{2} = 41.843\text{in}
\]

---

![Figure 3-8: \( e_g \) Detail](image-url)
\[ K_g = 1.380[734356.0 \text{in}^2 + (923.531 \text{in}^2)(41.843 \text{in})^2] = 3244494.4 \text{in}^4 \]

3.4.5.1 Number of Design Lanes

The number of design lanes is equal to the integer portion of the roadway width divided by 12 ft (LRFD 3.6.1.1.1).

\[ N_t = \left\lfloor \frac{39.833 \text{ft}}{12 \text{ft}} \right\rfloor = 3 \text{ Design Lanes} \]

3.4.5.2 Distribution of Live Loads per Lane for Moments in Exterior Beams

LRFD Table 4.6.2.2d-1 gives the live load distribution factors for moments in exterior beams. For two or more loaded lanes the exterior beam distribution factor is a function of the interior beam distribution factor. The interior beam distribution factor is given in LRFD Table 4.6.2.2b-1.

3.4.5.2.1 Compute Distribution Factor for Moment

Check the range of applicability for live load distribution factors.

<table>
<thead>
<tr>
<th>Range Variable</th>
<th>Value</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 \text{ft} \leq S \leq 16 \text{ft}</td>
<td>S = 7.0 \text{ft}</td>
<td>OK</td>
</tr>
<tr>
<td>4.5 \text{in} \leq t_s \leq 12 \text{in}</td>
<td>t_s = 7.0 \text{in}</td>
<td>OK</td>
</tr>
<tr>
<td>20 \text{ft} \leq L \leq 240 \text{ft}</td>
<td>L =162.995 \text{ft}</td>
<td>OK</td>
</tr>
<tr>
<td>N_b \geq 4</td>
<td>N_b = 6</td>
<td>OK</td>
</tr>
<tr>
<td>10,000 \text{in}^4 \leq K_s \leq 7,000,000 \text{in}^4</td>
<td>K_s =2288376 \text{in}^4</td>
<td>OK</td>
</tr>
<tr>
<td>-1.0 \leq d_e \leq 5.5</td>
<td>d_e = 2.855\text{ft}</td>
<td>OK</td>
</tr>
</tbody>
</table>

3.4.5.2.1.1 One Design Lane Loaded

The live load distribution factor for one loaded lane is computed by the lever rule.

\[ g M_1^e = \left( \text{mpf} \right) \frac{P \sum d_i}{(S)(P)} \]

Where mpf is the multiple presence factor, P is the axle load, di is the distance from the first interior girder (Girder B) to the wheel load, and S is the girder spacing.

Per the WSDOT BDM 3.9.3A, when the slab cantilever length exceeds 40% of the adjacent interior girder spacing a multiple presence factor of 1.0 is used in conjunction with the lever rule for computing the live load distribution for a single loaded lane.

AASHTO LRFD BDS 3.6.1.1.2 specifies a multiple presence factor of 1.2 for one loaded lane.

\[ g M_1^e = 1.0 \frac{P}{7}(7.854 \text{ft} + 1.854 \text{ft}) = 0.694 \]

3.4.5.2.1.2 Two or More Design Lanes Loaded

The live load distribution factor for two or more design lanes loaded is

\[ g M_2^e = e g M_2^1 \]
\[
\begin{align*}
\epsilon &= 0.77 + \frac{d_e}{9.1} \\
gM_{2+} &= 0.075 + \left( \frac{S}{9.5} \right)^{0.6} \left( \frac{S}{L} \right)^{0.2} \left( \frac{K_9}{12.0Lt_s^2} \right)^{0.1} \\
gM_{2+} &= 0.075 + \left( \frac{7}{9.5} \right)^{0.6} \left( \frac{7}{162.995} \right)^{0.2} \left( \frac{3244494.4}{12.0 \cdot 162.995 \cdot 7^3} \right)^{0.1} = 0.598 \\
\epsilon &= 0.77 + \frac{2.855}{9.1} = 1.084 \\
gM_{2+} &= 1.084(0.598) = 0.648
\end{align*}
\]

The girders are skewed with respect to the end of the bridge. The skew correction factor is given in LRFD Table 4.6.2.2e-1.

\[
f = 1 - c_1 (\tan \theta)^{1.5}
\]

\[
c_1 = 0.0 \text{ for } \theta < 30^\circ
\]

The skew correction factor is 1.0.

### 3.4.5.3 Distribution of Live Loads per Lane for Shear in Exterior Beams

LRFD Table 4.6.2.2.3b-1 gives the live load distribution factors for shear in exterior beams. For two or more loaded lanes the exterior beam distribution factor is a function of the interior beam distribution factor. The interior beam distribution factor is given in LRFD Table 4.6.2.2.3a-1.

#### 3.4.5.3.1 Compute Distribution Factor for Shear

Check the range of applicability for live load distribution factors.

<table>
<thead>
<tr>
<th>Range Variable</th>
<th>Value</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 ft ( \leq S \leq 16 ) ft</td>
<td>( S = 7.0 ) ft</td>
<td>OK</td>
</tr>
<tr>
<td>4.5 in ( \leq t_s \leq 12 ) in</td>
<td>( t_s = 7.0 ) in</td>
<td>OK</td>
</tr>
<tr>
<td>20 ft ( \leq L \leq 240 ) ft</td>
<td>( L = 162.995 ) ft</td>
<td>OK</td>
</tr>
<tr>
<td>( N_b \geq 4 )</td>
<td>( N_b = 6 )</td>
<td>OK</td>
</tr>
<tr>
<td>(-1.0 \leq d_e \leq 5.5 )</td>
<td>( d_e = 2.855) ft</td>
<td>OK</td>
</tr>
</tbody>
</table>

#### 3.4.5.3.1.1 One Design Lane Loaded

The live load distribution factor for one loaded lane is computed by the lever rule.

\[
gV_{1e} = 0.694
\]

#### 3.4.5.3.1.2 Two or More Design Lanes Loaded

The live load distribution factor for two or more loaded lanes is

\[
gV_{2+} = e gV_{2+} \\
e = 0.6 + \frac{d_e}{10} \\
gV_{2+} = 0.2 + \frac{S}{12} - \left( \frac{S}{35} \right)^{2.0} \\
gV_{2+} = 0.2 + \frac{7}{12} - \left( \frac{7}{35} \right)^{2.0} = 0.743
\]
\[ e = 0.6 + \frac{2.855}{10} = 0.889 \]

The exterior beam cannot be designed for less load than the interior beam so \( e = 1.0 \)

\[ gV_{2s}^E = 1.0(0.743) = 0.743 \]

The skew correction factor is given in LRFD Table 4.6.2.2.3c-1.

\[ f = 1.0 + 0.2 \left( \frac{12.0Lt^3}{K_g} \right)^{0.3} \tan \theta \]

The average skew angle is \( \theta = \frac{0.4 + 1.57}{2} = 0.79^\circ \)

\[ f = 1.0 + 0.2 \left( \frac{12.0 \cdot 162.995 \cdot 7^3}{3244494.4} \right)^{0.3} \tan(0.79) = 1.002 \]

\[ gV_{2s}^E = 1.002(0.743) = 0.744 \]

---

LRFD 4.6.2.2.3c - The skew correction factor is linearly interpolated between 1.0 at mid-span to its full value at the obtuse corner. Since the skew correction factor is 1.002, that level of detail will not be shown in these calculations. However, PGSuper does interpolate the skew correction factor as required by AASHTO LRFD.

---

### 3.4.5.4 Live Load Distribution Factor Summary

**Distribution Factor Summary for Strength and Service Limit States**

<table>
<thead>
<tr>
<th>Distribution Factor</th>
<th>1 Loaded Lane</th>
<th>2+ Loaded Lanes</th>
<th>Controlling Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment (gM)</td>
<td>0.694</td>
<td>0.648</td>
<td>0.694</td>
</tr>
<tr>
<td>Shear (gV)</td>
<td>0.694</td>
<td>0.744</td>
<td>0.744</td>
</tr>
</tbody>
</table>

### 3.4.5.5 Live Load Distribution Factor for Fatigue Limit State

The fatigue live load distribution uses the factor for one loaded lane (LRFD 3.6.1.4.3b). The single lane distribution factors include a multiple presence factor of 1.2 when computed by the equations in LRFD Section 4. The multiple presence factor for fatigue loading is 1.0 (LRFD 3.6.1.1.2). Typically, the one loaded lane distribution factors are divided by 1.2 to get the fatigue distribution factors. However, in this case, the one loaded lane distribution factors are computed by the lever rule with a multiple presence factor of 1.0 per WSDOT policy so dividing by 1.2 is not needed.

**Distribution Factor Summary for Fatigue Limit States**

<table>
<thead>
<tr>
<th>Distribution Factor</th>
<th>1 Loaded Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment (gM)</td>
<td>0.694</td>
</tr>
<tr>
<td>Shear (gV)</td>
<td>0.694</td>
</tr>
</tbody>
</table>

### 4 Flexure Design

WSDOT and local girder fabricators developed the design methodology used by PGSuper. The concept is to design for optimized fabrication of precast, prestressed concrete bridge girders. The primary goal is to determine the least required concrete strength at release and initial lifting while simultaneously placing the least possible demand on the stressing system and achieving adequate stability of the girder during handling operations. A detailed description and numerical example of optimized fabrication design is available in the PCI Journal\(^2\). Figure 4-1 gives a high-level summary of the design procedure.
Girder stresses and stability at initial lifting and hauling are integral elements of the design process. Lifting and hauling conditions often govern the design.

Designing precast, prestressed concrete bridge girders for lateral stability ensures safety and constructability. AASHTO LRFD 5.5.4.3 requires buckling and stability of precast members during handling, transportation, and erection to be investigated. PCI’s Aspire Magazine³ presents WSDOT’s perspective on stability design.
**Step 1 - Design for final service conditions**
- Determine precompression force

**Step 2 - Design for lifting without temporary top strands**
- Determine optimum permanent strand arrangement
- Determine lifting embedment locations for stability
- This step determines highest concrete strength at lifting

**Step 3 - Design for shipping**
- Estimate temporary top strand requirements
- Estimate truck support locations
- Estimate concrete strength required at shipping

**Step 4 - Design for lifting with temporary top strands**
- Determines lowest concrete strength at lifting
- Place lifting embedments as close to ends as possible while maintaining adequate lateral stability

**Temporary top strand required?**
- Yes
- No

**Step 5 - Revise shipping design**
- Confirm temporary top strand requirements
- Confirm truck support locations
- Confirm concrete strength required at shipping

**Step 6 – Check Erection Stresses**
- Check stresses in erected girder after temporary top strands are removed but before additional dead load is placed
- Check stresses in erected girder after additional dead load is placed

**Step 7 – Check Final Conditions**
- Check stresses in final service conditions
- Check strength conditions

*Figure 4-1: Optimized Fabrication Girder Design Procedure*
4.1 Step 1 Design for Final Service Conditions

Design for the Service III limit state for tensile stresses at the bottom of the girder at mid-span.

4.1.1 Stresses due to loads on non-composite section

\[ f = \frac{M}{S} \]

\[ S_t = -19152.5 \text{in}^3 \]
\[ S_b = 20594.8 \text{in}^3 \]

<table>
<thead>
<tr>
<th>Load</th>
<th>Moment (k-ft)</th>
<th>( f_t ) (ksi)</th>
<th>( f_b ) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>3368.43</td>
<td>-2.110</td>
<td>1.963</td>
</tr>
<tr>
<td>Diaphragms</td>
<td>140.17</td>
<td>-0.088</td>
<td>0.082</td>
</tr>
<tr>
<td>Slab</td>
<td>2465.61</td>
<td>-1.545</td>
<td>1.437</td>
</tr>
<tr>
<td>Haunch</td>
<td>760.28</td>
<td>-0.476</td>
<td>0.443</td>
</tr>
</tbody>
</table>

4.1.2 Stresses due to loads on the composite section

4.1.2.1 Stress due to dead loads

\[ f = \frac{M}{S} \]

\[ S_{t,ge} = -49599.7 \text{in}^3 \]
\[ S_{bc} = 25509.6 \text{in}^3 \]

<table>
<thead>
<tr>
<th>Load</th>
<th>Moment (k-ft)</th>
<th>( f_t ) (ksi)</th>
<th>( f_b ) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrier</td>
<td>732.12</td>
<td>-0.177</td>
<td>0.344</td>
</tr>
<tr>
<td>Future Overlay</td>
<td>739.63</td>
<td>-0.179</td>
<td>0.348</td>
</tr>
</tbody>
</table>

4.1.2.2 Live Load Stresses

The live load moments computed above are for a full lane of load. These values must be scaled with the live load distribution factor to get a per girder moment

\[ f = g \frac{M_{LLIM}}{S} \]

\[ S_{t,ge} = -49599.7 \text{in}^3 \]
\[ S_{bc} = 25509.3 \text{in}^3 \]

<table>
<thead>
<tr>
<th>Load</th>
<th>Moment (k-ft) per lane</th>
<th>g</th>
<th>Moment (k-ft per girder)</th>
<th>( f_t ) (ksi)</th>
<th>( f_b ) (ksi)</th>
<th>( f_b ) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design (HL93)</td>
<td>5485.09</td>
<td>0.694</td>
<td>3804.05</td>
<td>-0.920</td>
<td>0</td>
<td>1.789</td>
</tr>
<tr>
<td>Fatigue</td>
<td>2686.86</td>
<td>0.694</td>
<td>1863.40</td>
<td>-0.451</td>
<td>0</td>
<td>0.877</td>
</tr>
</tbody>
</table>
4.1.2.3 Stress due to slab shrinkage

Girder stresses must include those due to slab shrinkage when gains due to slab shrinkage are included in the effective prestress force. The equivalent force is illustrated in Figure 4-2.

![Figure 4-2: Equivalent deck shrinkage force](image)

\[
\Delta P_{ds} = \frac{-\varepsilon_{ds} f_{c} A_{c} E_{c,deck}}{1 + 0.7\psi_{d}(f_{c}, t_{d})}
\]

\[
f_{ss} = \Delta P_{ds} \left( \frac{1}{A_{c}} - \frac{e_{d}}{S_{c}} \right)
\]

\[
\varepsilon_{def} = K_{sh} k_{s} k_{hs} f_{c} k_{td} 0.48 \times 10^{-3}
\]

\[
k_{s} = 1.45 - 0.13 \left( \frac{V}{S} \right) \geq 1.0
\]

PGSuper computes the volume of the deck slab by integrating the full bridge width cross-sectional area over the length of the slab, measured along the alignment curve. This is the most general solution and accounts for deck slabs that vary in width. The surface area is twice the plan area (top and bottom of slab) less the common area with the girder top flanges plus the area of exposed slab edges. PGSuper computes the volume to surface area ratio to be:

\[
\frac{V}{S} = 4.154 \text{in}
\]

Use the gross slab depth when computing slab shrinkage effects. Shrinkage is an early age effect; therefore, the sacrificial depth is part of the deck slab that is shrinking.

\[
k_{s} = 1.45 - 0.13(4.154) = 0.910 < 1.0 \cdot 1.0
\]

\[
k_{hs} = 2.00 - 0.014H = 2.00 - 0.014(75) = 0.95
\]

Slab concrete age at time of initial loading is \(f'_{c} = 0.8f'_{c} \). (LRFD 5.4.2.3.1)

\[
f'_{c} = 0.8f'_{c} = 0.8(4\text{ksi}) = 3.2 \text{ksi}
\]

\[
k_{f} = \frac{5}{1 + f'_{c}} = \frac{5}{1 + 3.2} = 1.19
\]

\[
t = t_{f} - t_{d} = 2000 - 120 = 1880 \text{ days}
\]

\[
k_{td} = \frac{t}{12 \left( \frac{100 - 4f'_{c}}{f'_{c} + 20} \right) + t} = \frac{1880}{12 \left( \frac{100 - 4(3.2)}{3.2 + 20} \right) + 1880} = 0.977
\]
\[ K_{sh} = 0.5 \text{ (BDM 5.1.4.3.D – use 50% slab shrinkage strain)} \]

\[ \epsilon_{ddf} = (0.5)(1.0)(0.95)(1.19)(0.978)(0.48 \times 10^{-3}) = 0.265 \times 10^{-3} \]

\[ \psi_d(t_f, t_d) = 1.9k_{ch}k_fk_{td}t_i^{-0.118} \]

\[ k_{hc} = 1.56 - 0.008H = 1.56 - 0.008(75) = 0.96 \]

\[ t_i = 1 \text{ days} \]

\[ \psi_d(t_f, t_d) = 1.9(1.0)(0.96)(1.19)(0.978)(1)^{-0.118} = 2.12 \]

The deck overhang varies along the length of the girder. At the mid-span section, the area of the deck that contributes to shrinkage is

\[ A_d = 681.165 \text{in}^2 \]

The deck shrinkage force is

\[ \Delta P_{ds} = \frac{(-0.265 \times 10^{-3})(681.165 \text{in}^2)(4266.223 \text{ksi})}{[1 + 0.7(2.12)\Delta]} = -310.0 \text{ kip} \]

The composite girder area is

\[ A_c = 1349.614 \text{in}^2 \]

Recall that the effective overhang for section properties is limited to S/2.

The eccentricity of the center of the deck with respect to the composite section is

\[ e_d = (-28.883 \text{in}) \]

\[ S_{tge} = -49599.7\text{in}^3 \]

\[ S_{bc} = 25509.3\text{in}^3 \]

\[ f_{top} = (-310.0 \text{kip})\left(\frac{1}{1349.614 \text{in}^2} + \frac{-28.883 \text{in}}{49599.7\text{in}^3}\right) = -0.410 \text{ ksi} \]

\[ f_{bot} = (-310.0 \text{kip})\left(\frac{1}{1349.614 \text{in}^2} + \frac{-28.883 \text{in}}{25509.3\text{in}^3}\right) = 0.121 \text{ ksi} \]

4.1.2.4 Estimate Prestressing

Typically, the governing limit state for final service conditions is Service III

\[ \text{Service III} = 1.0 DC + 1.0 DW + 0.8(\text{LL} + \text{IM}) \]

\[ f_b = 1.963\text{ksi (girder)} + 0.082\text{ksi (diaphragm)} + 1.437\text{ksi (slab)} + 0.443\text{ksi (haunch)} + 0.344\text{ksi (barrier)} + 0.348\text{ksi (future overlay)} + 0.121\text{ksi (slab shrinkage)} + (0.8)(1.789\text{ksi}) (\text{Design LLIM}) \]

\[ = 6.169\text{ksi} \]

The stress due to prestressing is

\[ f_{ps} = \frac{P}{A} + \frac{Pe}{S} \]

The tension limit is 0.0 ksi per BDM 5.2.1.C.

\[ f_b + f_{ps} = 0.0 \text{ ksi} \]

For load rating, use the tension stress limit from the LRFD Bridge Design Specifications (BDM 13.2.4)
Assume the resultant prestress force is 3” above the bottom of the girder.

\[ e = Y_b - 3\text{in} = 35.657\text{in} - 3\text{in} = 33.657\text{in} \]

\[
6.169\text{ksi} + \frac{P}{923.531\text{in}^2} + \frac{P(33.657\text{in})}{20594.8\text{in}^3} = 0.0\text{ksi}
\]

\[ P = -2270.5\text{kip} \]

Assume a final effective prestress of 85% of the initial prestress (15% prestress loss)

\[ P = N(0.217\text{in}^2)(0.85)(0.75)f_{pu} \]

\[ 2270.5\text{kip} = N(0.217\text{in}^2)(0.85)(0.75)(270\text{ksi}) \]

\[ N = 60.8 \]

Use 61 strands.

\[ P = 61(0.217\text{in}^2)(0.85)(0.75)(270\text{ksi}) = 2278.4\text{ ksi} \]

### 4.1.3 Check Estimate of Final Concrete Strength

#### 4.1.3.1 Service limit state - compression with live load

Compression at the top of girder, Service I limit state with live load is limited to 0.6\(f'_c\).

Service I limit state for compression stresses at the top of the girder at mid-span.

\[
\text{Service I} = 1.0DC + 1.0DW + 1.0(LL + IM)
\]

\[
f_t = -2.110\text{ksi(girder)} - 0.088\text{ksi(diaphragm)} - 1.545\text{ksi(slab)} - 0.476\text{ksi(haunch)} - 0.177\text{ksi(barrier)}
- 0.179\text{ksi(future overlay)} - 0.410\text{ksi(slab shrinkage)} - 0.920\text{ksi(Design LLIM)} = -5.905\text{ ksi}
\]

The stress due to prestressing is

\[ f_{ps} = \frac{P_e}{A} + \frac{Pe}{S} \]

\[ f_{ps} = \frac{-2278.4\text{kip}}{923.531\text{in}^2} + \frac{(-2278.4\text{kip})(33.657\text{in})}{-20594.8\text{in}^3} = 1.256\text{ksi} \]

Compute the required concrete strength

\[ -0.6f'_c = 1.256\text{ksi} - 5.905\text{ksi} = -4.649\text{ksi} \]

\[ f'_c = 7.748\text{ksi} \]

#### 4.1.3.2 Service limit state - compression without live load

Compression at the top of girder, Service I limit state without live load is limited to 0.45\(f'_c\).

Service I limit state for compression stresses at the top of the girder at mid-span.

\[
\text{Service I} = 1.0DC + 1.0DW
\]

\[
f_t = -2.110\text{ksi(girder)} - 0.088\text{ksi(diaphragm)} - 1.545\text{ksi(slab)} - 0.476\text{ksi(haunch)} - 0.177\text{ksi(barrier)}
- 0.179\text{ksi(future overlay)} - 0.410\text{ksi(slab shrinkage)} = -4.985\text{ ksi}
\]

The stress due to prestressing is

\[ f_{ps} = 1.256\text{ksi} \]

Compute the required concrete strength

\[ -0.45f'_c = 1.256\text{ksi} - 4.985\text{ksi} = -3.729\text{ksi} \]

\[ f'_c = 8.29\text{ksi} \]
4.1.3.3 Fatigue limit state

Compression at the top of girder, Fatigue I limit state. Stress is limited to $0.4f'_c$.

Fatigue I limit state for compression stresses at the top of the girder at mid-span.

$$Fatigue I = 0.5DC + 0.5DW + 1.5(LL + IM)$$

$$f_I = 0.5(-2.110ksi(girder) - 0.088ksi(diaphragm) - 1.545ksi(slub) - 0.476ksi(haunch) - 0.177ksi(barrier) - 0.179ksi(future overlay) - 0.410ksi(slab shrinkage)) + 1.75(-0.451ksi)(Fatigue LLIM)$$

$$f_I = -3.282 ksi$$

The stress due to prestressing is

$$f_{ps} = \frac{-2367.44kip}{923.531in^2} + \frac{(-2367.44kip)(33.657in)}{-20594.8in^3} = 1.327 ksi$$

$$f_{ps} = 1.327 ksi$$

Compute the required concrete strength

$$-0.4f'_c = 0.5(1.327)ksi - 3.282ksi = -2.618ksi$$

$$f'_c = 6.55ksi$$

The required concrete strength does not exceed our assumed value of 8.7 ksi. OK

4.2 Step 2 - Design for Lifting without Temporary Top Strands

Temporary top strands are generally not required for lifting of girders. This step identifies the lifting locations that provide adequate lateral stability and the corresponding concrete strength. The optimum strand arrangement is also determined.

4.2.1 Proportion Strands

The optimum exit location for the permanent pretensioning results in the compressive stress at the transfer point or lift point being approximately equal to the compressive stress at the harp point. This provides the lowest exit eccentricity that does not increase the required concrete strength at lifting. Finding this location is an iterative process. Experience has shown that a good starting point is a ratio of straight-to-harped strands of approximately 2:1. Manipulation of the harped strands varies the exit eccentricity.

The harped strand exit location can be manipulated in one of two ways: 1) by lowering a fixed number of harped strands at the girder ends or, 2) by leaving the harped strands at their highest exit location and reducing the number of harped strands by dropping pairs of strands from the harped strand group into the straight strand group. Minimizing the number of harped strands held at their highest position at the ends of the girder provides space over the height of the web for longitudinal reinforcement that may protrude from the end of the girder for making a connection with diaphragm concrete.

The optimal arrangement of strands for this girder is 44 straight stands with an eccentricity of 31.447 in. and 17 harped strands with end eccentricity of −26.813 in and harp point eccentricity of 30.775 in. The permanent strand eccentricity at 0.5L is 31.447 in.

![Optimum Strand Arrangement](image-url)
4.2.2 Prestress losses

4.2.2.1 Initial relaxation before transfer

Prior to the 2005 interim revisions to the LRFD 3rd Edition, relaxation before prestress transfer was included in prestress loss calculations. Since the 2005 interim revisions, this is no longer required based on the idea that fabricators can overstress strands to achieve an effective prestress of $0.75f_{pu}$ at release. However, WSDOT retains the practice of including relaxation prior to prestress transfer because it reflects the production practices used by local fabricators.

\[
\Delta f_{PR0} = \frac{\log (24.0t)}{40.0} \left[ \frac{f_{pj}}{f_{py}} - 0.55 \right] f_{pj}
\]

\[
f_{pj} = 0.75f_{pu} = 0.75(270) = 202.5\text{ksi}
\]

\[
f_{py} = 0.9f_{pu} = 243\text{ksi}
\]

\[
t = 1 \text{ day}
\]

\[
\Delta f_{PR0} = \frac{\log (24.0 \cdot 1\text{day})}{40} \left[ \frac{202.5\text{ksi}}{243\text{ksi}} - 0.55 \right] (202.5\text{ksi}) = 1.980 \text{ksi}
\]

This calculation is for intrinsic relaxation of the strand. Intrinsic relaxation is associated with strand tensioned between two stationary points such as in a testing machine or between tensioning bulkheads.

4.2.2.2 Elastic Shortening

\[
\Delta f_{PES} = \frac{E_{ci}}{E_{cl}} f_{cp}
\]

\[
f_{cp} = \frac{P}{A} + \frac{Pe^2}{I} - \frac{Mge}{I}
\]

\[
P = N(a_{ps})(f_{pj} - \Delta f_{PR0} - \Delta f_{PES})
\]

Solve this equation iteratively for $P$.

Assume 10% initial loss and $f'_{ps} = 7.4 \text{ksi}$.

\[
E_{ci} = 120000(1.0)(0.155)^2(7.4)^{0.33} = 5580.731 \text{ksi}
\]

\[
P = 61(0.217)(1 - 0.10)(202.5) = 2412.4 \text{kip}
\]

\[
f_{cp} = \frac{2412.4 \text{kip}}{923.531\text{in}^2} + \frac{(2412.4\text{kip})(31.477\text{in})^2}{734356.0\text{in}^4} - \frac{(3514.22k \cdot ft)(12\text{in})}{734356.0\text{in}^4} = 4.059 \text{ksi}
\]

\[
\Delta f_{PES} = \frac{28500\text{ksi}}{5580.731\text{ksi}}(4.059\text{ksi}) = 20.68\text{ksi}
\]

\[
P = (61)(0.217\text{in}^2)(202.5\text{ksi} - 1.98\text{ksi} - 20.68\text{ksi}) = 2380.5 \text{kip}
\]

PGSuper performs this calculation with a very small convergence tolerance. The calculations are also performed at many points along the girder because the center of the prestress force changes due to harped strand location and the girder dead load moment changes along the length of the girder. There is less elastic shortening effect at the ends of the girder than at mid-span. Since stability tends to govern the design, and temporary strands require extra effort by the fabricator and the contractor, this refinement in the analysis is used to minimize the required concrete strength and temporary strand usage. The effective prestress force at release and initial lifting for various points (as determined by PGSuper) are given below. The initial loss converges to 11%.
4.2.3 Concrete strength for form stripping
This is not an essential design step; however, it helps to illustrate that the initial lifting case typically governs the initial concrete strength. Check the point of prestress transfer and the harp point.

4.2.3.1 Stress due to Prestressing

\[ f_{ps} = \frac{P}{A} + \frac{P_e}{S} \]

From PGSuper, the effective prestress force at the point of prestress transfer is \( P = 2423.16 \text{ kip} \) with an eccentricity of 16.166 in.

\[
\begin{align*}
  f_t &= \frac{- (2423.16 \text{kip})}{923.531 \text{in}^2} + \frac{(-2423.16 \text{kip})(16.166 \text{in})}{-19152.5 \text{in}^3} = -0.578 \text{ksi} \\
  f_b &= \frac{- (2423.16 \text{kip})}{923.531 \text{in}^2} + \frac{(-2423.16 \text{kip})(16.166 \text{in})}{20594.8 \text{in}^3} = -4.526 \text{ksi}
\end{align*}
\]

From PGSuper, the effective prestress force at the harp point is \( P = 2380.27 \text{ kip} \) with an eccentricity of 31.477 in.

\[
\begin{align*}
  f_t &= \frac{- (2380.27 \text{kip})}{923.531 \text{in}^2} + \frac{(-2380.27 \text{kip})(31.477 \text{in})}{-19152.5 \text{in}^3} = 1.335 \text{ksi} \\
  f_b &= \frac{- (2380.27 \text{kip})}{923.531 \text{in}^2} + \frac{(-2380.27 \text{kip})(31.477 \text{in})}{20594.8 \text{in}^3} = -6.215 \text{ksi}
\end{align*}
\]

4.2.3.2 Stress due to Girder self-weight

\[ M_g = \frac{w_g}{2} (L_g x - x^2) \]

At the point of prestress transfer

\[
\begin{align*}
  x &= 60d_b = 60(0.6 \text{in}) = 36 \text{in} = 3 \text{ft} \\
  M_g &= \frac{(1.058 klf)}{2} \left((162.995 \text{ft})(3 \text{ft}) - (3 \text{ft})^2 \right) = 253.96k \cdot \text{ft} \\
  f_t &= \frac{253.96k \cdot \text{ft}}{-19152.5 \text{in}^3} (\frac{12 \text{in}}{1 \text{ft}}) = -0.159 \text{ksi} \\
  f_b &= \frac{253.96k \cdot \text{ft}}{20594.8 \text{in}^3} (\frac{12 \text{in}}{1 \text{ft}}) = 0.148 \text{ksi}
\end{align*}
\]

At the harp point

\[
\begin{align*}
  x &= 0.4L_g = 0.4(162.995 \text{ft}) = 65.198 \text{ft} \\
  M_g &= \frac{(1.058 klf)}{2} \left((162.995 \text{ft})(65.198 \text{ft}) - (65.198 \text{ft})^2 \right) = 3373.66k \cdot \text{ft}
\end{align*}
\]
\[ f_t = \frac{3373.66k \cdot ft}{-19152.5in^3(1ft)} = -2.114 ksi \]
\[ f_b = \frac{3373.66k \cdot ft}{20594.8in^3(1ft)} = 1.966 ksi \]

### 4.2.3.3 Total stress
At the point of prestress transfer
\[ f_t = -0.578 + -0.159 = -0.738 ksi \]
\[ f_b = -4.526 + 0.148 = -4.378 ksi \]
At the harp point
\[ f_t = 1.335 - 2.114 = -0.779 ksi \]
\[ f_b = -6.215 + 1.996 = -4.219 ksi \]

### 4.2.3.4 Required strength at form stripping
At the point of prestress transfer
\[ -0.65f'_{cl} = -4.378 ksi \]
\[ f'_{cl} = 6.74 ksi \]
At the harp point
\[ -0.65f'_{cl} = -4.219 ksi \]
\[ f'_{cl} = 6.49 ksi \]

Theoretically, the concrete curing system and forms can be removed, and the prestress force can be imparted into the girder at this concrete strength. This would allow crews to begin work and reduce the time until construction of the next girder can begin. However, there must be a high degree of confidence that the lifting and final concrete strength targets can be attained.

### 4.2.4 Check girder stability
PGSuper performs many iterations to converge on an adequate lifting configuration. The solution converges for lifting devices located 11 ft from the ends of the girder.

The PGSuper Fabrication Options Report gives the required lift point and initial concrete strength for lifting without temporary strands as well as lifting with pretensioned or post-tensioned temporary strands.
4.2.4.1 **Vertical Location of Center of Gravity**

The location of the center of gravity is critical to the stability analysis. Girder camber, sweep, and lift loop offset all factor into the location of the center of gravity with respect to the lifting roll axis.

4.2.4.1.1 Estimate Camber

Compute camber for the girder in the hanging configuration. The stability analysis procedure needs the camber measured from a datum at the ends of the girder, not the lift points.

4.2.4.1.1.1 Girder

The total girder deflection relative to the end of the girder is the sum of the deflection at the girder ends and the mid-span deflection.

\[
L_s = L_g - 2a = 164.995\text{f}t - 2(11\text{f}t) = 140.995\text{f}t
\]

At girder ends

\[
\Delta_{g1} = \frac{w_g a}{24E_{ci}I_x} [3a^2(a + 2L_s) - L_s^3]
\]

\[
= \frac{(-1.058k\text{l})f(11\text{f}t)}{24(5580.731\text{kst})(734356.0\text{in}^4)} \left[3(11\text{f}t)^2(11\text{f}t + 2(140.995\text{f}t)) - (140.995\text{f}t)^3\right] \left(\frac{1728\text{in}^3}{1\text{ft}^3}\right)
\]

\[
= 0.551\text{ in}
\]

Mid-span

\[
\Delta_{g2} = \frac{5w_g L_s^4}{384E_{ci}I_x} - \frac{w_g a^2 L_s^2}{16E_{ci}I_x} = \left[\frac{5(-1.058k\text{l})f(140.995\text{f}t)^4}{384(5580.731\text{kst})(734356.0\text{in}^4)} - \frac{(-1.058k\text{l})f(11\text{f}t)^2(140.995\text{f}t)^3}{16(5580.731\text{kst})(734356.0\text{in}^4)}\right] \left(\frac{1728\text{in}^3}{1\text{ft}^3}\right)
\]

\[
= -2.296\text{in} + 0.067\text{in} = -2.229\text{in}
\]

4.2.4.1.2 Prestressing
Since the girder is unrestrained, the location of the supports does not affect the overall prestress deflection measured between the ends of the girder and the mid-point.

4.2.4.1.2.1 Straight Strands

\[ P = \left( \frac{44}{61} \right) (2384.47 \text{kip}) = 1719.95 \text{kip} \]

\[ \Delta_{ss} = \frac{PeI^2}{8E_{cl}I_x} = \frac{(1719.95 \text{kip})(31.447 \text{in})(162.995 \text{ft})^2}{8(5580.731 \text{ksi})(734356.0 \text{in}^4)} \left( \frac{144 \text{in}^2}{1 \text{ft}^2} \right) = 6.308 \text{in} \]

4.2.4.1.2.2 Harped Strands

\[ P = \left( \frac{17}{61} \right) (2384.47 \text{kip}) = 664.52 \text{kip} \]

\[ e' = e_{hp} - e_e = 30.775 \text{in} - (-26.813 \text{in}) = 57.588 \text{in} \]

\[ b = 0.4 \]

\[ N = \frac{Pe'}{bL} = \frac{(664.52 \text{kip})(57.588 \text{in})}{(0.4)(162.995 \text{ft})} \left( \frac{1 \text{ft}}{12 \text{in}} \right) = 48.91 \text{kip} \]

\[ \Delta_{hs} = \frac{b(3 - 4b^2)NL^3}{24E_{cl}I_x} + \frac{Pe_eI^2}{8E_{cl}I_x} \]

\[ = \frac{0.4(3 - 4(0.4)^2)(48.91 \text{kip})(162.995 \text{ft})^3}{24(5580.731 \text{ksi})(734356.0 \text{in}^4)} \left( \frac{1728 \text{in}^3}{1 \text{ft}^3} \right) \]

\[ + \frac{(664.52 \text{kip})(-26.813 \text{in})(162.995 \text{ft})^2}{8(5580.731 \text{ksi})(734356 \text{in}^4)} \left( \frac{144 \text{in}^2}{1 \text{ft}^2} \right) = 3.513 \text{in} - 1.973 \text{in} = 1.540 \text{in} \]

4.2.4.1.3 Camber at lifting

\[ \Delta_{ps} = \Delta_{ss} + \Delta_{hs} = 6.308 \text{in} + 1.540 \text{in} = 7.848 \text{in} \]

\[ \Delta_{camber} = -\Delta_{g1} + \Delta_{g2} + \Delta_{ps} = -0.551 \text{in} - 2.229 \text{in} + 7.848 \text{in} = 5.068 \text{in} \]

4.2.4.1.2 Location the roll axis above the top of girder

Lifting loops tend to be flexible laterally. Assume the rotation occurs at the top of the girder.

\[ y_{rc} = 0 \text{in} \]

4.2.4.1.3 Offset factor

The offset factor locates the center of mass of the girder with respect to the roll axis.
4.2.4.1.4 Location of CG below roll axis

\[ y_r = Y_{top} - F_o (\Delta_{camber}) + y_{rc} = 38.343\text{in} - (0.415)(5.068\text{in}) - 0\text{in} = 36.24\text{in} \]

4.2.4.2 Lateral Deflection Parameters

4.2.4.2.1 Lateral Sweep

Sweep tolerance is 1/8” per 10 ft

\[ \Delta_{sweep} = \frac{162.995\text{ft}}{10\text{ft}} \left( \frac{1}{8}\text{in} \right) = 2.037\text{in} \]

4.2.4.2.2 Initial Lateral Eccentricity

Initial lateral eccentricity of center of gravity of girder due to lateral sweep and eccentricity of lifting devices from CL girder

\[ e_{lift} = 0.25\text{in} \]

\[ e_t = F_o \Delta_{sweep} + e_{lift} = (0.415)(2.037\text{in}) + 0.25\text{in} = 1.095\text{in} \]

4.2.4.2.3 Lateral Deflection of CG

Lateral deflection of center of gravity due to total girder weight applied to weak axis

\[ W_g = w_g L_g = (1.058\text{klf})(162.995\text{ft}) = 172.45\text{kip} \]
\[
z_o = \left(\frac{W_g}{12E_{ct}I_{yy}L_y^2}\right) \left(\frac{L_s^5}{10} - a^2L_s^3 + 3a^4L_s + \frac{6a^5}{5}\right) \frac{172.45kip}{12(5580.731ksi)(72018.4in^4)(162.995ft^2)} \left(\frac{(140.995ft)^5}{10} - (11ft)^2(140.995ft)^3 + 3(11ft)^4(140.995ft) + \frac{4}{5}(11ft)^5\right) = 12.187 \text{ in}
\]

4.2.4.3 Equilibrium tilt angle
\[
\theta_{eq} = \frac{e_i}{y_r - z_o} = \frac{1.095in}{36.24in - 12.187in} = 0.04552 \text{ rad}
\]

4.2.4.4 Girder Stresses in Hanging Girder

4.2.4.4.1 Direct stress at Lift Point and Harp Point

4.2.4.4.1.1 Prestressing
\[
f_p = \frac{P}{A} + \frac{P_e}{S}
\]

At Lift Point

From PGSuper, the effective prestress force at the lift point is \( P = 1746.62 \text{ kip} \) straight strands and \( P = 674.83 \text{ kip} \) harped strands. The straight and harped strand eccentricities are 31.748 in and -17.097 in, respectively.
\[
f_t = \frac{-(1746.62kip + 674.83kip)}{923.531in^2} + \frac{(-1746.62kip)(31.748in) + (-674.83kip)(-17.097in)}{-19152.5in^3} = -0.329ksi
\]
\[
f_b = \frac{-(1746.62kip + 674.83kip)}{923.531in^2} + \frac{(-1746.62kip)(31.748in) + (-674.83kip)(-17.097in)}{20594.8in^3} = -4.754ksi
\]

At Harp Point

From PGSuper, the effective prestress force at the harp point is \( P = 1716.91 \text{ kip} \) straight strands and \( P = 663.35 \text{ kip} \) harped strands. The straight and harped strand eccentricities are 31.748 in and 30.775 in, respectively.
\[
f_t = \frac{-(1716.91kip + 663.35kip)}{923.531in^2} + \frac{(-1716.91kip)(31.748in) + (-663.35kip)(30.775in)}{-19152.5in^3} = 1.335ksi
\]
\[
f_b = \frac{-(1716.91kip + 663.35kip)}{923.531in^2} + \frac{(-1716.91kip)(31.748in) + (-663.35kip)(30.775in)}{20594.8in^3} = -6.215ksi
\]

4.2.4.4.1.2 Girder self-weight
\[
M_g = \frac{W_g}{2} (L_s x - x^2 - a^2)
\]

At Lift Point
\[
x = l_t - a = 11ft - 11ft = 0.0 ft
\]
\[
M_g = \frac{(1.058klf)}{2} ((140.995ft)(0ft) - (0ft)^2 - (11ft)^2) = -64.02k \cdot ft
\]
\[
f_t = \frac{-64.02k \cdot ft}{-19152.5in^3} = 0.040ksi
\]
\[
f_b = \frac{-64.02k \cdot ft}{20594.8in^3} = -0.037ksi
\]

At Harp Point
\[ x = 0.4L_g - a = 0.4(162.995\text{ft}) - 11\text{ft} = 54.198\text{ft} \]

\[ M_g = \frac{(1.058klf)}{2}((140.995\text{ft})(54.198\text{ft}) - (54.198\text{ft})^2 - (11\text{ft})^2) = 2425.0k \cdot \text{ft} \]

\[ f_t = \frac{2425.0k \cdot \text{ft}}{-19152.5\text{in}^3}\left(\frac{12\text{in}}{1\text{ft}}\right) = -1.519\text{ksi} \]

\[ f_b = \frac{2425.0k \cdot \text{ft}}{20594.8\text{in}^3}\left(\frac{12\text{in}}{1\text{ft}}\right) = 1.413\text{ksi} \]

### 4.2.4.1.3 Total Stress

**At Lift Point**

\[ f_t = -0.329\text{ksi} + 0.040\text{ksi} = -0.289\text{ksi} \]

\[ f_b = -4.754\text{ksi} - 0.037\text{ksi} = -4.792\text{ksi} \]

**At Harp Point**

\[ f_t = 1.335\text{ksi} - 1.519\text{ksi} = 0.184\text{ksi} - 0.184\text{ksi} \]

\[ f_b = -6.215\text{ksi} + 1.413\text{ksi} = -4.802\text{ksi} \]

### 4.2.4.2 Tilt induced stresses

**At Lift Point**

Top left flange tip at lift point

\[ f_{lt} = \frac{M_g W_{tf} \theta_{eq}}{2I_{yy}} \]

\[ f_{lt} = \frac{(-64.02k \cdot \text{ft})(49\text{in})}{2(72018.4\text{in}^4)}(0.04452\text{ rad})\left(\frac{12\text{in}}{1\text{ft}}\right) = -0.012\text{ksi} \]

Bottom right flange tip at lift point

\[ f_{br} = -\frac{M_g W_{bf} \theta_{eq}}{2I_{yy}} \]

\[ f_{br} = -\frac{(-64.02k \cdot \text{ft})(38.375\text{in})}{2(72018.4\text{in}^4)}(0.04452\text{ rad})\left(\frac{12\text{in}}{1\text{ft}}\right) = 0.009\text{ksi} \]

**At Harp Point**

Top left flange tip at Harp Point

\[ f_{lt} = \frac{M_g W_{tf} \theta_{eq}}{2I_{yy}} \]

\[ f_{lt} = \frac{(2425.0k \cdot \text{ft})(49\text{in})}{2(72018.4\text{in}^4)}(0.04452\text{ rad})\left(\frac{12\text{in}}{1\text{ft}}\right) = 0.451\text{ksi} \]

Bottom right flange tip at Harp Point

\[ f_{br} = -\frac{M_g W_{bf} \theta_{eq}}{2I_{yy}} \]

\[ f_{br} = -\frac{(2425.0k \cdot \text{ft})(38.375\text{in})}{2(72018.4\text{in}^4)}(0.04452\text{ rad})\left(\frac{12\text{in}}{1\text{ft}}\right) = -0.353\text{ksi} \]
4.2.4.3 Total stress
Top left flange tip at lift point
\[ f_{tl} = -0.329 ksi + 0.040 ksi - 0.012 ksi = -0.576 ksi \]

Bottom left flange tip at lift point
\[ f_{bl} = -4.754 ksi - 0.037 ksi + 0.009 ksi = -4.528 ksi \]

Top left flange tip at Harp Point
\[ f_{th} = 1.335 ksi - 1.519 ksi + 0.451 ksi = 0.266 ksi \]

Bottom left flange tip at Harp Point
\[ f_{bh} = -6.215 ksi + 1.413 ksi - 0.353 ksi = -5.155 ksi \]

4.2.4.4 Determine Concrete Strength at Lifting
For the general stress case, compression stress is limited to \(-0.65f'_{cl}\).

Bottom centerline at point of lift \(-0.65f'_{cl} = 4.792 ksi, f'_{cl} = 7.37 ksi\)

Bottom centerline at harp point \(-0.65f'_{cl} = 4.802 ksi, f'_{cl} = 7.39 ksi\)

Peak compression stress at the extremities of the section is limited to \(-0.7f'_{cl}\)

Bottom left flange tip at point of lift \(-0.7f'_{cl} = 4.801 ksi, f'_{cl} = 6.85 ksi\)

Bottom left flange time at harp point \(-0.7f'_{cl} = 5.155 ksi, f'_{cl} = 7.36 ksi\)

The controlling initial concrete strength matches our assumption. The required concrete strength at the lift point and the harp point are the same. This shows the proportioning of the strands is optimal. If two fewer strands were to be harped (46 straight, 15 harped), the required strength at the lift point would increase to 7.6 ksi which has the potential to increase curing time and could delay construction of the next girder.

4.2.4.5 Factor of Safety Against Cracking
Lateral cracking moment
\[ M_{cr} = \pm \frac{(f_r - f_{direct})2I_{yy}}{W_{top}} \]

Tilt angle at first crack
\[ \theta_{cr} = \frac{M_{cr}}{M_g} \leq 0.4 \text{ rad} \]

Cracking moment at lift point – cracking in top right corner
\[ f_{direct} = f_{ps} + f_g = -0.329 ksi + 0.040 ksi = -0.289 ksi \]
\[ f_r = 0.24\sqrt{f_{cl}'} = 0.24\sqrt{7.4 ksi} = 0.653 ksi \]
\[ M_{cr} = -\frac{(0.653 ksi - (-0.289 ksi))2(72018.4 in^4)}{49 in} \times \frac{1 in}{12 ft} = -230.71 k \cdot ft \]

Tilt angle at first crack at lift point
\[ \theta_{cr} = \frac{-230.71k \cdot ft}{-64.02k \cdot ft} = 3.6 \text{ rad} \div 0.4 \text{ rad} \]

Factor of Safety against Cracking at lift point
\[ FS_{cr} = \frac{y_r \theta_{cr}}{e_i + z_o \theta_{cr}} = \frac{(36.24in)(0.4)}{1.095in + (12.187in)(0.4)} = 2.429 \]
\[ FS_{cr} > 1.0 \quad \text{OK} \]

Cracking moment at Harp Point
\[ M_{cr} = \frac{(0.653ksi - (1.335ksi - 1.519ksi)2(72018.4in^4)}{49in} \cdot \frac{1in}{12ft} = 205.19 \text{ k \cdot ft} \]

Tilt angle at first crack at Harp Point
\[ \theta_{cr} = \frac{205.19k \cdot ft}{2425.0k \cdot ft} = 0.08461 \text{rad} \]

Factor of Safety against Cracking at Harp Point
\[ FS_{cr} = \frac{y_r \theta_{cr}}{e_i + z_o \theta_{cr}} = \frac{(36.24in)(0.08461)}{1.095in + (12.187in)(0.08461)} = 1.443 \]
\[ FS_{cr} > 1.0 \quad \text{OK} \]

4.2.4.6 Factor of Safety against Failure

\[ \theta_{max} = \sqrt{\frac{e_i}{2.5z_o}} \leq 0.4 \text{ rad} = \sqrt{\frac{1.095}{2.5(12.187)}} = 0.1896 \text{ rad} \]
\[ FS_f = \frac{y_r \theta_{max}}{e_i + (1 + 2.5\theta_{max})(z_o \theta_{max})} = \frac{(36.24in)(0.1896)}{1.095in + (1 + 2.5(0.1896))(12.187in)(0.1896)} = 1.527 \]
\[ FS_f > 1.5 \quad \text{OK} \]

4.3 Step 3 - Design for Shipping

Stresses and the factor of safety against cracking depend on, in part, the prestress force at the time of shipping. The prestress force is dependent on the strand configuration, including temporary top strands, and the initial concrete strength. The initial concrete strength is also dependent on the number of temporary top strands.

At this step in the design, the number of temporary top strands and the initial concrete strength for lifting with temporary top strands are unknown.

Using an assumed set of truck parameters and assuming the bunk points are located H (girder height) in from the ends of the girder, PGSuper uses the initial concrete strength determined in Step 2 and computes the factor of safety against rollover failure.

The location of the bunk points is iteratively moved inwards until the factor of safety against rollover failure is greater than 1.5. If an adequate factor of safety cannot be found, the truck parameters are revised, and the process starts over with bunk points at H from the end of the girder.

Once the factor of safety against rollover failure is satisfied, the factor of safety against cracking is evaluated. Temporary top strands are added until the factor of safety is greater than 1.0.

Finally, the compression stresses are evaluated and a concrete strength for shipping is estimated.

Recall that this analysis is based on the initial concrete strength for lifting without temporary top strands. If temporary top strands are required for shipping, it is advantageous to use them during initial lifting. Lift loops can be moved closer to the ends of the girder which reduces the initial concrete strength requirement. A lower initial concrete strength means the final steps in girder production can begin and production of the next girder can start sooner.
The result of the design iterations described above are bunk points need to be at least 13’-8” from the end of the girder to satisfy the rollover stability criteria and 4 temporary top strands are needed to satisfy the cracking criteria. The estimated concrete strength for hauling is 7.8 ksi.

Step 5 of the design procedure will illustrate the details of the shipping stability analysis. The calculations shown in Step 5 are also performed in this step to estimate the bunk points, temporary top strand requirements, and shipping concrete strength.

The non-composite girder, after temporary top strands are removed, carrying the weight of the wet deck concrete is analyzed in Section 4.6.2. This case has a required 28-day design concrete strength of 8.7 ksi. PGSuper will use the 7.8 ksi strength to check shipping, then find the 8.7 ksi requirement, and then begin the design iterations again starting with updating section properties and estimating the prestressing requirements. Therefore the 28-day design concrete strength of 8.7 ksi is assumed in Section 3.3.2.

4.4 Step 4 - Design for Lifting with Temporary Top Strands

Temporary top strands are generally not required for lifting of girders. However, if they are required for shipping it is advantageous to pretension them with the permanent strands as previously described.

From Step 3, 4 temporary top strands are required. These strands are located 2 inches below the top of the girder. Their eccentricity is -36.343 in.

4.4.1 Prestress losses

4.4.1.1 Initial relaxation before transfer

\[ \Delta f_{PR0} = \frac{\log (24.0 \cdot 1 \text{day})}{40} \left( \frac{202.5 \text{ksi}}{243.0 \text{ksi}} - 0.55 \right) (202.5 \text{ksi}) = 1.980 \text{ ksi} \]

4.4.1.2 Elastic Shortening

The temporary and permanent strands are at significantly different locations in the cross section. For this reason, compute the elastic shortening loss in the permanent and temporary strands separately.

\[ \Delta f_{PES} = \frac{E_p}{E_{cl}} f_{gp} \]

\[ f_{gp-perm} = \frac{P}{A} + \frac{Pe_{p}}{I} - \frac{M_ge_p}{I} \]

\[ f_{gp-temp} = \frac{P}{A} + \frac{Pe_{t}}{I} - \frac{M_ge_t}{I} \]

\[ P = N_p(a_{ps})(f_{pj} - \Delta f_{PR0} - \Delta f_{PES-perm}) + N_t(a_{ps})(f_{pj} - \Delta f_{PR0} - \Delta f_{PES-temp}) \]

Solve this equation iteratively for P.

The eccentricity of the resultant prestress force at mid-span is 27.304 in.

Assume 10% initial loss and \( f'_{cl} = 7.2 \text{ ksi} \).

\[ E_{cl} = 120000(1.0)(0.155)^2(7.2)^{0.33} = 5530.5 \text{ ksi} \]

\[ P = (61 + 4)(0.217)(1 - 0.10)(202.5) = 2570.6 \text{ kip} \]

At CG of permanent strands
\[ f_{cgp-perm} = \frac{2570.6 \text{ kip}}{923.531 \text{ in}^2} + \frac{(2570.6 \text{ kip})(27.304 \text{ in})(31.477 \text{ in})}{734356.0 \text{ in}^4} - \frac{(3514.22 \text{ kip} \cdot \text{ft})(12 \text{ in})}{734356.0 \text{ in}^4} (31.477 \text{ in}) = 3.984 \text{ ksi} \]

\[ \Delta f_{PES-perm} = \frac{28500 \text{ ksi}}{5530.5 \text{ kst}} (3.984 \text{ ksi}) = 20.53 \text{ ksi} \]

At CG of temporary strands

\[ f_{cgp-temp} = \frac{2570.6 \text{ kip}}{923.531 \text{ in}^2} + \frac{(2570.6 \text{ kip})(27.304 \text{ in})(-36.343 \text{ in})}{734356.0 \text{ in}^4} - \frac{(3514.22 \text{ kip} \cdot \text{ft})(12 \text{ in})}{734356.0 \text{ in}^4} (-36.343 \text{ in}) = 1.397 \text{ ksi} \]

\[ \Delta f_{PES-temp} = \frac{28500 \text{ ksi}}{5530.5 \text{ kst}} (1.397 \text{ ksi}) = 7.20 \text{ ksi} \]

\[
\begin{align*}
P &= (61)(0.217 \text{ in}^4)(202.5 \text{ ksi} - 1.98 \text{ ksi} - 20.53 \text{ ksi}) + 4(0.217 \text{ in}^4)(202.5 \text{ ksi} - 1.98 \text{ ksi} - 7.20 \text{ ksi}) = 2550.3 \text{ kip}
\end{align*}
\]

PGSuper performs this calculation with a very small convergence tolerance. The calculations are also performed at many points along the girder because the center of the prestress force changes due to harped strand location and the girder dead load moment changes along the length of the girder. There is less elastic shortening effect at the ends of the girder than at mid-span. The effective prestress force at release and initial lifting for various points (as determined by PGSuper) are given below. The initial loss converges to 11%.

<table>
<thead>
<tr>
<th>Location</th>
<th>Effective Prestress after release</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSXFR</td>
<td>2585.89 kip</td>
</tr>
<tr>
<td>HP</td>
<td>2549.04 kip</td>
</tr>
<tr>
<td>0.5Lg</td>
<td>2552.99 kip</td>
</tr>
</tbody>
</table>

### 4.4.2 Check girder stability

PGSuper performs many iterations to converge on an adequate lifting configuration. The solution converges for lifting devices located 8.25 ft from the ends of the girder when temporary top strands are used.

#### 4.4.2.1 Vertical Location of Center of Gravity

#### 4.4.2.1.1 Estimate Camber

The same calculations are made to estimate camber in the lifting without temporary top strand case. The differences are the modulus of elasticity is reduced because \( f_{c1}' \) reduced from 7.4 ksi to 7.2 ksi, the lift points moved from 11 ft to 8.25 ft, and temporary top strands are included in the prestress deflection. The resulting camber is

\[
\Delta \text{camber} = 4.025 \text{ in}
\]

The camber due to the temporary top strands is computed using the same equation as the straight strands. The temporary top strand eccentricity will be a negative value resulting in a negative, downward deflection

#### 4.4.2.1.2 Offset factor

\[
L_s = 162.995 - 2(8.25) = 146.495 \text{ ft}
\]

\[
F_o = \left(\frac{L_s}{L_g}\right)^2 - \frac{1}{3} = \left(\frac{146.495 \text{ ft}}{162.995 \text{ ft}}\right)^2 - \frac{1}{3} = 0.474
\]
4.4.2.1.3 Location the roll axis above the top of girder
\[ y_{rc} = 0\text{in} \]

4.4.2.1.4 Location of CG below roll axis
\[ y_t = Y_{top} - F_p(\Delta_{camber}) + y_{rc} = 38.343\text{in} - (0.474)(4.025\text{in}) - 0\text{in} = 36.433\text{in} \]

4.4.2.2 Lateral Deflection Parameters

4.4.2.2.1 Lateral Sweep
Sweep tolerance is 1/8” per 10 ft
\[ e_{sweep} = \frac{162.995\text{ft}}{10\text{ft}} \left(\frac{1}{8}\text{in}\right) = 2.037\text{in} \]

4.4.2.2.2 Initial Lateral Eccentricity
Initial lateral eccentricity of center of gravity of girder due to lateral sweep and eccentricity of lifting devices from CL girder
\[ e_{lift} = 0.25\text{in} \]
\[ e_i = F_p e_{sweep} + e_{lift} = (0.474)(2.037\text{in}) + 0.25\text{in} = 1.217\text{in} \]

4.4.2.2.3 Lateral Deflection of CG
Lateral deflection of center of gravity due to total girder weight applied to weak axis
\[ W_g = w_gL_g = (1.058\text{klf})(162.995\text{ft}) = 172.45\text{kip} \]
\[ z_o = \left(\frac{W_g}{12E_cl_{yy}L_g^2}\right)\left(\frac{L_g^5}{10} - a^2L_s^3 + 3a^4L_s + \frac{6a^5}{5}\right) \]
\[ = \left(\frac{172.45\text{kip}}{12(5530.5\text{ksi})(7201.84\text{in}^4)}\right)\left(\frac{(146.495\text{ft})^5}{10} - (8.25\text{ft})^2(146.495\text{ft})^3 \right.
\[ + 3(8.25\text{ft})^4(146.495\text{ft}) + \frac{6}{5}(8.25\text{ft})^5\left)\left(\frac{1728\text{in}^3}{1\text{ft}^3}\right) = 15.339\text{in} \]

4.4.2.3 Equilibrium tilt angle
\[ \theta_{eq} = \frac{e_i}{y_t - z_o} = \frac{1.217\text{in}}{36.433\text{in} - 15.339\text{in}} = 0.05768\text{rad} \]

4.4.2.4 Girder Stresses in Hanging Girder

4.4.2.4.1 Direct stress at Lift Point and Harp Point

4.4.2.4.1.1 Prestressing
\[ f_p = \frac{P}{A} + \frac{P_e}{S} \]
From PGSuper, the effective prestress force at the lift point is \( P = 1743.87\text{kip} \) straight strands, \( P = 673.64\text{kip} \) harped strands, and \( P = 167.83\text{kip} \) for temporary strands. The straight, harped, and temporary strand eccentricities are 31.748\text{in}, -19.526\text{in}, and -36.344\text{in}, respectively.
\[ f_t = -\left(1743.87\text{kip} + 673.64\text{kip} + 167.83\text{kip}\right) \]
\[ + \frac{923.531\text{in}^2}{-19152.5\text{in}^4} \]
\[ = -0.914\text{ksi} \]

38
\[ f_b = \frac{-(1743.87kip + 673.64kip + 167.83kip)}{923.531in^2} + \frac{(-1743.87kip)(31.748in) + (-673.64kip)(-19.526in) + (-167.83kip)(-36.343in)}{20594.8in^3} = -4.552ksi \]

From PGSuper, the effective prestress force at the harp point is \( P = 1717.36kip \) straight strands, \( P = 663.53kip \) harped strands, and \( P = 168.15kip \) temporary strands. The straight, harped, and temporary strand eccentricities are 31.748in, 30.775in, and -36.434in respectively.

\[ f_t = \frac{-(1717.36kip + 663.53kip + 168.15kip)}{923.531in^2} + \frac{(-1717.36kip)(31.748in) + (-663.53kip)(-31.748in) + (-168.15kip)(-36.343in)}{20594.8in^3} = 0.834ksi \]

\[ f_b = \frac{-(1717.36kip + 663.53kip + 168.15kip)}{923.531in^2} + \frac{(-1717.36kip)(31.748in) + (-663.53kip)(-31.748in) + (-168.15kip)(-36.343in)}{20594.8in^3} = -6.102ksi \]

4.4.2.4.1.2 Girder self-weight

\[ M_g = \frac{w_g}{2}(L_s x - x^2 - a^2) \]

At Lift Point

\[ x = l_t - a = 8.25ft - 8.25ft = 0ft \]

\[ M_g = \frac{(1.058kft)}{2}((146.495ft)(0ft) - (0ft)^2 - (11ft)^2) = -36.01k \cdot ft \]

\[ f_t = \frac{-36.01k \cdot ft}{-19152.5in^3} = 0.023ksi \]

\[ f_b = \frac{-36.01k \cdot ft}{20594.8in^3} = -0.021ksi \]

At Harp Point

\[ x = 0.4L_g - a = 0.4(162.995ft) - 8.25ft = 56.948ft \]

\[ M_g = \frac{(1.058kft)}{2}((146.495ft)(56.948ft) - (56.948ft)^2 - (8.25ft)^2) = 2689.16k \cdot ft \]

\[ f_t = \frac{2689.16k \cdot ft}{-19152.5in^3} = -1.685ksi \]

\[ f_b = \frac{2689.16k \cdot ft}{20594.8in^3} = 1.567ksi \]

4.4.2.4.2 Tilt induced stresses

Top left flange tip at lift point

\[ f_{it} = \frac{M_g W_{tf}}{2I_{yy}} \theta_{eq} \]

\[ f_{it} = \frac{(-36.01k \cdot ft)(49in)}{2(72018.4in^4)}(0.05768 rad)(\frac{12in}{1ft}) = -0.008ksi \]
Bottom right flange tip at lift point

\[ f_{br} = - \frac{M_g W_{bf} \theta_{eq}}{2I_{yy}} \]

\[ f_{br} = -\frac{(-36.01k \cdot ft)(38.375in)}{2(72018.4in^4)} \left(0.05768 \text{ rad} \right) \left(\frac{12\text{in}}{1\text{ft}}\right) = 0.007\text{ksi} \]

Top left flange tip at Harp Point

\[ f_{tt} = \frac{M_g W_{tf} \theta_{eq}}{2I_{yy}} \]

\[ f_{tt} = \frac{(2662.16k \cdot ft)(49in)}{2(72018.4in^4)} \left(0.05768 \text{ rad} \right) \left(\frac{12\text{in}}{1\text{ft}}\right) = 0.627\text{ksi} \]

Bottom right flange tip at Harp Point

\[ f_{br} = - \frac{M_g W_{bf} \theta_{eq}}{2I_{yy}} \]

\[ f_{br} = -\frac{(2662.16k \cdot ft)(38.375in)}{2(72018.4in^4)} \left(0.05768 \text{ rad} \right) \left(\frac{12\text{in}}{1\text{ft}}\right) = -0.491\text{ksi} \]

4.4.2.4.3 Total stress

Top left flange tip at lift point

\[ f_{tt} = -0.914\text{ksi} + 0.023\text{ksi} - 0.008\text{ksi} = -0.900\text{ksi} \]

Bottom left flange tip at lift point

\[ f_{br} = -4.401\text{ksi} - 0.021\text{ksi} + 0.007\text{ksi} = -4.566\text{ksi} \]

Top left flange tip at Harp Point

\[ f_{tt} = 0.834\text{ksi} - 1.668\text{ksi} + 0.627\text{ksi} = -0.207\text{ksi} \]

Bottom left flange tip at Harp Point

\[ f_{br} = -6.102\text{ksi} + 1.551\text{ksi} - 0.491\text{ksi} = -5.042\text{ksi} \]

4.4.2.4 Determine Concrete Strength at Lifting

For the general stress case, compression stress is limited to \(-0.65f'_{ct}\).

Bottom centerline at point of lift \(-0.65f'_{ct} = -4.573\text{ksi}, f'_{ct} = 7.04\text{ksi}\)

Bottom centerline at harp point \(-0.65f'_{ct} = -4.551\text{ksi}, f'_{ct} = 7.00\text{ksi}\)

Peak compression stress at the extremities of the section is limited to \(-0.7f'_{ct}\).

Bottom left flange tip at point of lift point \(-0.70f'_{ct} = -4.580\text{ksi}, f'_{ct} = 6.54\text{ksi}\)

Bottom left flange time at harp point \(-0.70f'_{ct} = -5.042\text{ksi}, f'_{ct} = 7.2\text{ksi} \leftarrow \text{Controls} \)

The controlling concrete strength matches the assumed value.

4.4.2.5 Factor of Safety Against Cracking

Lateral cracking moment
Precast, Prestressed Girder Design Example – PGSuper Training (11/16/2022)

\[ M_{cr} = \pm \frac{(f_r - f_{direct})2l_{yy}}{W_{top}} \]

Tilt angle at first crack

\[ \theta_{cr} = \frac{M_{cr}}{M_g} \leq 0.4 \text{ rad} \]

Cracking moment at lift point – cracking in top right corner

\[ f_{direct} = f_{ps} + f_g = -0.914\text{ksi} + 0.023\text{ksi} = -0.892\text{ksi} \]

\[ f_r = 0.24\sqrt{f'_{ct}} = 0.24\sqrt{7.2\text{ksi}} = 0.644 \text{ ksi} \]

\[ M_{cr} = -\left( \frac{0.644\text{ksi} - (-0.892\text{ksi})}{49\text{in}} \right)2(72018.4\text{in}^4)(\frac{1\text{in}}{12\text{ft}}) = -376.15 \text{ k \cdot ft} \]

Tilt angle at first crack at lift point

\[ \theta_{cr} = \frac{-376.151 \text{k \cdot ft}}{-36.01 \text{k \cdot ft}} = 10.44 \text{ rad} \div 0.4 \text{ rad} \]

Factor of Safety against Cracking at lift point

\[ FS_{cr} = \frac{y_r\theta_{cr}}{e_i + z_o\theta_{cr}} = \frac{(36.433\text{in})(0.4)}{1.217\text{in} + (15.339\text{in})(0.4)} = 1.98 \]

\[ FS_{cr} > 1.0 \ \text{OK} \]

Cracking moment at Harp Point

\[ M_{cr} = \left( \frac{0.644\text{ksi} - (0.834\text{ksi} - 1.685\text{ksi})}{49\text{in}} \right)2(72018.4\text{in}^4)(\frac{1\text{in}}{12\text{ft}}) = 362.09 \text{ k \cdot ft} \]

Tilt angle at first crack at Harp Point

\[ \theta_{cr} = \frac{362.09 \text{k \cdot ft}}{2662.16 \text{k \cdot ft}} = 0.13601 \text{rad} \]

Factor of Safety against Cracking at Harp Point

\[ FS_{cr} = \frac{y_r\theta_{cr}}{e_i + z_o\theta_{cr}} = \frac{(36.433\text{in})(0.13601)}{1.217\text{in} + (15.339\text{in})(0.13601)} = 1.5 \]

\[ FS_{cr} > 1.0 \ \text{OK} \]

4.4.2.6 Factor of Safety against Failure

\[ \theta_{max} = \frac{e_i}{2.5z_o} \leq 0.4 \text{ rad} = \frac{\sqrt{1.217}}{2.5(15.339)} = 0.17812 \text{ rad} \]

\[ FS_f = \frac{y_r\theta_{max}}{e_i + (1 + 2.5\theta_{max})(z_o\theta_{max})} = \frac{(36.433\text{in})(0.17812)}{1.217\text{in} + (1 + 2.5(0.17812))(165.339\text{in})(0.17812)} = 1.256 \]

When \( FS_f < FS_{cr} \), \( FS_f = FS_{cr} \ ∴ FS_f = 1.5 \]

\[ FS_f \geq 1.5 \ \text{OK} \]
4.5 Step 5 – Revise Shipping Design

Up to this point initial concrete strength, final concrete strength, and temporary strand requirements have been estimated based on assumed values. This design step verifies the shipping stability design.

Per the WSDOT Standard Specifications, WF girders can be shipped as soon as 10 days and as late as 90 days. The effective prestress force and camber vary during this time range. The analysis is conservative when using the effective prestress force at 10 days and the camber at 90 days (BDM 5.6.3.D.6).

4.5.1 Estimate Prestress Losses at Shipping

Assume hauling to occur as soon as possible (10 days)

4.5.1.1 Initial Relaxation

See 4.4.1.1

4.5.1.2 Elastic Shortening

See 4.4.1.2

4.5.1.3 Shrinkage of Girder Concrete

\[
\Delta f_{SRH} = \varepsilon_{bih} E_p K_{th}
\]

\[
\varepsilon_{bih} = k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3}
\]

\[
K_{th}(Permanent Strands) = K_{th-perm} = \frac{1}{1 + \frac{E_p A_{ps}}{E_c A_g} \left(1 + \frac{A_g e_{ps} e_p}{l_y}ight) \left(1 + 0.7 \psi_b(t_f, t_i)\right)}
\]

\[
K_{th}(Temporary Strands) = K_{th-temp} = \frac{1}{1 + \frac{E_p A_{ps}}{E_c A_g} \left(1 + \frac{A_g e_{ps} e_t}{l_y}ight) \left(1 + 0.7 \psi_b(t_f, t_i)\right)}
\]

\[
\psi_b(t_f, t_i) = 1.9 k_s k_{hc} k_f k_{td} e_{t_i}^{-0.118}
\]

\[
k_s = 1.45 - 0.13 \left(\frac{V}{S}\right) \geq 1.0
\]

\[
V = \frac{A L_g}{P L_g + 2A} = \frac{(923.531 in^2)(162.995 ft)}{(289.284 in)(162.995 ft) + 2(923.531 in^2)} = 3.182 \text{in}
\]

\[
k_s = 1.45 - 0.13(3.187) = 1.04
\]

\[
k_{hs} = 2.00 - 0.014 H = 2.00 - 0.014(75) = 0.95
\]

\[
k_{hc} = 1.56 - 0.008 H = 1.56 - 0.005(75) = 0.96
\]

\[
k_f = \frac{5}{1 + f_c'} = \frac{5}{1 + 7.2} = 0.61
\]

\[
t_i = 1 \text{day}, t_h = 10 \text{days}, t = t_h - t_i = 9 \text{days}
\]

\[
k_{td}(t = 9\text{days}) = \frac{9}{12 \left(\frac{100 - 4(7.2)}{7.2 + 20}\right)} = 0.223
\]

\[
t_i = 1 \text{day}, t_f = 2000 \text{days}, t = t_f - t_i = 1999 \text{days}
\]
\[ k_{td}(t = 1999\text{days}) = \frac{1999}{12(100 - 4(7.2)) + 1999} = 0.985 \]

\[ \psi_p(t_f, t_i) = 1.9(1.04)(0.96)(0.61)(0.985)(1)^{-0.118} = 1.135 \]

\[ \varepsilon_{bh} = (1.04)(0.95)(0.61)(0.223)(0.48 \times 10^{-3}) = 0.000064 \]

\[ A_{ps} = (61 + 4)(0.217\text{in}^2) = 14.105\text{in}^2 \]

\[ K_{th-perm} = \frac{1}{1 + \frac{28500\text{ksi}}{5530.5\text{ksi}} \frac{14.105\text{in}^2}{923.531\text{in}^3} \left( 1 + \frac{923.531\text{in}^2}{734356.0\text{in}^4} \right) \left[ 1 + 0.7(1.135) \right]} = 0.773 \]

\[ K_{th-temp} = \frac{1}{1 + \frac{28500\text{ksi}}{5530.5\text{ksi}} \frac{14.105\text{in}^2}{923.531\text{in}^3} \left( 1 + \frac{923.531\text{in}^2}{734356.0\text{in}^4} \right) \left[ 1 + 0.7(1.135) \right]} = 1.04 \]

\[ \Delta f_{PSRH} (\text{Permanent Strands}) = (0.000064)(28500\text{ksi})(0.773) = 1.414\text{ksi} \]

\[ \Delta f_{PSTH} (\text{Temporary Strands}) = (0.000064)(28500\text{ksi})(1.04) = 1.895\text{ksi} \]

### 4.5.1.4 Creep of Girder Concrete

\[ \Delta f_{PCRH} = \frac{E_p}{E_{ct}} f_{gp} \psi_p(t_h, t_i) K_{th} \]

\[ \psi_p(t_h, t_i) = 1.9(1.04)(0.96)(0.61)(0.985)(1)^{-0.118} = 0.257 \]

From the elastic shortening loss calculated in 4.4.1.2

\[ f_{gp-perm} = 3.984\text{ksi} \]

\[ f_{gp-temp} = 1.397\text{ksi} \]

After multiple iterations in PGSuper

\[ f_{gp-perm} = 3.945\text{ksi} \]

\[ f_{gp-temp} = 1.402\text{ksi} \]

\[ \Delta f_{CRH-perm} = \frac{28500\text{ksi}}{5530.5\text{ksi}} (3.945\text{ksi})(0.257)(0.773) = 4.033\text{ksi} \]

\[ \Delta f_{CRH-temp} = \frac{28500\text{ksi}}{5530.5\text{ksi}} (1.402\text{ksi})(0.257)(1.04) = 1.921\text{ksi} \]

### 4.5.1.5 Relaxation of Prestressing Strands

The girder concrete holds the prestressing strand in tension. The concrete undergoes creep and shrinkage deformations. The strands are between two points that move toward one another. Relaxation occurs at a reduced rate compared to intrinsic relaxation. The relaxation equations given by the AASHTO LRFD BDS are for reduced relaxation.

\[ \Delta f_{PR1H} = \frac{f_{pt} \log (24t_h)}{K_p' \log (24t_i)} \left( \frac{f_{pt}}{f_y} - 0.55 \right) \left[ 1 - \frac{3(\Delta f_{PSRH} + \Delta f_{PCRH})}{f_{pt}} \right] K_{th} \]

\[ K_{th} = 45 \]

\[ f_{pt} = f_{pj} - \Delta f_{PR0} - \Delta f_{PES} \]

\[ f_{pt-perm} = 202.5\text{ksi} - 1.98\text{ksi} - 20.33\text{ksi} = 180.19\text{ksi} \]
Precast, Prestressed Girder Design Example – PGSuper Training (11/16/2022)

\[
f_{pt-temp} = 202.5 \text{ ksi} - 1.98 \text{ ksi} - 7.22 \text{ ksi} = 193.3 \text{ ksi}
\]

\[
\Delta f_{pR1H-perm} = \left[\frac{180.19 \text{ ksi} \log (24 \cdot 10)}{243 \text{ ksi}} \left(\frac{180.19 \text{ ksi}}{243 \text{ ksi}} - 0.55\right)\right] \left[1 - \frac{3(1.414 \text{ ksi} + 4.098 \text{ ksi})}{180.19 \text{ ksi}}\right] \left(0.773\right) = 0.920 \text{ ksi}
\]

\[
\Delta f_{pR1H-temp} = \left[\frac{193.3 \text{ ksi} \log (24 \cdot 10)}{243 \text{ ksi}} \left(\frac{193.3 \text{ ksi}}{243 \text{ ksi}} - 0.55\right)\right] \left[1 - \frac{3(1.895 \text{ ksi} + 1.921 \text{ ksi})}{193.3 \text{ ksi}}\right] \left(1.04\right) = 1.773 \text{ ksi}
\]

PGSuper supports all three methods of computing relaxation described in the AASHTO LRFD BDS (LRFD 5.9.3.4.2c, C5.9.3.4.2c)

4.5.1.6 Losses at Shipping

\[
\Delta f_{PH} = \Delta f_{PRO} + \Delta f_{PES} + \Delta f_{PRLH}
\]

\[
\Delta f_{PRLH} = \Delta f_{PSRH} + \Delta f_{PCRH} + \Delta f_{PR1H}
\]

\[
\Delta f_{pLTH-perm} = 1.414 \text{ ksi} + 4.098 \text{ ksi} + 0.920 \text{ ksi} = 6.376 \text{ ksi}
\]

\[
\Delta f_{pLTH-temp} = 1.895 \text{ ksi} + 1.921 \text{ ksi} + 1.773 \text{ ksi} = 5.589 \text{ ksi}
\]

4.5.2 Check Girder Stability

Assume bunk points at 13’-8”.

Per WSDOT BDM 5.6.3 the hauling stability design is based on the least stiff haul truck from BDM Table 5.6.3-1 that allows the stress and stability factors of safety to be achieved.

<table>
<thead>
<tr>
<th>Table 5.6.3-1 Shipping Support Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shipping Support Rotational Spring Constant, (K_s) (Kip-in/radian)</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>40,000</td>
</tr>
<tr>
<td>50,000</td>
</tr>
<tr>
<td>60,000</td>
</tr>
<tr>
<td>70,000</td>
</tr>
<tr>
<td>80,000</td>
</tr>
</tbody>
</table>

Assume shipping support parameters for 40,000 kip-in/radian rotational spring constant. That equals to the HT40-72 haul truck configuration in PGSuper.
4.5.2.1 Stability Analysis Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational Stiffness</td>
<td>$K_\theta = 40000 \text{ kips \cdot in / rad}$</td>
</tr>
<tr>
<td>Center-to-center wheel spacing</td>
<td>$W_{cc} = 72 \text{ in}$</td>
</tr>
<tr>
<td>Height of the roll center above the roadway surface</td>
<td>$H_{rc} = 24\text{in}$</td>
</tr>
<tr>
<td>Height of the bottom of the girder above roadway</td>
<td>$H_{bg} = 72 \text{ in}$</td>
</tr>
<tr>
<td>Bunk placement tolerance</td>
<td>$e_{bunk} = 1.0 \text{ in}$</td>
</tr>
<tr>
<td>Normal Crown Slope</td>
<td>$\alpha = 0.02 \text{ ft/ft}$</td>
</tr>
<tr>
<td>Maximum Superelevation</td>
<td>$\alpha = 0.06 \text{ ft/ft}$</td>
</tr>
<tr>
<td>Impact for Normal Crown Slope Case</td>
<td>$IM = \pm 20%$</td>
</tr>
<tr>
<td>Impact for Superelevation Case</td>
<td>$IM = 0%$</td>
</tr>
<tr>
<td>Modulus of Rupture</td>
<td>$f_r = 0.24\lambda\sqrt{f'_{c}} = (0.24)(1.0)\sqrt{8.7\text{ksi}} = 0.708\text{ksi}$</td>
</tr>
</tbody>
</table>

4.5.2.2 Vertical Location of Center of Gravity

4.5.2.2.1 Camber at Hauling

Assume girder transportation occurs as late as possible to maximum camber grown while in storage. Assume transportation occurs at 90 days.

The camber at hauling is equal to the camber at the end of storage plus the change in dead load deflection due to the different support conditions between storage and hauling.

From before, the prestress deflection measured from the ends of the girder is

$$\Delta_{ps} = 7.160\text{in}$$
Changing the datum to the storage support location

\[ \Delta_{ps1} = 6.897\text{in at mid-span} \]

\[ \Delta_{ps2} = -0.263\text{in at girder end} \]

---

**Figure 4-7: Prestress induced Deflection based on Storage Datum**

The dead load deflection at mid-span during storage is

\[ L_s = L_g - 2a = 162.995\text{ft} - 2(1.708\text{ft}) = 159.578\text{ft} \]

The dead load deflection at the girder ends during storage is

\[ \Delta_{g1} = \frac{w_g a}{24E_{ci}I_x} [3a^2(a + 2L_s) - t_s^2] \]

\[ = \frac{(-1.058kfl)(1.708ft)}{24(5530.5ksi)(734356.0in^4)} \left[ 3(1.708ft)^2(1.708ft + 2(159.578ft)) - (159.578ft)^3 \right] \left( \frac{1728\text{in}^3}{1ft^3} \right) \]

\[ = 0.130\text{in} \]

At mid span

\[ \Delta_{g2} = \frac{5w_g L_s^4}{384E_{ci}I_x} - \frac{w_g a^2 I_s^2}{16E_{ci}I_x} \] \[ = -3.80\text{in} \]

---

**Figure 4-8: Girder Self-Weight Deflection during Storage**

Creep deflection during storage is

\[ \Delta_{creep} = \Psi_b(t_h, t_i) (\Delta_{ps} + \Delta_{g}) \]

\[ k_{td}(t = 89\text{days}) = \frac{89}{12 \left( \frac{100 - 4(7.2)}{7.2 + 20} + 89 \right)} = 0.738 \]

\[ \Psi_b(t_h, t_i) = 1.9(1.04)(0.96)(0.61)(0.738)(1)^{-0.118} = 0.852 \]

At mid-span

\[ \Delta_{creep} = (0.852)(6.897\text{in} - 3.80\text{in}) = 2.639\text{in} \]

At end of girder
**Precast, Prestressed Girder Design Example – PGSuper Training (11/16/2022)**

\[
\Delta_{\text{creep}} = (0.852)(-0.263\text{in} + 0.130\text{in}) = -0.113\text{in}
\]

Girder deflection in the hauling configuration

\[
L_s = 162.995\text{ft} - 2(13.67\text{ft}) = 135.66\text{ft}
\]

Mid-span deflection

\[
\Delta = \frac{5w_gL_s^4}{384E_cI_x} - \frac{w_ga^2L_s^2}{16E_cI_x} = \left[ \frac{5(-1.058\text{klf})(135.66\text{ft})^4}{384(5886.891\text{ksi})(734356.0\text{in}^4)} - \frac{(-1.058\text{klf})(13.67\text{ft})^2(135.66\text{ft})^2}{16(5886.891\text{ksi})(734356.0\text{in}^4)} \right] \left( \frac{1728\text{in}^3}{1\text{ft}^3} \right)
\]

Deflection at girder ends

\[
\Delta = \frac{w_ga}{24E_cI_x} \left[ 3a^2(a + 2L_g) - L_s^2 \right]
\]

We want the total camber measured between the girder ends and mid-span

\[
\Delta_{\text{camber}} = (\Delta_g + \Delta_{ps} + \Delta_{\text{creep}})_{\text{mid-span}} - (\Delta_g + \Delta_{ps} + \Delta_{\text{creep}})_{\text{end}}
\]

PGSuper computes \( \Delta_{\text{camber}} = 7.338\text{in} \). Use this value going forward.

**4.5.2.2 Offset Factor**

\[ F_o = \left( \frac{L_s}{L_g} \right)^2 - \frac{1}{3} = \left( \frac{135.66\text{ft}}{162.995\text{ft}} \right)^2 - \frac{1}{3} = 0.359 \]

**4.5.2.3 Location of roll axis below top of girder**

\[ y_{rc} = H_{bg} + H_g - H_{rc} = 72.0\text{in} + 74.0\text{in} - 24.0\text{in} = 122.0\text{in} \]

**4.5.2.4 Location of center of gravity above roll axis**

\[ y_r = y_{rc} - Y_{top} + F_o(\Delta_{\text{camber}}) = 122.0\text{in} - 38.343\text{in} + 0.359(7.338\text{in}) = 86.295\text{in} \]

**4.5.2.3 Lateral Deflection Parameters**

**4.5.2.3.1 Lateral Sweep**

Sweep tolerance = 1/8” per 10 ft

\[ \Delta_{\text{sweep}} = \left( \frac{162.995\text{ft}}{10\text{ft}} \right) \left( \frac{1}{8}\text{in} \right) = 2.037\text{in} \]

**4.5.2.3.2 Initial Lateral Eccentricity**

Initial lateral eccentricity of center of gravity of girder due to lateral sweep and eccentricity of bunking devices from CL girder

\[ e_t = F_o\Delta_{\text{sweep}} + e_{\text{bunk}} = (0.359)(2.037\text{in}) + 1.000\text{in} = 1.732\text{in} \]

**4.5.2.3.3 Lateral Deflection of CG**

Lateral deflection of center of gravity due to total weight of girder applied to the weak axis

\[ z_o = \frac{W_g}{12E_cI_yL_g^3} \left( \frac{L_s^5}{10} - a^2L_s^3 + 3a^4L_s + \frac{6}{5}a^5 \right) \]
Precast, Prestressed Girder Design Example – PGSuper Training (11/16/2022)

\[ z_o = \frac{172.48 \text{kip}}{12(5886.891 \text{ksi})(72018.4 \text{in}^4)(162.995 \text{ft})^2} \left( \frac{(135.66 \text{ft})^5}{10} - \frac{(13.67 \text{ft})^2(135.66 \text{ft})^3 + 3(13.67 \text{ft})^4(135.66 \text{ft})}{1728 \text{in}^3} \right) = 9.137 \text{in} \]

Impact up
\[ z_o = (0.8)(9.137 \text{in}) = 7.309 \text{in} \]

Impact down
\[ z_o = (1.2)(9.137 \text{in}) = 10.964 \text{in} \]

4.5.2.3.4 Girder Stresses at Harping Point

4.5.2.3.4.1 Stress due to prestressing
\[ f_t = \frac{-\left(1655.95 \text{kip} + 640.23 \text{kip} + 163.38 \text{kip}\right)}{923.531 \text{in}^2} + \frac{\left(-1655.95 \text{kip}\right)(31.748 \text{in}) + \left(-640.23 \text{kip}\right)(30.775 \text{in}) + \left(-163.38 \text{kip}\right)(-36.343 \text{in})}{-19152.5 \text{in}^3} = 0.800 \text{ksi} \]
\[ f_b = \frac{-\left(1655.95 \text{kip} + 640.23 \text{kip} + 163.38 \text{kip}\right)}{20594.85 \text{in}^3} \]

4.5.2.3.4.2 Stress due to girder self-weight (without impact)
\[ M_g = \frac{w_g}{2} \left(L_o x - x^2 - a^2\right) \]
\[ x = 0.4L_o - a = 0.4(162.995 \text{ft}) - 13.67 \text{ft} = 54.265 \text{ft} \]
\[ M_g = \frac{1.058 \text{kfl}}{2} \left((135.66 \text{ft})(54.265 \text{ft}) - (54.265 \text{ft})^2 - (13.67 \text{ft})^2\right) = 2195.02 \text{k} \cdot \text{ft} \]
\[ f_t = \frac{-2195.02 \text{k} \cdot \text{ft}}{-19152.5 \text{in}^3} \left(\frac{12 \text{in}}{1 \text{ft}}\right) = -1.375 \text{ksi} \]
\[ f_b = \frac{-2195.02 \text{k} \cdot \text{ft}}{20594.85 \text{in}^3} \left(\frac{12 \text{in}}{1 \text{ft}}\right) = 1.279 \text{ksi} \]

4.5.2.4 Analyze normal crown slope, no impact case

4.5.2.4.1 Equilibrium tilt angle
\[ \theta_{eq} = \frac{(K_g a + (IM)W_g e_i)}{K_g - (IM)W_g (y_r + z_o)} = \frac{\left(40000 \frac{\text{k}\cdot\text{in}}{\text{rad}}\right)\left(0.02 \frac{\text{ft}}{\text{ft}}\right) + (1.0)(172.48 \text{kip})(1.732 \text{in})}{\left(40000 \frac{\text{k}\cdot\text{in}}{\text{rad}}\right) - (1.0)(172.48 \text{kip})(86.295 \text{in} + 9.137 \text{in})} = 0.04668 \text{ rad} \]

4.5.2.4.2 Stress due to lateral loading from tilt
Top left flange tip
\[ f_{lt} = \frac{(IM)(M_g)\theta_{eq}W_{top}}{2I_y} = \frac{(1.0)(2195.02 \text{k} \cdot \text{ft})(0.04668)(49 \text{in})}{(2)(72018.4 \text{in}^4)} \left(\frac{12 \text{in}}{1 \text{ft}}\right) = 0.418 \text{ ksi} \]
Bottom right flange tip
\[ f_{br} = -\left(\frac{IM(M_g)\theta_{eq} W_{bot}}{2I_y}\right) = -\left(1.0\right)(2195.02 k \cdot ft)(0.04668)(38.375 in) \left(\frac{12 in}{1 ft}\right) = -0.328 \text{ksi} \]

### 4.5.2.4.3 Factor of Safety against Cracking

Lateral cracking moment
\[ f_{direct} = f_p + (IM)f_g = 0.800 \text{ksi} + (1.0)(-1.375 \text{ksi}) = -0.575 \text{ksi} \]

\[ M_{cr} = \frac{(f_r - f_{direct})2I_y}{W_{top}} = \left(\frac{0.708 \text{ksi} - (-0.575 \text{ksi})}{49 \text{in}}\right)(2)(72018.4 \text{in}^4) \left(\frac{1 \text{ft}}{12 \text{in}}\right) = 314.26 k \cdot ft \]

Tilt angle at first crack
\[ \theta_{cr} = \frac{M_{cr}}{(IM)M_g} \leq 0.4 \]

\[ \theta_{cr} = \frac{314.26 k \cdot ft}{(1.0)(2195.02 k \cdot ft)} = 0.14323 \text{ rad} \]

Factor of Safety against Cracking
\[ FS_{cr} = \frac{K_g(\theta_{cr} - \alpha)}{(IM)W_g[(\frac{(IM)z_o + y_r}{\theta_{cr} + e_i}]]} \]

\[ FS_{cr} = \left(\frac{(40000 \frac{\text{kip}}{\text{rad}})\left(0.14323 \text{ rad} - 0.02 \frac{ft}{ft}\right)}{(1.0)(172.48 kip)}\right) = 1.86 \]

\[ FS_{cr} > 1.0 \text{ OK} \]

### 4.5.2.4.4 Factor of Safety against Failure

\[ \theta_{max} = \alpha + \sqrt{\frac{\alpha^2}{2.5(\frac{IM}{2})}} \leq 0.4 \text{ rad} \]

\[ \theta_{max} = 0.02 \frac{ft}{ft} + \left(\frac{0.02 \frac{ft}{ft}}{2.5(1.0)(9.137 \text{in})}\right) \left(1.732 \text{ in} + (1.0)(9.137 \text{in}) + (86.295 \text{in})\right) \left(\frac{0.02 \frac{ft}{ft}}{\text{ft}}\right) = 0.399 \text{ rad} \]

\[ FS_f = \frac{K_g(\theta_{max} - \alpha)}{(IM)W_g\theta_{max}(1 + 2.5\theta_{max}) + y_r \theta_{max} + e_i}] \]

\[ FS_f = \left(\frac{(40000 \frac{\text{kip}}{\text{rad}})\left(0.399 - 0.02 \frac{ft}{ft}\right)}{(1.0)(172.48 kip)}\right) = 2.023 \]

If \( FS_f < FS_{cr} \), \( FS_f = FS_{cr} \)

\[ FS_f = 2.023 \]

\[ FS_f > 1.5 \text{ OK} \]

### 4.5.2.4.5 Factor of Safety against Rollover

\[ \theta_{ro} = \frac{(IM)W_g\left(\frac{W_{bc}}{2} - H_{rc}\alpha - e_{bunk} \right)}{K_g} + \alpha \]
50 \\[ \theta_{ro} = \frac{(1.0)(172.48kip)(\frac{72in}{2} - (24in)(0.02) - 1.0in)}{(40000 \frac{kip}{in \cdot rad})} + 0.02 = 0.17316 \text{ rad} \]

\[ F_{Sr} = F_{Sr} = \frac{K_0 (\theta_{ro} - \alpha)}{(IM)W_g[(IM)z_o + (1 + 2.5\theta_{ro}) + y_r] \theta_{ro} + e_i] \]

\[ F_{Sr} = \frac{(40000 \frac{kip}{in \cdot rad})(0.17316 - 0.02)}{(1.0)(172.48kip)\left[\left((1.0)(9.137in) \left(1 + 2.5(0.17316)\right) + 86.295in \right)(0.17316) + 1.732in\right]} = 1.875 \]

\[ F_{Sr} > 1.5 \text{ OK} \]

4.5.2.5 Analyze normal crown slope, impact up

4.5.2.5.1 Equilibrium tilt angle

\[ \theta_{eq} = \frac{(K_0 \alpha + (IM)W_0 e_i)}{K_0 - (IM)W_0(y_r + z_o)\left(\frac{40000 \frac{kip}{in \cdot rad}}{0.02} + (0.8)(172.48kip)(1.732in)\right)} = 0.03836 \text{ rad} \]

4.5.2.5.1.2 Stress due to lateral loading from tilt

Top left flange tip

\[ f_{lt} = \frac{(IM)(M_g)\theta_{eq}W_{top}}{2I_y} = \frac{(0.8)(2195.02k \cdot ft)(0.03836)(49in)}{(2)(72018.4in^4)} \left(\frac{12in}{1ft}\right) = 0.275 \text{ ksi} \]

Bottom right flange tip

\[ f_{br} = -\frac{(IM)(M_g)\theta_{eq}W_{bot}}{2I_y} = -\frac{(0.8)(2195.02k \cdot ft)(0.03836)(38.375in)}{(2)(72018.4in^4)} \left(\frac{12in}{1ft}\right) = -0.215 \text{ ksi} \]

4.5.2.5.2 Factor of Safety against Cracking

Cracking moment

\[ f_{direct} = f_{ps} + (IM)f_g = 0.800ksi + (0.8)(-1.375ksi) = -0.300ksi \]

\[ M_{cr} = \frac{(f_r - f_{direct})2l_y}{W_{top}} = \frac{(0.708ksi - (-0.300ksi))(2)(72018.4in^4)}{49in \left(\frac{12in}{12in}\right)} = 246.88k \cdot ft \]

Tilt angle at first crack

\[ \theta_{cr} = \frac{M_{cr}}{(IM)M_g} \leq 0.4 \]

\[ \theta_{cr} = \frac{246.88k \cdot ft}{(0.8)2195.02k \cdot ft} = 0.14059 \text{ rad} \]

Factor of Safety against Cracking

\[ F_{Scr} = \frac{K_0 (\theta_{cr} - \alpha)}{(IM)W_g[\left((IM)z_o + y_r\right) \theta_{cr} + e_i] \left(\frac{40000 \frac{kip}{in \cdot rad}}{0.02} + (0.8)(172.48kip)(1.732in)\right]} \]

\[ F_{Scr} = \frac{(0.8)(172.48kip)\left[\left((0.8)(9.137in) + 86.295in\right)(0.14059rad) + 1.732in\right]}{2.35} \]

\[ F_{Scr} > 1.0 \text{ OK} \]
4.5.2.5.3 Factor of Safety against Failure

\[
\theta_{\text{max}} = \alpha + \sqrt{\alpha^2 + \frac{e_t + ((IM)z_o + y_r)\alpha}{2.5z_o}} \leq 0.4 \text{ rad}
\]

\[
\theta'_{\text{max}} = 0.02 \frac{ft}{ft} + \sqrt{\left(0.02 \frac{ft}{ft}\right)^2 + \frac{1.73in + \left((0.8)(9.137in) + 86.295in\right)0.02}{2.5(0.8)(9.317in)}} = 0.46024 \text{ rad} \div 0.4 \text{ rad}
\]

\[
FS_f = \frac{K_0(\theta'_{\text{max}} - \alpha)}{(IM)W_g[(IM)z_o\theta'_{\text{max}}(1 + 2.5\theta'_{\text{max}}) + y_r\theta'_{\text{max}} + e_t]}
\]

\[
FS_f = \frac{(40000 \frac{kip}{rad})(0.4 - 0.02 \frac{ft}{ft})}{(0.8)(172.48kip)((0.8)(9.317in)(0.4)(1 + 2.5(0.4)) + (86.295in)(0.4) + 1.732in) = 2.617}
\]

\[
FS_f > 1.5 \text{ OK}
\]

4.5.2.5.4 Factor of Safety against Rollover

\[
\theta_{r_o} = \frac{(IM)W_g\left(\frac{W_c}{2} - H_{r,c}\alpha\right)}{K_0} + \alpha
\]

\[
\theta_{r_o} = \frac{(0.8)(172.48kip)\left(\frac{72in}{2} - (24in)(0.02)\right)}{(40000 \frac{kip}{rad})} + 0.02 = 0.14253 \text{ rad}
\]

\[
FS_r = \frac{K_0(\theta_{r_o} - \alpha)}{(IM)W_g[(IM)z_o(1 + 2.5\theta_{r_o}) + y_r\theta_{r_o} + e_t]}
\]

\[
FS_r = \frac{(40000 \frac{kip}{rad})(0.14253 - 0.02)}{(0.8)(172.48kip)((0.8)(9.317in)(1 + 2.5(0.14253)) + 86.295in)(0.14253) + 1.732in} = 2.30
\]

\[
FS_r > 1.5 \text{ OK}
\]

4.5.2.6 Analyze normal crown slope, impact down

4.5.2.6.1.1 Equilibrium tilt angle

\[
\theta_{eq} = \frac{(K_0\alpha + (IM)W_0e_l)}{K_0 - (IM)W_0(y_r + z_o)} = \frac{(40000 \frac{kip}{rad})(0.02 \frac{ft}{ft}) + (1.2)(172.48kip)(1.732in)}{(40000 \frac{kip}{rad}) - (1.2)(172.48kip)(86.295in + (1.2)(9.137in))} = 0.05831 \text{ rad}
\]

4.5.2.6.1.2 Stress due to lateral loading from tilt

Top left flange tip

\[
f_{tl} = \frac{(IM)(M_d)\theta_{eq}W_{top}}{2I_y} = \frac{(1.2)(2195.02k \cdot ft)(0.05831)(49in)(12in)}{(2)(72018.4in^4)} = 0.627 \text{ ksi}
\]

Bottom right flange tip

\[
f_{br} = -\frac{(IM)(M_d)\theta_{eq}W_{bot}}{2I_y} = -\frac{(1.2)(2195.02k \cdot ft)(0.05831)(38.375in)(12in)}{(2)(72018.4in^4)} = -0.491 \text{ ksi}
\]

4.5.2.6.2 Factor of Safety against Cracking

51
Cracking moment

\[ f_{\text{direct}} = f_{ps} + (IM)f_g = 0.800\text{ksi} + (1.2)(-1.375\text{ksi}) = -0.850\text{ksi} \]

\[ M_{cr} = \frac{(f_r - f_{\text{direct}})2l_r}{W_{top}} = \frac{(0.708\text{ksi} - (-0.850\text{ksi}))(2)(72018.4\text{in}^3)}{49\text{in}} \left( \frac{1\text{ft}}{12\text{in}} \right) = 381.64\text{k} \cdot \text{ft} \]

Tilt angle at first crack

\[ \theta_{cr} = \frac{M_{cr}}{(IM)M_g} \leq 0.4 \]

\[ \theta_{cr} = \frac{381.64 \text{k} \cdot \text{ft}}{(1.2)(2195.02 \text{k} \cdot \text{ft})} = 0.14489 \text{ rad} \]

Factor of Safety against Cracking

\[ FS_{cr} = \frac{K_\theta (\theta_{cr} - \alpha)}{(IM)W_g[((IM)z_o + y_r)\theta_{cr} + e_i]} \]

\[ FS_{cr} = \frac{(40000 \text{ k in rad})(0.14489 \text{ rad} - 0.02 \text{ ft} / \text{ft})}{(1.2)(172.48\text{kip})\{(1.2)(9.137\text{in}) + 86.295\text{in})(0.14489\text{ rad}) + 1.732\text{in}]} = 1.53 \]

\[ FS_{cr} > 1.0 \text{ OK} \]

4.5.2.6.3 Factor of Safety against Failure

\[ \theta_{max} = \alpha + \sqrt{\frac{e_l + ((IM)z_o + y_r)\alpha}{2.5z_o}} \leq 0.4 \text{ rad} \]

\[ \theta'_{max} = 0.02 \frac{ft}{ft} + \sqrt{(0.02 \frac{ft}{ft})^2 + \frac{1.732\text{in} + ((1.2)(9.137\text{in}) + 86.295\text{in})0.02}{2.5(1.2)(9.317\text{in})}} = 0.38683 \text{ rad} \]

\[ FS_f = \frac{K_\theta (\theta'_{max} - \alpha)}{(IM)W_g[(IM)z_o \theta'_{max} (1 + 2.5\theta'_{max}) + y_r \theta'_{max} + e_i]} \]

\[ FS_f = \frac{(40000 \text{ k in rad})(0.38683 - 0.02 \text{ ft} / \text{ft})}{(1.2)(172.48\text{kip})\{(1.2)(9.137\text{in})0.38683)(1 + 2.5(0.38683)) + (86.295\text{in})(0.38683) + 1.732\text{in}]} = 1.631 \]

\[ FS_f > 1.5 \text{ OK} \]

4.5.2.6.4 Factor of Safety against Rollover

\[ \theta_{ro} = \frac{(IM)W_g \left( \frac{W_{cc}}{2} - H_{rc} \alpha \right)}{K_\theta} + \alpha \]

\[ \theta_{ro} = \frac{(1.2)(172.48\text{kip}) \left( \frac{72\text{in}}{2} - (24\text{in})(0.02) \right)}{(40000 \text{ k-in rad})} + 0.02 = 0.20380 \text{ rad} \]

\[ FS_r = \frac{K_\theta (\theta_{ro} - \alpha)}{(IM)W_g[[((IM)z_o(1 + 2.5\theta_{ro}) + y_r)\theta_{ro} + e_i]} \]

52
\[ FS_r = \frac{\left(40000 \text{ kips/lin}(0.20380 - 0.02)\right)}{(1.2)(172.48\text{kip})\left[((1.2)(9.317\text{in})(1 + 2.5(0.20380)) + 86.295\text{in}(0.20380) + 1.732\text{in}\right]} = 1.565 \]

\[ FS_r > 1.5 \text{ OK} \]

4.5.2.7 Analyze at maximum superelevation, no impact

4.5.2.7.1 Equilibrium tilt angle

\[ \theta_{eq} = \frac{(K_g + (IM)W_g e_t)}{K_g - (IM)W_g (y_r + (IM)x_o)} = \left(\frac{40000\text{ kips/lin}(0.06\text{ft}) + (1.0)(172.48\text{kip}) (1.732\text{in})}{40000\text{ kips/lin} - (1.0)(172.48\text{kip})(86.295\text{in} + (1.0)(9.317\text{in})}\right) = 0.11465 \text{ rad} \]

4.5.2.7.2 Stress due to lateral loading from tilt

Top left flange tip

\[ f_{tl} = \frac{(IM)(M_g)\theta_{eq}W_{top}}{2I_y} = \frac{(1.0)(2195.02\text{ kips}) (0.11465)(49\text{ in})(12\text{ in} \text{ ft}^{-1})}{(2)(72018.4\text{ ft}^4)} = 1.027 \text{ kips/ft} \]

Bottom right flange tip

\[ f_{tr} = \frac{(IM)(M_g)\theta_{eq}W_{bot}}{2I_y} = \frac{-(1.0)(2195.02\text{ kips}) (0.11465)(38.375\text{ in})(12\text{ in} \text{ ft}^{-1})}{(2)(72018.4\text{ ft}^4)} = -0.856 \text{ kips/ft} \]

4.5.2.7.3 Factor of Safety against Cracking

Cracking moment

\[ M_{cr} = \frac{(f_t - f_{direct})2I_y}{W_{top}} = \frac{(0.708\text{ kips} - (-0.575\text{ kips}))(2)(72018.4\text{ ft}^4)}{49\text{ in} \text{ ft}^{-1}} = 314.26 \text{ kips-ft} \]

Tilt angle at first crack

\[ \theta_{cr} = \frac{M_{cr}}{(IM)M_g} \leq 0.4 \]

\[ \theta_{cr} = \frac{314.26 \text{ kips-ft}}{(1.0)2195.02 \text{ kips-ft}} = 0.14317 \text{ rad} \]

Factor of Safety against Cracking

\[ FS_{cr} = \frac{K_g(\theta_{cr} - \alpha)}{(IM)W_g[(IM)x_o + y_r, \theta_{cr} + e_t]} \]

\[ FS_{cr} = \left(\frac{40000\text{ kips/lin}(0.14317\text{ rad} - 0.06\text{ft})}{(1.0)(172.48\text{kip})[(1.0)(9.137\text{in}) + 86.295\text{in})(0.14317\text{rad}) + 1.732\text{in}]\right) = 1.253 \]

\[ FS_{cr} > 1.0 \text{ OK} \]

4.5.2.7.4 Factor of Safety against Failure

\[ \theta_{max} = \alpha + \sqrt{\frac{e_t + (IM)x_o + y_r}{2.5(IM)x_o}} \alpha \leq 0.4 \text{ rad} \]
\[ \theta_{\text{max}} = 0.06 \frac{ft}{ft} + \sqrt{(0.06 \frac{ft}{ft})^2 + \frac{1.732in + (1.0)(9.137in) + (86.295in)}{2.5(1.0)(9.137in)}} \left(0.06 \frac{ft}{ft}\right) = 0.634 \text{ rad} \times 0.4 \text{ rad} \]

\[ FS_f = \frac{K_0(\theta_{\text{max}} - \alpha)}{(IM)W_g[(IM)z_r \theta_{\text{max}} (1 + 2.5 \theta_{\text{max}}) + y_r \theta_{\text{max}} + e_i]} \]

\[ FS_f = \frac{(40000 \text{ kips})}{(1.0)(172.48\text{kip})} \left(0.4 - 0.06 \frac{ft}{ft}\right) = 1.810 \]

\[ FS_f > 1.5 \text{ OK} \]

4.5.2.7.5 Factor of Safety against Rollover

\[ \theta_{r_o} = \frac{(IM)W_g}{K_0} \left(\frac{W_{c}}{2} - H_{r_c} \alpha - e_{bunk}\right) + \alpha \]

\[ \theta_{r_o} = \frac{(1.0)(172.48\text{kip})}{(40000 \text{ kips})} \left(\frac{72in}{2} - (24in)(0.06) - 1.0in\right) + 0.06 = 0.20903 \text{ rad} \]

\[ FS_r = \frac{K_0(\theta_{r_o} - \alpha)}{(IM)W_g[(IM)z_r (1 + 2.5 \theta_{r_o}) + y_r \theta_{r_o} + e_i]} \]

\[ FS_r = \frac{(40000 \text{ kips})(0.20903 - 0.06)}{(1.0)(172.48\text{kip})} \left(\frac{(1.0)(9.137in)(1 + 2.5(0.20903)) + 86.295in(0.20903) + 1.732in}{1 + 2.5(0.20903)}\right) = 1.52 \]

\[ FS_r > 1.5 \text{ OK} \]

4.5.3 Check concrete strength

4.5.3.1 Compression

Compression stress is limited to \(-0.65f'_c\) when tilt is not considered and \(-0.70f'_c\) when tilt is considered.

<table>
<thead>
<tr>
<th>Case</th>
<th>(f_{ps}(\text{ksi}))</th>
<th>IM</th>
<th>(f_g(\text{ksi}))</th>
<th>(f_{\text{direct}} = \frac{f_{ps}}{(IM)}f_g(\text{ksi}))</th>
<th>(f'_c)</th>
<th>(f'_{\text{ult}}(\text{ksi}))</th>
<th>(f'<em>{\text{c}} = \frac{f</em>{\text{direct}} + f'_{\text{ult}}}{0.65}) (ksi)</th>
<th>(f'<em>{\text{c}} = \frac{f</em>{\text{direct}} + f'_{\text{ult}}}{0.70}) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2% slope, no impact</td>
<td>-5.886</td>
<td>1.0</td>
<td>1.279</td>
<td>-4.607</td>
<td>7.1</td>
<td>-0.318</td>
<td>7.1</td>
<td></td>
</tr>
<tr>
<td>2% slope, impact up</td>
<td>-5.886</td>
<td>0.8</td>
<td>1.279</td>
<td>-4.863</td>
<td>7.5</td>
<td>-0.215</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>2% slope, impact down</td>
<td>-5.886</td>
<td>1.2</td>
<td>1.279</td>
<td>-4.351</td>
<td>6.2</td>
<td>-0.491</td>
<td>6.9</td>
<td></td>
</tr>
<tr>
<td>6% slope, no impact</td>
<td>-5.886</td>
<td>1.0</td>
<td>1.279</td>
<td>-4.607</td>
<td>7.1</td>
<td>-0.856</td>
<td>7.8</td>
<td></td>
</tr>
</tbody>
</table>

4.5.3.2 Tension

Check tension stress limit

Maximum tension stress occurs at harp point top left corner of girder on with superelevated slope of 6\%
Stress limit

\[ 0.24\lambda \sqrt{\frac{f_t}{f_c'}} = 0.24(1.0)\sqrt{8.7\text{kpsi}} = 0.708 \text{kpsi} \]

Tension stress is within limit. However, use of this tensile stress limit requires reinforcement sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of \( \frac{1}{2}f_y \), no exceeding 30 ksi (LRFD Table 5.9.2.3.1b-1).

4.6 Step 6 – Check Erection Stresses

Evaluate the stress in the girder after the temporary strands are removed and the wet deck is cast. This is the maximum loading condition for the bare girder.

This check is not specified in the AASHTO LRFD BDS. This check is a WSDOT design policy. See WSDOT BDM Table 5.2.1-1

4.6.1 Losses between Transfer to Deck Placement

4.6.1.1 Temporary strand removal

Removing the temporary strands changes the state of stress in the girder and causes additional elastic shortening loss in the permanent strands.

Assuming the temporary strands are removed shortly after erection, the force in the temporary strands at time of removal is

\[ P_{tr} = A_i\left(f_{pj} - f_{ptr}\right) \]

The change of stress in the concrete at the level of the permanent strand due to the removal of the temporary strands is

\[ f_{ptr} = -\frac{P_{tr}e_p}{A_g} \]

The change in stress in the permanent strands is

\[ \Delta f_{ptr} = E_p \frac{f_{ptr}}{E_c} \]

At the mid-span section

\[ P_{tr} = 4(0.217\text{in}^2)(202.5\text{ksi} - 7.569\text{ksi}) = 162.93\text{ kip} \]

\[ f_{ptr} = -\frac{162.93\text{kip}}{923.531\text{in}^2} - \frac{(162.93\text{kip})(-36.343\text{in})(31.477\text{in})}{734356.0\text{in}^4} = 0.077\text{ksi} \]

\[ \Delta f_{ptr} = \frac{28500\text{ksi}}{5886891\text{ksi}}(0.077\text{ksi}) = 0.375\text{ ksi} \]

4.6.1.2 Shrinkage of Girder Concrete

\[ \Delta f_{psr} = \epsilon_{bid} E_p K_{id} \]

\[ \epsilon_{bid} = k_s k_{hs} k_f k_{td} 0.48\times10^{-3} \]

\[ K_{id} = \frac{1}{1 + \frac{E_p A_{ps}}{E_c A_g} \left(1 + \frac{A_g e_p}{A_g}ight) \left[1 + 0.7\psi_b(t_f, t_i)\right]} \]

\[ \psi_b(t_f, t_i) = 1.9k_s k_{he} k_f k_{td} t_i^{-0.118} \]
\[ k_s = 1.45 - 0.13 \left( \frac{V}{S} \right) \geq 1.0 = 1.04 \]
\[ k_{hs} = 2.00 - 0.014H = 0.95 \]
\[ k_{hc} = 1.56 - 0.008H = 0.96 \]
\[ k_f = \frac{1}{1 + f_{cl}'} = 0.61 \]

\[ k_{id} = \frac{t}{12\left( \frac{100 - 4f_{cl}'}{f_{cl}'} + t \right)} = \begin{cases} 0.791 & \text{with } t = (t_d - t_i) = 119 \text{ day} \\ 0.985 & \text{with } t = (t_f - t_i) = 1999 \text{ day} \end{cases} \]

\[ t_i = 1 \text{ day} \]
\[ t_d = 120 \text{ day} \]
\[ t_f = 2000 \text{ day} \]

\[ \varepsilon_{bid} = (1.04)(0.95)(0.61)(0.791)(0.48 \times 10^{-3}) = 0.000228 \]
\[ \psi_b(t_f, t_i) = 1.9(1.04)(0.96)(0.61)(0.985)(1)^{-0.118} = 1.135 \]

\[ K_{id} = \frac{1}{1 + \left( \frac{28500 \text{ksi}}{5530.5 \text{ksi}} \right) \left( \frac{14.105 \text{in}^2}{923.531 \text{in}^2} \right) \left( \frac{1 + (923.531 \text{in}^2)(27.304 \text{in})(31.477 \text{in})}{734356.0 \text{in}^4} \right) (1 + 0.7(1.135))} = 0.773 \]

\[ \Delta f_{PSR} = (0.000228)(28500 \text{ksi})(0.773) = 5.022 \text{ ksi} \]

### 4.6.1.3 Creep of Girder Concrete

\[ \Delta f_{PCR} = \frac{E_p}{E_{ct}} f_{cap} \psi_b(t_d, t_i) K_{id} \]

\[ \psi_b(t_d, t_i) = 1.9(1.04)(0.96)(0.61)(0.791)(1)^{-0.118} = 0.912 \]

\[ \Delta f_{PCR} = \frac{28500 \text{ksi}}{4935.632 \text{ksi}} (3.945 \text{ksi})(0.912)(0.773) = 14.327 \text{ ksi} \]

### 4.6.1.4 Relaxation of Prestressing Strands

\[ \Delta f_{pr1} = \left[ \frac{f_{pt} \log \left( 24t_d \right)}{K_{i}'} \left( \frac{f_{pt}'}{f_{py}'} - 0.55 \right) \right] \left[ 1 - \frac{3(\Delta f_{PSR} + \Delta f_{PCR})}{f_{pt}} \right] K_{id} \]

\[ f_{pt} = f_p - \Delta f_{pro} - \Delta f_{pES} = 202.5 \text{ksi} - 1.98 \text{ksi} - 20.328 \text{ksi} = 180.193 \text{ksi} \]

\[ \Delta f_{pr1} = \left[ \frac{180.193 \text{ ksi} \log \left( 24 \cdot 120 \right)}{45 \log \left( 24 \cdot 1 \right)} \left( \frac{180.193 \text{ksi}}{243 \text{ksi}} - 0.55 \right) \right] \left( 1 - \frac{3(5.022 \text{ksi} + 14.327 \text{ksi})}{180.193 \text{ksi}} \right) (0.773) = 1.007 \text{ ksi} \]

### 4.6.1.5 Time dependent losses

\[ \Delta f_{PLT_id} = \Delta f_{PSR} + \Delta f_{PCR} + \Delta f_{pr1} \]

\[ \Delta f_{PLT_id} = 5.022 \text{ksi} + 14.327 \text{ksi} + 1.007 \text{ksi} = 20.355 \text{ ksi} \]

### 4.6.1.6 Elastic gains

Added dead load on noncomposite section

\[ M_{adl} = M_{diaphragm} + M_{stabil} + M_{haunch} \]

\[ M_{adl} = 140.17k \cdot ft + 2465.61k \cdot ft + 760.28k \cdot ft = 3366.05k \cdot ft \]
\[
\Delta f_{cd}' = \frac{M_{act}}{f_y}
\]
\[
\Delta f_{cd} = (3366.05k \cdot ft) \left( \frac{12in}{1ft} \right) \left( \frac{31477in}{734356.0in^4} \right) = 1.731 \text{ ksi}
\]
\[
\Delta f_{PED} = \frac{E_p}{E_c} \Delta f_{cd}'
\]
\[
\Delta f_{PED} = \left( \frac{28500\text{ksi}}{5886.891\text{ksi}} \right) (1.731\text{ksi}) = 8.382 \text{ ksi}
\]

### 4.6.2 Stresses

**Effective prestress**

\[
P = A_{ps}(f_p - \Delta f_{p0} - \Delta f_{PES} - \Delta f_{PLT_{id}} - \Delta f_{ptr} + \Delta f_{PED})
\]
\[
P = 13.237in^2(202.5ksi - 1.98ksi - 20.328ksi - 20.355ksi - 0.375ksi + 8.382ksi) = 2221.8\text{ kip}
\]

**Stress in girder due to effective prestress**

\[
f_p = \frac{P}{A} + \frac{P e}{S}
\]
\[
f_t = \frac{-2221.8\text{kip}}{923.531in^2} + \frac{(-2221.8\text{kip})(31.447\text{in})}{-19152.5in^3} = 1.246\text{ksi}
\]
\[
f_b = \frac{-2221.8\text{kip}}{923.531in^2} + \frac{(-2221.8\text{kip})(31.447\text{in})}{20594.8in^3} = -5.801\text{ksi}
\]

**Stress due to loading**

\[
f = \frac{M_{girder} + M_{diaphragm} + M_{slab} + M_{haunch}}{S}
\]
\[
f_t = \frac{(3368.43k \cdot ft + 140.17k \cdot ft + 2465.61k \cdot ft + 760.28k \cdot ft) \left( \frac{12in}{1ft} \right)}{-19152.5in^3} = -4.219 \text{ ksi}
\]
\[
f_b = \frac{(3368.43k \cdot ft + 140.17k \cdot ft + 2465.61k \cdot ft + 760.28k \cdot ft) \left( \frac{12in}{1ft} \right)}{20594.8in^3} = 3.924 \text{ ksi}
\]

**Stress Limits**

**Compression**

\[-0.45f_c' = -0.45(8.7 ksi) = -3.915 ksi\]

**Tension**

\[0.19\sqrt{f_c'} = 0.19(1.0)\sqrt{8.7 ksi} = 0.560 ksi\]

**Stress Demand**

\[
f_t = 1.246 ksi - 4.219 ksi = -2.974 ksi
\]
\[
f_b = -5.801 ksi + 3.924 ksi = -1.877 ksi
\]

Stresses at mid-span satisfy the limit state.

These calculations are repeated at the point of prestress transfer. The results are

\[
f_t = -0.648 ksi
\]
\[
f_b = -3.892 ksi
\]

The required concrete strength is
−0.45f'_c = −3.892ksi

f'_c = 8.65 ksi

This is the overall governing 28-day design concrete strength.

4.7 Step 7 – Check Final Conditions

4.7.1 Losses from Deck Placement to Final

4.7.1.1 Shrinkage of Girder Concrete

\[ \Delta f_{PSD} = \varepsilon_{bf} E_p K_{df} \]

\[ \varepsilon_{bf} = \varepsilon_{bf} - \varepsilon_{bid} \]

\[ \varepsilon = k_s k_{hs} k_f k_{id} 0.48 \times 10^{-3} \]

\[ K_{df} = \frac{1}{1 + \frac{E_p A_{ps}}{E_c A_c} \left( 1 + \frac{A_c e^2}{I_c} \right) \left[ 1.0 + 0.7 \psi_b (t_f, t_i) \right]} \]

From before

\[ k_s = 1.04 \]

\[ k_{hs} = 0.95 \]

\[ k_{hc} = 0.96 \]

\[ k_f = 0.61 \]

\[ \psi_b (t_f, t_i) = 1.135 \]

\[ \varepsilon_{bid} = 0.000228 \]

\[ k_{id} (t = t_f - t_i) = 0.985 \]

\[ \varepsilon_{bf} = (1.04)(0.95)(0.61)(0.985)(0.48 \times 10^{-3}) = 0.000284 \]

\[ e_c = e + y_{bc} - y_b = 31.477in + 48.867in - 35.657in = 44.687in \]

\[ K_{df} = \frac{1}{1 + \frac{(28500kksi) (13.237in^2)}{(5530.5kksi) (1349.614in^4)} \left( 1 + \frac{(1349.614in^4)(44.687in^2)}{1246570.6in^4} \right) (1 + 0.7(1.135))} = 0.777 \]

\[ \Delta f_{PSD} = (0.000284)(28500kksi)(0.777) = 1.234 ksi \]

4.7.1.2 Creep of Girder Concrete

\[ \Delta f_{PSC} = \frac{E_p}{E_c} \int_{cog} \left[ \psi_b (t_f, t_i) - \psi_b (t_d, t_i) \right] K_{af} + \frac{E_p}{E_c} (\Delta f_{cd}) \psi_b (t_f, t_d) K_{af} \]

\[ \Delta f_{cd} = - (\Delta f_{PSR} + \Delta f_{PCR} + \Delta f_{PRL}) \left( \frac{A_{ps}}{A_g} \right) \left( 1 + \frac{A_c e^2}{I_c} \right) - (\Delta f'_{cd} + \Delta f''_{cd}) \]

\[ \Delta f'_{cd} = \frac{M_{ad} t e}{l_g} = 1.731 ksi \]

\[ \Delta f''_{cd} = \frac{M_{sat}(y_{bc} - y_{bg} + e)}{l_c} \]
\[
M_{sidt} = M_{Barrier} + M_{overlay}
\]
\[
M_{sidt} = 732.83k \cdot ft + 739.63k \cdot ft = 1471.75k \cdot ft
\]
\[
\Delta f'_{cd} = \frac{(1471.75k \cdot ft)(48.867in - 35.657in + 31.477in)}{12in} \left( \frac{12in}{1ft} \right) = 0.633 ksi
\]
\[
\Delta f_{cd} = -(5.022 + 14.732 + 1.007 ksi) \left( \frac{14.105in^2}{923.531in^2} \left( 1 + \frac{(923.531in^2)(31.477in)^2}{734356.0in^4} \right) - (1.731ksi + 0.633ksi) \right)
\]
\[
= -3.020 ksi
\]
\[
k_{cd} = \frac{t}{12 (\frac{100 - 4f_{cd}^2}{f_{ct} + 20}) + t} = 0.984 \text{ with } t = (t_f - t_d) = 2000 - 120 = 1880 day
\]
\[
\psi_b(t_f, t_d) = 1.9((1.04)(0.96)(0.61)(0.984)(120)^{-0.118} = 0.644
\]
\[
\Delta f_{PCD} = \left( \frac{28500ksi}{5530.5ksi} \right)(3.945ksi)(1.135 - 0.912)(0.777) + \left( \frac{28500ksi}{5886.891ksi} \right)(-3.020ksi)(0.644)(0.777) = -2.892 ksi
\]

### 4.7.1.3 Relaxation of Prestressing Strands

\[\Delta f_{PR2} = \Delta f_{PR1} = 1.007 ksi\]

### 4.7.1.4 Shrinkage of Deck Concrete

\[\Delta f_{PSS} = \frac{E_p}{E_c} \Delta f_{cd} K_d [1 + 0.7\psi_b(t_f, t_d)]\]

\[\Delta f_{cd} = \epsilon_{def} A_d E_{c, deck} \left( \frac{1}{A_c} \frac{e_d}{l_c} \right)\]

\[\Delta f_{PSS} = \left( \frac{28500ksi}{5886.891ksi} \right)(0.091ksi)(0.777)(1 + 0.7(0.644)) = 0.498 ksi\]

### 4.7.1.5 Time Dependent Losses

\[\Delta f_{PLT, cd} = \Delta f_{PSD} + \Delta f_{PCD} + \Delta f_{PR1} - \Delta f_{PSS} = 1.234ksi - 2.892ksi + 1.007ksi - 0.498ksi = -1.149 ksi\]

\[\Delta f_{PLT} = \Delta f_{PLT, cd} + \Delta f_{PLT, cd} = 20.355ksi - 1.149ksi = 19.206ksi\]

\[\Delta f_{PT} = \Delta f_{PR0} + \Delta f_{PES} + \Delta f_{PT} + \Delta f_{PLT} - \Delta f_{PED} - \Delta f_{PSID} = 1.98ksi + 20.328ksi + 0.375ksi + 19.206ksi - 8.382ksi - 3.065ksi = 30.441ksi\]

### 4.7.1.6 Elastic Gains

#### 4.7.1.6.1 Superimposed dead loads

\[\Delta f_{PSIDL} = \frac{E_p}{E_c} \Delta f''_{cd} = \left( \frac{28500ksi}{5886.891ksi} \right)(0.633ksi) = 3.065 ksi\]

#### 4.7.1.6.2 Live Loads

\[\Delta f_{PLL} = \frac{E_p}{E_c} \Delta f''_{cd}\]

\[\Delta f''_{cd} = \frac{M_{lim}(Y_{bc} - Y_{bg} + e)}{I_c}\]
\[
\Delta f_{dL}''' = \left\{ \begin{array}{ll}
\frac{(3804.05 \, k \cdot ft)(48.867 \, in - 35.657 \, in + 31.477 \, in)}{1243570.6 \, in^4} & (12 \, in \, 1 \, ft) = 1.636 \, ksi \ (Design \ Live \ Load) \\
\frac{(1863.4 \, k \cdot ft)(48.867 \, in - 35.657 \, in + 31.477 \, in)}{1243570.6 \, in^4} & (12 \, in \, 1 \, ft) = 0.802 \, ksi \ (Fatigue \ Live \ Load)
\end{array} \right.
\]
\[
\Delta f_{PLL} = \left\{ \begin{array}{ll}
\frac{(28500 \, ksi)}{5886.891 \, ksi} & (1.636 \, ksi) = 7.922 \, ksi \ (Design \ Live \ Load) \\
\frac{(28500 \, ksi)}{5886.891 \, ksi} & (0.802 \, ksi) = 3.881 \, ksi \ (Fatigue \ Live \ Load)
\end{array} \right.
\]

4.7.2 Stresses

4.7.2.1 Final Stresses without Live Load

This analysis is for the Service I limit state.

Effective Prestress

\[
P = A_p (f_p - \Delta f_{pro} - \Delta f_{pes} - \Delta f_{ptr} - \Delta f_{plt} + \Delta f_{ped} + \Delta f_{psidl})
\]

\[
P = 13.237 \, in^2 (202.5 \, ksi - 1.98 \, ksi - 20.328 \, ksi - 0.375 \, ksi - 19.206 \, ksi + 8.382 \, ksi + 3.065 \, ksi)
\]

\[
P = (13.237 \, in^2)(172.059 \, ksi) = 2277.5 \, kip
\]

Stress in girder due to effective prestress

\[
f_{ps} = \frac{P}{A} + \frac{P_e}{S}
\]

\[
f_t = \frac{-2277.5 \, kip}{923.531 \, in^2} + \frac{(-2277.5 \, kip)(31.477 \, in)}{-19152.5 \, in^3} = 1.277 \, ksi
\]

\[
f_b = \frac{-2277.5 \, kip}{923.531 \, in^2} + \frac{(-2277.5 \, kip)(31.477 \, in)}{20594.8 \, in^3} = -5.947 \, ksi
\]

Stress due to loading

\[
f = \left( \frac{M_{girder} + M_{diaphragm} + M_{slab} + M_{haunch}}{S} \right) + \left( \frac{M_{barrier} + M_{overlay}}{S_c} \right) + f_{ss}
\]

\[
f_t = \frac{(3368.43 + 140.17 + 2465.61 + 760.28 \, k \cdot ft)(12 \, in \, 1 \, ft)}{-19152.5 \, in^3} + \frac{(732.12 + 739.63 \, k \cdot ft)(12 \, in \, 1 \, ft)}{-49599.7 \, in^3} - 0.410 \, ksi
\]

\[
f_t = -4.986 \, ksi
\]

\[
f_b = \frac{(3368.43 + 140.17 + 2465.61 + 760.28 \, k \cdot ft)(12 \, in \, 1 \, ft)}{20594.8 \, in^3} + \frac{(732.12 + 0.0 \, k \cdot ft)(12 \, in \, 1 \, ft)}{25509.3 \, in^3} + 0.121 \, ksi = 4.390 \, ksi
\]

This limit state evaluation is for compression stress. Minimize the stress at the bottom of the girder by considering the case before future overlay installation.

Stress Limits

Compression

\[-0.45f_c' = -0.45(8.7 \, ksi) = -3.915 \, ksi\]

Stress Demand

\[f_t = 1.277 \, ksi - 4.986 \, ksi = -3.709 \, ksi\]
Stresses satisfy the limit state criteria.

4.7.2.2 Final Stresses with Live Load

4.7.2.2.1 Service I Limit State
This is a compression stress limit state.

Effective Prestress with Service I live load

\[ P = A_{ps}(f_{pj} - \Delta f_{pR0} - \Delta f_{pES} - \Delta f_{ptr} - \Delta f_{pLT} + \Delta f_{pED} + \Delta f_{PSIDL} + \Delta f_{PLL}) \]

\[ P = 13.237 \text{in}^2 (202.5 \text{ksi} - 1.98 \text{ksi} - 20.328 \text{ksi} - 0.375 \text{ksi} - 19.206 \text{ksi} + 8.382 \text{ksi} + 3.065 \text{ksi} + 7.922 \text{ksi}) \]

\[ = (13.237 \text{in}^2)(179.98 \text{ksi}) = 2382.4 \text{kip} \]

Stress in girder due to effective prestress

\[ f_{ps} = \frac{P}{A} + \frac{P_e}{S} \]

\[ f_t = \frac{-2382.4 \text{kip}}{923.531 \text{in}^2} + \frac{(13.237 \text{kip})(31.477 \text{in})}{-19152.5 \text{in}^3} = 1.324 \text{ksi} \]

Stress due to loading

\[ f = \frac{(M_{girder} + M_{diaphragm} + M_{stab} + M_{haunch}) + (M_{barrier} + M_{overlay} + M_{tlm})}{S_c} + f_{ss} \]

\[ f_t = \frac{(3368.43 + 140.17 + 2465.61 + 760.28 \text{ kips} \cdot \text{ft})}{12 \text{ in}} \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) + \frac{(732.12 + 739.63 + 3804.05 \text{ kips} \cdot \text{ft})}{-49599.7 \text{ in}^3} \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) - 0.410 \text{ksi} \]

\[ f_t = -5.906 \text{ ksi} \]

This limit state evaluation is for compression stress. Minimize the stress at the bottom of the girder by considering the case before future overlay installation and live load not on the structure.

Stress Limits

Compression

\[ -0.6 f'_c = -0.6(8.7 \text{ksi}) = -5.220 \text{ksi} \]

Stress Demand

\[ f_t = (1.325 \text{ksi} - 5.906 \text{ksi}) = -4.570 \text{ksi} \]

Stresses are within limits

4.7.2.2.2 Service III Limit State
This is a tension stress limit state.

Effective Prestress with Service III live load

\[ P = A_{ps}(f_{pj} - \Delta f_{pR0} - \Delta f_{pES} - \Delta f_{ptr} - \Delta f_{pLT} + \Delta f_{pED} + \Delta f_{PSIDL} + 0.8 \Delta f_{PLL}) \]

\[ P = 13.237 \text{in}^2 (202.5 \text{ksi} - 1.98 \text{ksi} - 20.328 \text{ksi} - 0.375 \text{ksi} - 19.206 \text{ksi} + 8.382 \text{ksi} + 3.065 \text{ksi} + (0.8)7.922 \text{ksi}) \]

\[ = (13.237 \text{in}^2)(178.397 \text{ksi}) = 2361.4 \text{kip} \]

Stress in girder due to effective prestress

\[ f_{ps} = \frac{P}{A} + \frac{P_e}{S} \]
\[ f_b = \frac{-2361.4 kip}{923.531 in^2} + \frac{(-2361.4 kip)(31.477 in)}{20594.8 in^3} = -6.166 ksi \]

Stress due to loading

\[ f = \left( \frac{M_{\text{girder}} + M_{\text{diaphragm}} + M_{\text{slab}} + M_{\text{haunch}}}{S} \right) \left( \frac{12 in}{1 ft} \right) + \left( \frac{M_{\text{barrier}} + M_{\text{overlay}} + 0.8M_{\text{lim}}}{S_c} \right) + f_s = \]

\[ = 6.169 ksi \]

Stress Limits

The tension stress limit is 0.0 ksi per WSDOT BDM

Stress Demand

\[ f_t = (-6.166 ksi - 6.169 ksi) = 0.003 ksi \approx 0 ksi \]

Stresses are within limits

Check the assumed effective prestress of 85% (15% loss).

\[ \frac{f_{pe}}{f_{pj}} = \frac{178.397}{202.5} = 0.88 = 88\% \approx 85\% \]

\[ Loss = 1 - \frac{f_{pe}}{f_{pj}} = 1 - 0.88 = 0.12 = 12\% \approx 15\% \]

4.7.2.2.3 Fatigue I Limit State

Effective Prestress with Fatigue I live load

\[ P = A_{ps}(f_{pj} - \Delta f_{PRO} - \Delta f_{DES} - \Delta f_{PER} - \Delta f_{ED} + \Delta f_{PSIL} + 1.5\Delta f_{LL}) \]

\[ P = 13.237 in^2(202.5 ksi - 1.98 ksi - 20.324 ksi - 0.375 ksi - 19.206 ksi + 8.382 ksi + 3.065 ksi + (1.5)3.881 ksi) \]

\[ = (13.237 in^2)(177.88 ksi) = 2354.6 kip \]

Stress in girder due to effective prestress

\[ f_{ps} = \frac{P}{A} + \frac{P_{ps}}{S} \]

\[ f_t = \frac{-2354.6 kip}{923.531 in^2} + \frac{(-2354.6 kip)(31.477 in)}{-19152.5 in^3} = 1.327 ksi \]

Stress due to loading

\[ f = \frac{0.5(M_{\text{girder}} + M_{\text{diaphragm}} + M_{\text{slab}} + M_{\text{haunch}})}{S} + \frac{0.5(M_{\text{barrier}} + M_{\text{overlay}})}{S_c} + \frac{0.5f_s}{S_c} + \frac{1.5M_{\text{lim}}}{S_c} \]

\[ f_t = \frac{0.5(3368.43 + 140.17 + 2465.61 + 760.28 k \cdot ft)(12 in)}{-19152.5 in^3} \left( \frac{12 in}{1 ft} \right) + \frac{0.5(732.12 + 739.63 k \cdot ft)(12 in)}{-49599.7 in^3} \left( \frac{12 in}{1 ft} \right) \]

\[ + (0.5)(-0.410 ksi) + \frac{1.5(1863.4 k \cdot ft)(12 in)}{-49599.7 in^3} \left( \frac{12 in}{1 ft} \right) = -3.282 ksi \]

Stress Demand

\[ f_t = 0.5(1.327 ksi) - 3.282 ksi = -2.619 ksi \]

Stress limit

\[ -0.4f'_c = -0.4(8.7 ksi) = -3.480 ksi \]

Stresses are within limits
4.7.3 Moment Capacity

4.7.3.1 Compute Nominal Moment Capacity at 0.5Lg.

Strength I limit state

\[ \text{Strength I} = 1.25DC + 1.5DW + 1.75(LL + IM) \]

\[ M_u = 1.25(3368.43 + 140.17 + 2465.61 + 760.28 + 732.12) + 1.50(739.63) + 1.75(3804.05) = 17099.8 \text{k} \cdot \text{ft} \]

\[ c = \frac{A_{ps}f_{pu} - \alpha_1f'_c(b - b_w)h_f}{\alpha_1f'_c\beta_1 b_w + kA_{ps}\frac{f_{pu}}{d_p}} \]

\[ k = 2 \left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 2 \left(1.04 - \frac{243}{270}\right) = 0.28 \]

\[ f_{ps} = f_{pu} \left(1 - \frac{c}{d_p}\right) \]

\[ a_t = 0.85 \]

\[ d_p = Y_t + e + t_s = 38.343\text{in} + 31.477\text{in} + 7\text{in} = 76.82\text{in} \]

\[ c = \frac{(13.237\text{in}^2)(270\text{ksi}) - 0.85(4\text{ksi})(84\text{in} - 6.125\text{in})(7\text{in})}{0.85(4\text{ksi})(0.85)(6.125\text{in}) + (0.28)(13.237\text{in}^2)\left(\frac{270\text{ksi}}{76.82\text{in}}\right)} = \frac{1720.565\text{kip}}{17.7 \frac{\text{k}}{\text{in}} + 13.0 \frac{\text{k}}{\text{in}}} = 56.04\text{in} \]

\[ f_{ps} = 270\text{ksi} \left(1 - \frac{0.28}{\frac{56.04\text{in}}{76.82\text{in}}}\right) = 214.85\text{ksi} \]

\[ a = \beta_1 c = 0.85(56.04\text{in}) = 47.63\text{in} \]

\[ M_n = A_{ps}f_{ps} \left(d_p - \frac{a}{2}\right) + \alpha_1f'_c(b - b_w)h_f \left(\frac{a}{2} - \frac{h_f}{2}\right) \]

\[ M_n = (13.237\text{in}^2)(214.85\text{ksi}) \left(76.82\text{in} - \frac{47.63\text{in}}{2}\right) + 0.85(4\text{ksi})(84\text{in} - 6.125\text{in})(7\text{in}) \left(\frac{47.63\text{in}}{2} - \frac{7\text{in}}{2}\right) \]

\[ = 188396.9 \text{k} \cdot \text{ft} \]

\[ d_t = 74\text{in} + 7\text{in} - 2\text{in} = 79\text{in} \]

\[ \varepsilon_t = 0.003 \frac{d_t - c}{c} = 0.003 \frac{79 - 56.04}{56.04} = 0.00123 \]

\[ 0.75 \leq \phi = 0.75 + 0.25 \left(\frac{\varepsilon_t - \varepsilon_{ct}}{\varepsilon_{tt} - \varepsilon_{ct}}\right) \leq 1.0 \]

\[ \phi = 0.75 + \frac{0.25(0.00123 - 0.002)}{0.005 - 0.002} = 0.686, \text{use 0.75} \]

\[ M_r = \phi M_n = 0.75(15699.7 \text{k} \cdot \text{ft}) = 11774.8\text{k} \cdot \text{ft} \]

\[ M_r < M_u \textbf{No Good} \]

The AASHTO method for computing moment capacity does not account for the large compression flange in the girder or the higher strength of the girder concrete. See Reference 7 for more information. PGSUPER uses strain compatibility analysis to compute the moment capacity.

Stress-strain relationship for prestressing strands:
\[ f_{ps} = \varepsilon_{ps} \left[ 877 + \frac{27,613}{1 + (112.4\varepsilon_{ps})^{7.36}} \right] \leq 270 ksi \]

Stress-strain relationship for concrete:

\[ f_c = f'_c \frac{n \left( \varepsilon_{cf} \right)}{n - 1 + \left( \frac{\varepsilon_{cf}}{\varepsilon'_c} \right)^{nk}} \]

where

\[ n = 0.8 + \frac{f'_c}{2500} \]
\[ k = 0.67 + \frac{f'_c}{9000} \]
\[ \text{if } \frac{\varepsilon_{cf}}{\varepsilon'_c} < 1.0, k = 1.0 \]

\[ E_c = \frac{40,000\sqrt{f'_c} + 1,000,000}{1000} \]
\[ \varepsilon'_c \times 1000 = \frac{f'_c}{E_c} \frac{n}{n - 1} \]

Effective prestress, \( f_{pe} = f_{pj} - \Delta f_{pt} = 202.5 ksi - 30.441 ksi = 172.059 ksi \)

Initial strain in prestressing strand, sometimes referred to as the decompression strain, is \( \varepsilon_{psi} = \frac{f_{pe}}{E_p} = \frac{172.059 ksi}{28500 ksi} = 6.037 \times 10^{-3} \)

Discretize the composite girder section into “slices”. Compute the strain at the centroid of each slice. The stress in the slice is determined from the stress-strain relationship for the slice material. Finally, compute the axial force and moment contribution for each slice. Sum the contribution of each slice to determine the capacity of the section.
### Table 4-1: Strain compatibility analysis details

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<thead>
<tr>
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<td>Ytop  (in)</td>
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<td>Ipc   (KSI)</td>
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<td>dM = (df)(Yeg) (kip-ft)</td>
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**Girder**

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<th>Slice</th>
<th>Piece</th>
<th>Area  (in²)</th>
<th>Ytop  (in)</th>
<th>Ybot  (in)</th>
<th>Yeg   (in)</th>
<th>Ipc   (KSI)</th>
<th>Initial Strain</th>
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<th>Total Strain</th>
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Precast, Prestressed Girder Design Example – PGSuper Training (11/16/2022)
### Precast, Prestressed Girder Design Example – PGSuper Training (11/16/2022)

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Resultant Force = $\sum(\delta F) = 0.00$ kip

Resultant Moment = $\sum(\delta M) = -20526.2$ kip-ft

Depth to neutral axis, $c = 15.124$ in
Compression Resultant, $C = -3464.24$ kip
Depth to Compression Resultant, $d_c = 5.720$ in
Tension Resultant, $T = 3464.24$ kip
Depth to Tension Resultant, $d_e = 76.822$ in
Net tensile strain, $\varepsilon_t = 12.7x10^{-3}$
Nominal Capacity, $M_n = 20526.2$ kip-ft
Moment Arm, $d_e - d_c = M_n/T = 71.102$ in

\[
0.75 \leq \phi = 0.75 + 0.25 \left( \frac{\varepsilon_t - \varepsilon_{ct}}{\varepsilon_{ct} - \varepsilon_{cl}} \right) \leq 1.0
\]

\[
\phi = 0.75 + 0.25(0.0127 - 0.002) \leq 1.64, \text{ use } 1.0
\]

\[
M_r = 1.0(20526.2)k \cdot ft \geq M_u = 17099.8k \cdot ft \text{ OK}
\]

The Moment Capacity Detail report from PGSuper provides the details of the moment capacity strain compatibility analysis.

The total strain in the reinforcement is the sum of the net tensile strain $= 12.7x10^{-3}$ and the initial strain $6.037x10^{-3}$. The total strain is $18.7x10^{-3} = 1.87\%$. This is less than the minimum required elongation of $3.5\%$ for A416 strand.

#### 4.7.3.2 Minimum Reinforcement and the Cracking Moment

To insure there is sufficient reinforcement in the section to achieve ductile behavior, a minimum amount of reinforcement is required. The minimum reinforcement is such that any section in the girder shall have adequate prestressed reinforcement to develop a factored flexural resistance, $M_r$, which is at least the lesser of the cracking strength or $133\%$ of the ultimate moment. (LRFD 5.6.3.3)

\[
M_{r_{\text{min}}} = \text{lesser of } \left( \frac{M_{cr}}{1.33M_u} \right)
\]

The cracking moment is

\[
M_{cr} = \gamma_3 \left[ (\gamma_1f_r + \gamma_2f_{\text{cpe}})S_c - M_{dnc} \left( \frac{S_c}{S_b} - 1 \right) \right]
\]

where:

- $f_r$ = Modulus of rupture
- $f_{\text{cpe}}$ = Compressive stress due to prestressing at the bottom of the girder
- $S_c$ = Bottom section modulus of the composite section
4.7.3.2.1 Compute cracking moment at 0.5Lg.

\[
\begin{align*}
M_{cr} &= \gamma_1 \cdot M_{dg} + \gamma_2 \cdot M_{dc} + \gamma_3 \cdot M_{ck} \\
&= 1.6 \cdot (0.708 ksi) + (5.947 ksi) + 1.0 = 14707.1 k \cdot ft
\end{align*}
\]

4.7.3.2.2 Evaluate Minimum Reinforcement Requirement

\[
M_r = \frac{1}{k} \cdot f_p \cdot A_s \cdot f_s \geq M_{cr}
\]

The splitting zone is \( \frac{h}{4} = 1.542 ft \).
Splitting resistance is \( P_r = f_s A_s \)

The required area of splitting reinforcement is

\[
103.44\,\text{kip} = (20\,\text{ksi})A_s \\
A_s = 5.172\,\text{in}^2 \\
5.172\,\text{in}^2 / 1.542\,\text{ft} = 3.62\,\text{in}^2 / \text{ft}
\]

The standard stirrup layout has 2-legs #5 at 3.5” for the end 1’-9” of the beam.

\[
2(0.31\,\text{in}^2) / (3.5\,\text{in}) = 2.126\,\text{in}^2 / \text{ft}
\]

Try 2-legs #5 at 2.5”.

\[
2(0.31\,\text{in}^2) / (2.5\,\text{in}) = 2.976\,\text{in}^2 / \text{ft}
\]

WSDOT BDM 5.6.2F permits the total splitting reinforcement to extend beyond H/4 at a spacing not greater than 2.5”.

\[
5.172\,\text{in}^2 = 2.976\,\text{in}^2 X \\
X = 1.74\,\text{ft}
\]

Use 2-legs #5 at 2.5” for 1’-9”

PGSuper does not currently apply the provisions of WSDOT BDM 5.6.2F.

4.9 Check Confinement Zone Reinforcement

For the distance of \( 1.5d \) from the ends of the girder, reinforcement shall be placed to confine the prestressing steel in the bottom flange (LRFD 5.9.4.4.2). The reinforcement shall not be less than #3 deformed bars with spacing not exceeding 6 in.

The length of the confinement zone is \( 1.5d = 1.5(74\,\text{in}) = 111\,\text{in} = 9.25\,\text{ft} \).

Provide #3 bars spaced at 6” for the end 9.25ft of the girder.

5 Shear Design

Ensure the girder has sufficient capacity to resist shear in the Strength I limit state. Verify that shear reinforcement is adequately detailed.

These computations and checks demonstrate shear design at the critical section (LRFD 5.7.3.2 and 5.7.3.3). A complete design will also evaluate shear at locations where abrupt changes to the shear force diagram occur and at changes in reinforcement size and spacing.

5.1 Locate Critical Section for Shear

The critical section for shear is located at \( d_v \), from the face of support where \( d_v \) is determined based on critical section. For purposes of design, the ultimate shear between the support and the critical section is equal to the shear at the critical section.

Determining the location of the critical section can be challenging because \( d_v \) varies with position along the girder. To find the critical section plot \( d_v \), along the length of the girder and draw a 45° line from the face of support towards the center of the girder. The intersection of the 45° line and the \( d_v \) curve is the location of the critical section. Figure 5-1 illustrates this technique.
For this girder, the critical section is located 5.782 ft from the face of support. The table that follows show the details for finding the critical sections.

**Table 5-1: Critical Section Calculation Details for Abutment 1**

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<tr>
<th>Location from Left Support (ft)</th>
<th>Assumed C.S. Location (in)</th>
<th>d_v (in)</th>
<th>CS Intersects?</th>
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</tr>
<tr>
<td>6.282</td>
<td>69.384</td>
<td>69.384</td>
<td>*Yes</td>
</tr>
<tr>
<td>6.542</td>
<td>72.498</td>
<td>69.384</td>
<td>No</td>
</tr>
<tr>
<td>(H) 6.667</td>
<td>74.000</td>
<td>69.384</td>
<td>No</td>
</tr>
<tr>
<td>(1.5H) 9.750</td>
<td>111.000</td>
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<tr>
<td>11.958</td>
<td>137.498</td>
<td>69.384</td>
<td>No</td>
</tr>
<tr>
<td>(SZB) 12.170</td>
<td>140.037</td>
<td>69.384</td>
<td>No</td>
</tr>
<tr>
<td>(SZB) 12.920</td>
<td>149.037</td>
<td>69.384</td>
<td>No</td>
</tr>
<tr>
<td>14.591</td>
<td>169.092</td>
<td>69.384</td>
<td>No</td>
</tr>
</tbody>
</table>

* - Intersection values are linearly interpolated

5.2 Check Ultimate Shear Capacity

5.2.1 Compute Nominal Shear Resistance

The nominal shear resistance, $V_n$, is the lesser of:

$$V_n = V_c + V_p + V_s$$

$$V_n = 0.25 f'_c b_w d_v + V_p$$

for which
\[ V_c = 0.0316 \beta \sqrt{f'_c b_v d_v} \]
\[ V_s = \frac{A_v f_d d_v \cot \theta}{s} \]

where

- \( b_v \) = Effective web width taken as the minimum web width within the depth \( d_v \).
- \( d_v \) = Effective shear depth
- \( s \) = Stirrup spacing
- \( \beta \) = Factor indicating ability of diagonally cracked concrete to transmit tension
- \( \theta \) = Angle of inclination of diagonal compressive stresses
- \( A_v \) = Area of shear reinforcement within a distance \( s \)
- \( V_p \) = Component in the direction of the applied shear of the effective prestressing force, positive if resisting the applied shear.

### 5.2.1 Determination of \( \beta \) and \( \theta \)

#### Step 1: Determine \( b_v \)

\( b_v \) is the effective web width. For this girder \( b_v = 6.125 \text{in} \).

#### Step 2: Determine \( d_v \)

\( d_v \) is the distance measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (internal moment arm), but it need not be taken less than the greater of \( 0.9d_e \) or \( 0.72h \).

From a flexural capacity analysis at the critical section the Moment Arm = 59.470 in, \( d_e = 77.094 \text{in} \), and \( h = 81 \text{in} \).

\[ d_v = \text{greatest of} \left\{ \begin{array}{l} \text{Moment Arm} = 59.470 \text{in} \\ 0.9d_e = 0.9(77.094 \text{in}) = 69.384 \text{in} \\ 0.72h = 0.72(81 \text{in}) = 58.32 \text{in} \end{array} \right\} \]

#### Step 3: Compute stress in prestressing steel when the stress in the surrounding concrete is 0.0 ksi

\[ f_{ps} = 0.70 f_{pu} = 189 \text{ksi} \]

#### Step 4: Compute the longitudinal strain on the flexural tension side of the beam

\[ \varepsilon_s = \left( \frac{\mid M_u \mid + 0.5N_u \mid V_{V_p} \mid - A_p f_{ps}}{E_s A_s + E_p A_{ps} + E_c A_{ct}} \right) \text{for } \varepsilon_s < 0 \]

At the critical section the flexural tension side is the bottom half of the beam. The area of concrete and area of steel on the flexural tension side are identified in Figure 5-2.
The longitudinal strain parameters are:

\[ M_u = 2603.98 \text{ k \cdot ft} \]
\[ N_u = 0 \text{ kip} \]
\[ V_u = 413.73 \text{ kip} \]
\[ V_p = 43.18 \text{ kip} \]
\[ |V_u - V_p| = 370.54 \text{ kip} \]
\[ d_o = 69.384 \text{ in} \]
\[ A_s = 0 \text{ in}^2 \]
\[ E_s = 29000 \text{ ksi} \]
\[ A_{ps} = 9.143 \text{ in}^2 \]
\[ E_{ps} = 28500 \text{ ksi} \]
\[ A_{ct} = 507.406 \text{ in}^2 \]
Precast, Prestressed Girder Design Example – PGSuper Training (11/16/2022)

\[ E_c = 5886.891 ksi \]

\[ \varepsilon_s = \frac{\left( \frac{1263.98 kftl}{69.3 ksi} \right) \cdot 0.5 \text{ksi} + 370.54 kip \right)}{29000 ksi (0 \text{in}^2) + (28500 ksi) (9.143 \text{in}^2) + (5886.891 ksi) (507.406 \text{in}^2)} = -0.279 \times 10^{-3} < 0 \]

**Step 5: Compute \( \beta \) and \( \theta \)**

\[ \beta = \frac{4.8 (1 + 750 \varepsilon_s)}{(1 + 750)(−0.279 \times 10^{-3})} = 6.07 \]

\[ \theta = 29 + 3500 \varepsilon_s = 29 + (3500)(−0.279 \times 10^{-3}) = 28.02° \]

5.2.1.2 **Compute Shear Capacity of Concrete**

\[ V_c = 0.0316 \beta \lambda \sqrt{f_y} b_n d_v = 0.0316 (6.07)(1.0) \sqrt{8.7 ksi (6.125 in)} (69.384 in) = 240.51 kip \]

5.2.1.3 **Compute Shear Capacity of Transverse Reinforcement**

For #5 stirrups at 6”, \( A_v = 0.62 \text{ in}^2 \).

\[ V_v = A_v f_y d_v \cot \theta = \frac{(0.62 \text{ in}^2) (60 \text{ ksi}) (69.384 \text{ in}) \cot 28.02°}{6 \text{ in}} (1.0) = 808.29 kip \]

5.2.1.4 **Compute Nominal Shear Capacity of Section**

\[ V_n = V_c + V_v + V_t = 240.51 kip + 43.18 kip + 808.29 kip = 1091.99 kip \]

\[ V_t = 0.25 f'_c b_n d_v + V_p = 0.25 (8.7 ksi) (6.125 in) (69.384 in) + 43.18 kip = 967.51 kip \]

\[ V_t = \phi V_n = 0.9 (967.51 kip) = 870.76 kip \]

5.2.1.5 **Check Ultimate Shear Capacity**

\[ V_u = 413.73 kip \leq V_t = 870.76 \text{ kip} \]

**OK**

Repeat these calculations at all locations where stirrup size or spacing changes or where the applied shear abruptly changes.

5.2.2 **Check Requirement for Transverse Reinforcement**

Transverse reinforcement is required when \( V_u > 0.5 \phi (V_c + V_p) \). (LRFD 5.8.2.4)

\[ 0.5 \phi (V_c + V_p) = 0.5 (0.9) (240.51 kip + 43.18 kip) = 127.66 kip < 413.73 kip \]

\( V_u \) exceeds the limiting value; therefore, transverse reinforcement is required at this section. Transverse reinforcement is provided. **OK**

5.2.3 **Check Minimum Transverse Reinforcement**

Where transverse reinforcement is required, as specified in LRFD 5.8.2.4, the area of steel shall not be less than \( A_{vo min} = 0.0316 \sqrt{f'_c b_n d_v} = 0.0316 \sqrt{8.7 ksi (6.125 in) (60 ksi)} = 0.057 \text{ in}^2 < 0.62 \text{ in}^2 \)

**OK**

This can also be represented as \( A_{vo min} = 0.0316 \sqrt{f'_c b_n d_v} = 0.0316 \sqrt{8.7 ksi (6.125 in) (60 ksi)} = 0.0095 \frac{\text{in}^2}{\text{in}} = 0.114 \frac{\text{in}^2}{f't'} \).

5.2.4 **Check Maximum Spacing of Transverse Reinforcement**

The spacing of the transverse reinforcement shall not exceed the following:

- If \( v_v < 0.125 f'_c \) then \( s \leq 0.8 d_v \leq 24 \text{ in} \)
• If $v_u \geq 0.125f'_c$ then $s \leq 0.4d_v \leq 12$ in

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_d d_v} = \frac{|413.73 - 0.9(43.18)|}{0.9(6.125)(69.384)} = 0.98 \text{ ksi}$$

$$0.125f'_c = 0.125(8.7 \text{ ksi}) = 1.09 \text{ ksi}$$

$$v_u = 0.98 \text{ ksi} < 0.125f'_c = 1.09 \text{ ksi}$$

$$s_{max} = 0.8d_v = 0.8(69.384 \text{ in}) = 55.5 \text{ in} \leq 24 \text{ in (18 in per WSDOT)} \rightarrow s_{max} = 18 \text{ in}$$

The actual spacing is 6.0 in. **OK**

---

Per WSDOT BDM 5.2.2.B.4, the maximum spacing of transverse reinforcement is limited to 18 inches

### 5.3 Check Longitudinal Reinforcement for Shear

At each section, the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy:

$$A_s f_y + A_{ps} f_{ps} \geq \left[ \frac{M_u}{d_v \phi_f} + 0.5 \frac{N_u}{\phi_a} + \left( \frac{V_u}{\phi_v} - V_p \right) \cot \theta \right]$$

At the inside edge of the bearing area of simple end supports to the section of critical shear, the longitudinal reinforcement on the flexural tension side of the member shall satisfy:

$$A_s f_y + A_{ps} f_{ps} \geq \left( \frac{V_u}{\phi_v} - 0.5V_s - V_p \right) \cot \theta$$

At the critical section, all the harped strands are above the mid-height of the girder. The harped strands are not on the flexural tension side (See LRFD Figure 5.7.3.4.2-2)

$$A_{ps} = (44)(0.217in^2) = 9.548in^2$$

From the moment capacity analysis, $f_{ps,avg} = 126.855\text{ksi}$ which takes into account the lack of full development per LRFD 5.9.4.3.2. Do not apply the reduction again in these calculations.

$$d_v = 69.382\text{in}$$
$$V_u = 413.73\text{kip}$$
$$V_s = 459.70\text{kip}$$
$$V_p = 43.18\text{kip}$$
$$\theta = 28.02^\circ$$

$$\left( \frac{V_u}{\phi_v} - 0.5V_s - V_p \right) \cot \theta = \left( \frac{413.73\text{kip}}{0.9} - 0.5(459.7\text{kip}) - 43.18\text{kip} \right) \cot 28.02^\circ = 350.7\text{kip}$$

$$A_{ps} f_{ps} = (9.548in^2)(126.885\text{ksi}) = 1211.5\text{kip}$$

$$1211.5\text{kip} \geq 350.7\text{kip} \quad \text{OK}$$

### 5.4 Check Horizontal Interface Shear

This entire design assumes that the slab and girder work together to form a composite section. Verify the slab-girder interface has adequate capacity to develop this composite action.
5.4.1 Check Nominal Capacity

The critical section for shear location is used to demonstrate these calculations. A complete design will verify the slab-girder interface capacity at various sections along the girder.

5.4.1.1 Compute Nominal Capacity

The nominal shear resistance at the slab-girder interface is

\[ V_{ni} = cA_{cv} + \mu A_{vf}f_y + P_c \]

\[ \leq \min \left( \frac{K_1f'_cA_{cv}}{K_2A_{cv}} \right) \]

where

- \( V_{ni} \) = Nominal shear resistance (kip)
- \( A_{cv} \) = Area of concrete engaged in shear transfer (in²)
- \( A_{vf} \) = Area of shear reinforcement crossing the shear plane (in²)
- \( f_y \) = Yield strength of reinforcement (ksi)
- \( c \) = Cohesion factor
- \( \mu \) = Friction factor
- \( P_c \) = Permanent net compressive force normal to the shear plane, or 0.0 kip if tensile (kip)
- \( f'_c \) = Specified 28-day strength of the weaker concrete (ksi)
- \( K_1 = 0.3 \)
- \( K_2 = 1.8 \)

The top flange of the girder, which is a roughened surface, supports the deck slab. For this situation \( c = 0.280 \) ksi and \( \mu = 1.0 \).

The area of concrete engaged in the shear transfer: \( A_{cv} = b_{vl}L_{vl} = (49 \text{ in}) \left( \frac{1 \text{ m}}{39.37 \text{ in}} \right) = 49 \text{ in}^2 \text{ / in} \).

The area of shear reinforcement consists of the stirrups extending from the web into the slab (#5 @ 3.5 in): \( A_{vf} = \frac{0.62 \text{ in}^2}{3.5 \text{ in}} = 0.177 \frac{\text{in}^2}{\text{in}} = 2.126 \frac{\text{in}^2}{\text{ft}} \).

It is conservative to take \( P_c = 0 \) klf.

\[ V_{ni} = cA_{cv} + \mu A_{vf}f_y + P_c = (0.280 \text{ ksi}) \left( \frac{49 \text{ in}^2}{39.37 \text{ in}} \right) + 1.0 \left( \frac{2.126 \text{ in}^2}{\text{ft}} \right) (60 \text{ ksi}) + 0 \text{ klf} \]

\[ = 292.183 \text{ kip/ft} \]

\[ K_1f'_cA_{cv} = 0.3(4\text{ ksi}) \left( \frac{49 \text{ in}^2}{39.37 \text{ in}} \right) = 705.6 \text{ kip/ft} \]

\[ K_2A_{cv} = 1.8 \left( \frac{49 \text{ in}^2}{39.37 \text{ in}} \right) = 1058.4 \text{ kip/ft} \]

\[ V_{ni} = 292.183 \text{ kip/ft} \]

\[ V_r = 0.9(292.183 \text{ kip/ft}) = 262.965 \text{ kip/ft} \]

5.4.1.2 Compute Demand

The factored interface shear stress for a concrete girder/slab bridge may be determined as \( V_{ud} = \frac{V_u}{b_{vl}d_{vl}} \). The factored interface shear force for a concrete girder/slab bridge may be determined as \( V_{ul} = v_{ul}A_{cv} \). Substituting Equation 5.8.4.2-1 into 5.8.4.2-2 the interface shear force is \( V_{uh} = \frac{V_u}{d_{vl}} \).

At the critical section, \( V_u = 413.73 \text{ kip} \).

\[ V_{uh} = \frac{V_u Q}{l} = \frac{(413.73\text{ kip})(12199.9\text{ in}^3)}{(1246570.6\text{ in}^4)} = 48.589 \text{ kip/ft} \]

\[ V_{uh} \leq V_r \]
5.4.2 Check Minimum Reinforcement

The LRFD specification requires a minimum amount of shear reinforcement in the slab-girder interface. Check to make sure this requirement is satisfied.

The cross-sectional area, $A_{vf}$, of the reinforcement per unit length should not be less than $0.05 s v e y$.

For a cast-in-place concrete slab on clean concrete girder surface free of laitance:

- The minimum interface shear reinforcement need not exceed the lessor of the amount determined using Eqn. 5.8.4.4-1 and the amount needed to resist $1.33 V u r r \phi$ as determined using Eqn 5.8.4.1-3
- The minimum reinforcement provisions shall be waived for girder/slab interfaces with surface roughened to an amplitude of 0.25 in where the factored interface shear stress, $v_{ul}$ of Eqn 5.8.4.2-1 is less than 0.210 ksi, and all vertical (transverse) shear reinforcement required by the provisions of Article 5.8.1.1 is extended across the interface and adequately anchored into the slab.

$$v_{ul} = V_{ul} = \frac{48.589 {kip}}{49 {in^2}} = 0.083 \frac{ksi}{ft} < 0.210 \frac{ksi}{ft}$$

This requirement is waived.

The maximum allowable spacing of the transverse reinforcement is 24.0 in. The actual spacing at this section is 3.5 in. The maximum spacing along the length of the girder is 18.0 in. OK

6 Check the “A” Dimension

The slab offset, or “A” dimension in WSDOT terminology, was assumed to be 12.5 in. Verify the haunch is large enough to accommodate the camber, but not too large that the girder has to carry unnecessary dead load. For such a large girder, an extra inch of concrete over the top flange can add up to a considerable amount of weight.

The haunch depth is to be such that at the mid-span the distance between the bottom of the slab and the top of the girder is equal to the slab fillet dimension, 0.75 in. Account for geometric effects due to the roadway and camber. The haunch depth at the bearing is $A_{haunch} = A_{slab+fillet} + A_{profile \text{ effect}} + A_{girder \text{ orientation \text{ effect}}} + A_{excess \text{ camber}}$.

6.1 Slab and Fillet

The slab and fillet is the gross slab depth plus the fillet dimension. If the actual camber is exactly equal to the predicted value, and all deflections are as predicted, the top of the girder will be exactly $t_{fillet}$ below the bottom of the deck at its closest point.
\[ A_{slab+fillet} = 7.5 \text{ in} + 0.75 \text{ in} = 8.25 \text{ in} \]

### 6.2 Profile Effect

PGSuper uses a general approach to determine the profile effect. Draw a chord line from the point where a vertical line passing through the CL Bearings intersect the deck. Then the profile effect is the maximum difference in elevation between this chord line and the roadway surface.
The entire bridge is within the limits of the horizontal and vertical curves. Also, there are no superelevation transitions within the limits of the bridge. Therefore, the simplified method of computing the profile effect can be used as described in WSDOT BDM Appendix 5-B1.

6.2.1 Vertical Curve

Figure 6-3: Vertical Curve Effect

\[ A_{vc} = \frac{1.5(g_2 - g_1)L_g^2}{100L_{vc}} \text{ (in)} = \frac{1.5(-1.5\% - (-2\%))(124.58\text{ft})^2}{100(600\text{ft})} = 0.194 \text{ in} \]
6.2.2 Horizontal Curve

\[ A_{hc} = \frac{1.55^2 m}{R} (in) = \frac{1.5(827.42 - 702.71)^2 (0.04 \text{ ft})}{6000 \text{ ft}} = 0.156\text{in} \]

6.2.3 Profile Effect

\[ A_{profile} = A_{vc} + A_{hc} = 0.194\text{in} + 0.156\text{in} = 0.350\text{in} \]

6.3 Girder Orientation Effect

The girder orientation effect accounts for the crown slope and the orientation of the girder. \( A_{girder\ orientation\ effect} = \frac{m \cdot w_{tf}}{2} \).
The excess camber is the camber that remains in the girder after all of the loads are applied.

The graphic below illustrates how the girder deflects over time.
Assume time-dependent deformations end after deck casting

\[ \Delta_{girder} = \text{deflection due to girder self} \]
\[ \Delta_{ps} = \text{deflection due to permanent prestressing, based on inplace span length} \]
\[ \Delta_{\text{creep1}} = \psi(t_e, t_i)(\Delta_{\text{girder}} + \Delta_{ps}) \]
\[ \Delta_{\text{diaphragm}} = \text{deflection due to diaphragm self weight} \]
\[ \delta_{\text{girder}} = \text{incremental girder deflection due to change in support location between storage and erection} \]
\[ \Delta_{\text{creep2}} = [\psi(t_d, t_f) - \psi(t_e, t_f)](\Delta_{\text{girder}} + \Delta_{ps}) + \psi(t_d, t_e)(\Delta_{\text{tpr}} + \Delta_{\text{diaphragm}} + \delta_{\text{girder}}) \]
\[ \Delta_{\text{tpr}} = \text{temporary prestressing strand removal} \]
\[ \Delta_{\text{deck}} = \text{deflection due to deck self weight} \]
\[ \Delta_{\text{haunch}} = \text{deflection due to haunch self weight} \]
\[ \Delta_{\text{barrier}} = \text{deflection due to traffic barrier self weight} \]
\[ \Delta_{\text{excess}} = \text{excess camber} \]
\[ \Delta_1 = (\Delta_{\text{girder}} + \Delta_{ps}) \]
\[ \Delta_2 = \Delta_1 + \Delta_{\text{creep1}} \]
\[ \Delta_3 = \Delta_2 + \Delta_{\text{tpr}} + \Delta_{\text{diaphragm}} \]
\[ \Delta_4 = \Delta_3 + \Delta_{\text{creep2}} \]
\[ \Delta_5 = \Delta_4 + \Delta_{\text{deck}} + \Delta_{\text{haunch}} \]
\[ \Delta_6 = \Delta_{\text{excess}} = \Delta_5 + \Delta_{\text{barrier}} \]
6.4.1 Compute Creep Coefficients

The creep coefficients for release until erection and deck casting are computed above.

Prestress release until erection \( \psi(t_h = 90, t_i = 1) = \psi(t_e = 90, t_i = 1) = 0.852 \)

Prestress release until deck casting \( \psi(t_d = 120, t_e = 1) = 0.912 \)

Compute creep coefficient for erection to deck casting

\[
f'_e = 7.4 \text{ ksi} \\

k_f = \frac{5}{(1 + 7.4)} = 0.610 \\

k_{td} = \frac{(120 - 90)}{12 \left( \frac{100 - 4(7.4)}{7.4 + 20} \right) + (120 - 90)} = 0.488 \\

\psi(t_d = 120, t_e = 90) = 1.9(1.04)(0.96)(0.610)(0.488)(90)^{-0.118} = 0.331
\]

6.4.2 Compute Deflections

Girder Deflection, for the erected girder

\[
\Delta_g = \frac{5wL^4}{384Ec_{il}l_x} = \frac{5(-1.058kklf)(159.578ft)^4}{384(5530.5ksi)(734356.0in^4)} \left( \frac{1728in^3}{1ft^3} \right) = -3.802\text{in}
\]

Prestress Deflection, \( \Delta_{ps} = 7.161\text{in} \). This is the deflection measured relative to the ends of the girder. The deflection at the CL Bearing based on the release datum is \( \Delta_{psbrg} = 0.263\text{in} \). The prestress deflection measured relative to the bearings is \( \Delta_{ps} = 7.161\text{in} - 0.263\text{in} = 6.897\text{in} \)

Creep Deflection during Storage, \( \Delta_{creep1} = 0.852(6.897\text{in} - 3.802\text{in}) = 2.639\text{in} \)

Diaphragm Deflection, \( \Delta_{diaphragm} = -0.141\text{in} \)

Slab Deflection, \( \Delta_{slab} = -2.610\text{in} \)

Haunch Deflection, \( \Delta_{haunch} = -0.808\text{in} \)

Creep Deflection between diaphragm and deck casting, \( \Delta_{creep2} = (0.912 - 0.852)(6.897\text{in} - 3.802\text{in}) + (0.331)(-0.141\text{in}) = 0.347\text{in} \). Since the girder is stored at the locations of the bearings \( \delta_{girder} = 0 \text{in} \)

Traffic Barrier Deflection, \( \Delta_{barrier} = -0.457\text{in} \)

Because the girder concrete has gained strength, not all of the deflection due to the installation of the temporary top strands is recoverable. The elastic rebound due to temporary strand removal is

\[
P = 162.93\text{kip}
\]

\[
\Delta_{tpsr} = \frac{Pe^2}{8EIx} = \frac{(162.93kip)(-36.343\text{in})(162.995\text{ft})^2}{8(5886.891ksi)(734356.0in^4)} \left( \frac{144in^2}{1ft^2} \right) = 0.655\text{in}
\]

Measured relative to the bearings, \( \Delta_{tpsr} = 0.628\text{in} \)

\[
\Delta_1 = -3.802\text{in} + 6.897\text{in} = 3.096\text{in} \\
\Delta_2 = 3.096\text{in} + 2.639\text{in} = 5.735\text{in} \\
\Delta_3 = 5.735 - 0.141\text{in} + 0.628\text{in} = 6.222\text{in} \\
\Delta_4 = 6.222\text{in} + 0.347\text{in} = 6.569\text{in} = D_{120} \\
\Delta_5 = 6.569\text{in} - 2.610\text{in} - 0.808\text{in} = 3.150\text{in} \\
\Delta_6 = 3.150 - 0.457\text{in} = 2.693\text{in} = \Delta_{excess}
\]
The screed camber, $C = \Delta_4 - \Delta_6 = 3.876 \text{ in}$

### 6.5 Check Required Haunch

The required haunch is

$$A_{\text{haunch}} = A_{\text{slab+fllet}} + A_{\text{top flange effect}} + A_{\text{profile effect}} + A_{\text{excess camber}}$$

$$A_{\text{haunch}} = 8.25\text{in} + 0.98\text{in} + 0.35\text{in} + 2.693\text{in} = 12.27\text{ in}$$

The provided haunch is 12.5 in. **OK**

### 6.6 Compute Lower Bound Camber at 40 days

#### 6.6.1 Creep Coefficients

Creep coefficients are computed the same as before, assuming erection at 10 days and deck casting at 40 days.

$$\psi_b(t_d = 10, t_i = 1) = 0.257$$

$$\psi_b(t_f = 40, t_i = 1) = 0.638$$

$$\psi_b(t_d = 40, t_e = 10) = 0.429$$

#### 6.6.2 Compute Deflections

Creep Deflection during Storage,

$$\Delta_{\text{creep1}} = 0.638(6.897\text{in} - 3.802\text{in}) = 0.795\text{in}$$

Creep Deflection between diaphragm and deck casting,

$$\Delta_{\text{creep2}} = (0.638 - 0.257)(6.897\text{in} - 3.802\text{in}) + (0.429)(-0.141\text{in}) = 1.391\text{in}$$

$$\Delta_1 = 3.096\text{in}$$

$$\Delta_2 = 3.891\text{in}$$

$$\Delta_3 = 4.378\text{in}$$

$$\Delta_4 = 5.769\text{in} = D_{40}$$

This is an upper bound value for camber at 40 days. There is a ±25% natural variation in camber from the mean value. Therefore, lower bound camber at 40 days = $0.5D_{40} = 2.884\text{ in}$

### 6.7 Check for Possible Girder Sag

When the screed camber, $C$, exceeds the deflection at slab casting, $D$, the girder will have a net downward deflection, also known as sag. The sag condition is most likely to occur for rapidly constructed bridges.

Compare the screed camber to the average value of camber at 40 days to determine the potential for sag. The average value is

$$75\% \Delta_{40} = (0.75)(5.769\text{in}) = 4.327\text{in}$$

$$\Delta_{\text{excess}} = D - C$$

$$\Delta_5 = 5.769 - 2.610 - 0.808 = 2.351\text{in}$$

$$\Delta_e = 2.351 - 0.457 = 1.894\text{in} = \Delta_{\text{excess}}$$

$$C = \Delta_4 - \Delta_6 = 5.769 - 1.894 = 3.875\text{in}$$

$$C < 75\%\Delta_{40} \text{ OK}$$

### 7 Bearing Seat Elevations

From the PGSuper Bridge Geometry Report, the roadway surface elevations at the CL Bearing points for Girder A are:

- Abutment 1, Sta. 7+02.70, Offset 17.463ft L, Elev. 104.688ft
- Abutment 2, Sta. 8+61.82, Offset 17.457ft L, Elev. 101.748ft
The slope of the girder is \( \frac{104.688 \text{ ft} - 101.748 \text{ ft}}{576 \text{ ft}} = -0.0184 \text{ ft/ft} \)

The slope-adjusted height of the girder is \( 74 \text{ in} \left( \sqrt{(-0.0184)^2 + (1)^2} \right) = 74.013 \text{ in} \)

Deduct the sloped adjusted girder height and the slab offset from the roadway surface elevation to get the bottom of girder elevation.

Bottom of girder elevation at Abutment 1: \( \text{Elev} = 104.688 \text{ ft} - 74.013 \text{ in} \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) - 12.5 \text{ in} \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) = 97.479 \text{ ft} \)

Bottom of girder elevation at Abutment 2: \( \text{Elev} = 101.748 \text{ ft} - 74.013 \text{ in} \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) - 12.5 \text{ in} \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) = 94.538 \text{ ft} \)

After designing the bearings, add the bearing recess (typically \( \frac{1}{2}'' \)) and deduct the bearing depth from the bottom of girder elevation to get the bearing seat elevation.

8 Design Summary

\[ f'_{ct} = 7.2 \text{ ksi} \]
\[ f'_{c} = 8.7 \text{ ksi} \]

# Permanent Strands = 61 (44 Straight, 17 Harped)

# Temporary Top Strands = 4

Slab Offset ("A"Dimension) = 12.5 in

Pick Point = 8'3"

Bunk Point = 13'8"

Haul Truck Configuration = 40,000 kip-in/ rad with 72" wheel base (HT40 – 72)

Use stirrups per WSDOT standard

9 Load Rating

The bridge opens for traffic without the future overlay installed. For this reason, take the DW force effects associated with the overlay as zero. Installing the overlay necessitates updating the load rating analysis.

9.1 Inventory Rating

9.1.1 Moment

\[ RF = \left( \phi_c \phi_s \phi_n KM_n - \gamma_{DC} M_{DC} - \gamma_{DW} M_{DW} \right) \frac{\gamma_{LL} M_{LLIM}}{M_{LL}} \]

\[ \phi_c \phi_s \geq 0.85 \]

\[ K = \frac{M_r}{M_{min}} \leq 1.0 \]

At 0.5L

\[ \phi_c = \phi_s = \phi_n = 1.0 \]

\[ M_n = 20526.2 \text{ k} \cdot \text{ft} \]

\[ M_{DC} = 7466.6 \text{ k} \cdot \text{ft} \]

\[ M_{DW} = 0.0 \text{ k} \cdot \text{ft} \]

\[ M_{LLIM} = 3804.05 \text{ k} \cdot \text{ft/girder} \]
\[ M_{cr} = 14707.13k \cdot ft \]
\[ M_u = 17099.79k \cdot ft \]
\[ M_{min} = \min\left\{ \frac{M_{cr}}{1.33M_u} \right\} = 14707.13k \cdot ft \]
\[ K = \frac{(1.0)(20526.2k \cdot ft)}{14707.13k \cdot ft} = 1.39 \div 1.0 \]
\[ \gamma_D = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.75 \]
\[ RF = \frac{(1)(1)(1)(1)(20526.2k \cdot ft) - (1.25)(7466.6k \cdot ft) - (1.5)(0k \cdot ft)}{(1.75)(3804.05k \cdot ft)} = 1.68 \]

### 9.1.2 Shear

\[ RF = \frac{\phi_c \phi_s \phi_n V_n - \gamma_D V_{DC} - \gamma_{DW} V_{DW}}{\gamma_{LL} V_{LLIM}} \]

At 10.388ft and 142.908ft (location where stirrup spacing increases)

\[ \phi_c = \phi_s = 1.0, \phi_n = 0.9 \]
\[ V_n = 518.10kip \]
\[ V_{DC} = 147.84kip \]
\[ V_{DW} = 0.0k \]
\[ V_{LLIM} = 90.13 \frac{kip}{girder} \]
\[ \gamma_D = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.75 \]
\[ RF = \frac{(1)(1)(0.9)(518.10kip) - (1.25)(147.84kip) - (1.5)(0kip)}{(1.75)(90.13kip)} = 1.78 \]

### 9.1.3 Bending Stress – Service III limit state

\[ RF = \frac{f_R - \gamma_D f_{DC} - \gamma_{DW} f_{DW}}{\gamma_{LL} f_{LLIM}} \]

Per WSDOT BDM 13.2.4, use the AASHTO LRFD specifications tension stress limit and live load factor for load rating purposes.

\[ f_R = f_{limit} - f_{ps} = 0.19(1.0)\sqrt{f_c^*} - f_{ps} \]
\[ f_R = 0.19(1.0)\sqrt{8.7ksi} - (-6.221ksi) = 0.560ksi - (-6.221ksi) = 6.781ksi \]

Since we are using refined losses and elastic gains, the live load factor is 1.0 (See AASHTO LRFD Table 3.4.1-4).

\[ \gamma_{LL} = 1.0 \]
\[ RF = \frac{6.781ksi - (1.0)(4.268ksi) - 1.0(0ksi)}{(1.0)(1.789ksi)} = 1.40 \]

### 9.2 Operating Rating

#### 9.2.1 Moment

\[ RF = \frac{\phi_c \phi_s \phi_n K M_n - \gamma_D M_{DC} - \gamma_{DW} M_{DW}}{\gamma_{LL} M_{LLIM}} \]
\[ \phi_c \phi_s \geq 0.85 \]
\[ K = \frac{M_c}{M_{\text{min}}} \leq 1.0 \]

At 0.5L

\[ \phi_c = \phi_s = \phi_n = 1.0 \]
\[ M_n = 20526.2 \text{k} \cdot \text{ft} \]
\[ M_{DC} = 7466.6 \text{k} \cdot \text{ft} \]
\[ M_{DW} = 0.0 \text{k} \cdot \text{ft} \]
\[ M_{LLIM} = 3804.05 \text{k} \cdot \text{ft/girder} \]
\[ M_{cr} = 14707.13 \text{k} \cdot \text{ft} \]
\[ M_u = 17099.79 \text{k} \cdot \text{ft} \]
\[ M_{\text{min}} = \min \left\{ \frac{M_{cr}}{1.33M_u} \right\} = 14707.13 \text{k} \cdot \text{ft} \]
\[ K = \frac{(1.0)(20526.2 \text{k} \cdot \text{ft})}{14707.13 \text{k} \cdot \text{ft}} = 1.39 \div 1.0 \]
\[ \gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.35 \]
\[ RF = \frac{(1)(1)(1)(20526.2 \text{k} \cdot \text{ft}) - (1.25)(7466.6 \text{k} \cdot \text{ft}) - (1.5)(0 \text{k} \cdot \text{ft})}{(1.35)(3804.05 \text{k} \cdot \text{ft})} = 2.18 \]

9.2.2 Shear

\[ RF = \frac{\phi_c \phi_s \phi_n V_n - \gamma_{DC} V_{DC} - \gamma_{DW} V_{DW}}{\gamma_{LL} V_{LLIM}} \]

At 10.388ft and 142.908ft (location where stirrup spacing increases)

\[ \phi_c = \phi_s = 1.0, \phi_n = 0.9 \]
\[ V_n = 518.10 \text{kip} \]
\[ V_{DC} = 147.84 \text{kip} \]
\[ V_{DW} = 0.0 \text{kip} \]
\[ V_{LLIM} = 90.13 \text{kip/girder} \]
\[ \gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.35 \]
\[ RF = \frac{(1)(1)(0.9)(518.10 \text{kip}) - (1.25)(147.84 \text{kip}) - (1.5)(0 \text{kip})}{(1.35)(90.13 \text{kip})} = 2.38 \]

9.3 Legal Loads

Type 3, \( M_{LLIM} = 1404.25 \text{k} \cdot \text{ft} \)

Type 3S2, \( M_{LLIM} = 1826.63 \text{k} \cdot \text{ft} \)

Type 3-3, \( M_{LLIM} = 1931.27 \text{k} \cdot \text{ft} \)

Type 3-3 rating will govern so we will show calculations of the rating factors for this loading. The rating factor calculations for the other loadings will be similar. The rating factor calculations for NRL, EV2 and EV3 are similar.
9.3.1 Moment

\[ RF = \frac{\phi_e \phi_s \phi_n K M_n - \gamma_{DC} M_{DC} - \gamma_{DW} M_{DW}}{\gamma_{LL} M_{LLIM}} \]
\[ \phi_e \phi_s \geq 0.85 \]
\[ K = \frac{M_c}{M_{min}} \leq 1.0 \]

At 0.5L

\[ \phi_e = \phi_s = \phi_n = 1.0 \]
\[ M_n = 20526.2k \cdot ft \]
\[ M_{DC} = 7466.6k \cdot ft \]
\[ M_{DW} = 0.0k \cdot ft \]
\[ M_{LLIM} = 1931.27k \cdot ft \]
\[ M_c = 14707.13k \cdot ft \]
\[ M_u = 17099.79k \cdot ft \]
\[ M_{min} = \min \left\{ \frac{M_c}{1.33 M_u} = 14707.13k \cdot ft \right\} \]
\[ K = \frac{(1.0)(20526.2k \cdot ft)}{14707.13k \cdot ft} = 1.39 : 1.0 \]
\[ \gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.45 \]
\[ RF = \frac{(1)(1)(1)(1)(20526.2k \cdot ft) - (1.25)(7466.6k \cdot ft) - (1.5)(0k \cdot ft)}{(1.45)(1931.27k \cdot ft)} = 4.0 \]

9.3.2 Shear

\[ RF = \frac{\phi_e \phi_s \phi_n V_n - \gamma_{DC} V_{DC} - \gamma_{DW} V_{DW}}{\gamma_{LL} V_{LLIM}} \]

At 10.388ft and 142.908ft (location where stirrup spacing increases)

\[ \phi_e = \phi_s = 1.0, \phi_n = 0.9 \]
\[ V_n = 518.10kip \]
\[ V_{DC} = 147.84kip \]
\[ V_{DW} = 0.0k \]
\[ V_{LLIM} = 90.13k \cdot ft \]
\[ \gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.45 \]
\[ RF = \frac{(1)(1)(0.9)(518.10kip) - (1.25)(147.84kip) - (1.5)(0kip)}{(1.45)(90.13kip)} = 4.29 \]

9.3.3 Bending Stress – Service III limit state

In accordance with WSDOT BDM 13.1.1 Service III ratings for legal loads can be ignored at the discretion of the state Load Rating Engineer. Without a variance for this structure, the Service III rating factor will be computed.
RF = \frac{f_R - \gamma_{DC}f_{DC} - \gamma_{DW}f_{DW}}{\gamma_{LL} f_{LLIM}}

For load rating we use the AASHTO specified tension limit and live load factor

\[ f_R = f_{\text{limit}} - f_{ps} = 0.19\sqrt{\frac{f_{ps}}{f_{ps}}} \]

Before we can compute the stress in the girder due to the prestressing, we must compute the effective prestress accounting for the elastic gain for the Type 3-3 loading.

\[ \Delta f_{pLL} = \frac{E_p M_{LLIM} (Y_{EC} - Y_{bg} + e)}{I_c} \cdot \frac{28500 \text{ksi}}{5886.891 \text{ ksi}} \cdot \frac{1931.27k \cdot ft}{1243508.6 \text{ in}^4} \cdot \frac{(12 \text{ in})}{1 \text{ ft}} \]

\[ P = (13.237 \text{ in}^2)(172.059 \text{ ksi} + 4.032 \text{ ksi}) = 2330.92 \text{ kip} \]

\[ f_{ps} = \frac{-2330.92 \text{ kip}}{923.531 \text{ in}^2} + \frac{(-2330.92 \text{ kip})(31.477 \text{ in})}{20594.8 \text{ in}^3} = -6.086 \text{ ksi} \]

\[ f_R = 0.19(1.0)\sqrt{8.7 \text{ ksi}} = 0.560 \text{ ksi} - (-6.086 \text{ ksi}) = 6.647 \text{ ksi} \]

\[ \gamma_{LL} = 1.0 \]

\[ RF = \frac{6.647 \text{ ksi} - (1.0)(4.268 \text{ ksi}) - (1.0)(0 \text{ ksi})}{(1.0)(0.909 \text{ ksi})} = 2.62 \]

9.3.4 Special Hauling Vehicles (SHV)

WSDOT policy (BDM 13.2.19) requires bridges to be load rated for SHVs if the rating factor for Type 3, Type 3S2, or Type 3-3 are less than 1.35. The legal load rating factors exceed this threshold, so the additional load rating analysis is not required.

9.4 Permit Loads

WSDOT load rates bridges for two standard overload vehicles, Overload 1 (OL1) and Overload 2 (OL2). The vehicle details are presented in WSDOT BDM 13.1.5.

The load ratings for the permit loads are the same as the legal loads (with the obvious exception of the live load effects and load factors being different).

In accordance with WSDOT BDM 13.1.1 Service III ratings for permit loads can be ignored at the discretion of the state Load Rating Engineer. Without a variance for this structure, the Service III rating factor is computed.

The rating factors are summarized in Table 9-1.

Table 9-1: Load rating summary for permit vehicles.

<table>
<thead>
<tr>
<th>Permit Vehicle</th>
<th>Moment Rating</th>
<th>Shear Rating</th>
<th>Service III Stress Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>OL1</td>
<td>3.58</td>
<td>3.98</td>
<td>1.98</td>
</tr>
<tr>
<td>OL2</td>
<td>1.95</td>
<td>2.18</td>
<td>1.15</td>
</tr>
</tbody>
</table>

WSDOT also evaluates the optional reinforcement yielding check (MBE 6A.5.4.2.2b). The stress in the prestressing steel nearest the extreme tension fiber should not exceed 0.9\( f_y \). The analysis method used by PGSuper follows MBE A3.13.4.2b.

\[ f_R = 0.9 f_y = (0.9)(0.9) f_{pu} = (0.9)(0.9)(270 \text{ ksi}) = 218.7 \text{ ksi} \]

Moment beyond cracking

\[ M_{DCr} = \gamma_{DC} M_{DC} + \gamma_{DW} M_{DW} + \gamma_{LL} M_{LLIM} - M_{cr} \]
Unlike the other permit rating cases where the one loaded lane live load distribution factor is used (MBE 6A.4.5.4.2b), use the governing of one loaded lane and two or more loaded lanes for these calculations (MBE C6A.5.4.2.2b).

For OL1, \( M_{LLIM} = 2605.89 \text{k}\cdot\text{ft} \) per girder.

For OL2, \( M_{LLIM} = 4794.03 \text{k}\cdot\text{ft} \) per girder

\[
M_{bcr} = (1.0)(7466.6\text{k}\cdot\text{ft}) + (1.0)(0) + (1.0)(4794.03\text{k}\cdot\text{ft}) - 14707.13\text{k}\cdot\text{ft} = -2446.5\text{k}\cdot\text{ft}
\]

Because \( M_{bcr} < 0 \), the loads aren’t enough to cause cracking, so take \( M_{bcr} = 0.0 \text{k}\cdot\text{ft} \)

The additional stress transferred to the reinforcement due to cracking is

\[
f_{bcr} = \frac{E_s M_{bcr}(d_s - c)}{I_{cr}} = 0.0 \text{ksi}
\]

Compute the effective prestress

For OL1

\[
\Delta f_{p,LL} = \frac{E_p}{E_c} \frac{M_{LLIM}(Y_{bc} - Y_{bg} + e)}{I_c} = \frac{28500 \text{ksi}}{5886.891 \text{ksi}} \frac{(2065.89 \text{k}\cdot\text{ft})(48.867\text{in} - 35.657\text{in} + 31.477\text{in})}{12\text{in}} \frac{12\text{in}}{1\text{ft}} = 4.312 \text{ksi}
\]

\[
f_{pe} = 172.056 \text{ksi} + 4.312 \text{ksi} = 176.37 \text{ksi}
\]

\[
f_s = f_{pe} + f_{bcr} = 176.37 \text{ksi} + 0 \text{ksi} = 176.37 \text{ksi}
\]

For OL2

\[
\Delta f_{p,LL} = \frac{E_p}{E_c} \frac{M_{LLIM}(Y_{bc} - Y_{bg} + e)}{I_c} = \frac{28500 \text{ksi}}{5886.891 \text{ksi}} \frac{(4794.03 \text{k}\cdot\text{ft})(48.867\text{in} - 35.657\text{in} + 31.477\text{in})}{12\text{in}} \frac{12\text{in}}{1\text{ft}} = 10.01 \text{ksi}
\]

\[
f_{pe} = 172.056 \text{ksi} + 10.01 \text{ksi} = 182.07 \text{ksi}
\]

\[
f_s = 182.07 \text{ksi}
\]

Yield stress ratio

\[
SR = \frac{f_r}{f_s}
\]

OL1

\[
SR = \frac{218.7 \text{ksi}}{176.37 \text{ksi}} = 1.24
\]

OL2

\[
SR = \frac{218.7 \text{ksi}}{182.07 \text{ksi}} = 1.20
\]

10 Software

PGSuper is precast-prestressed girder design, analysis, and load rating software. PGSuper is part of the BridgeLink Bridge Engineering Application Suite jointly developed by the Washington State and Texas Departments of Transportation.

Download from [http://www.wsdot.wa.gov/eesc/bridge/software](http://www.wsdot.wa.gov/eesc/bridge/software)

This design example is based on version 7.0.
11 References


2. Brice, R., Khalegh, B., Seguirant, S., “Design optimization for fabrication of pretensioned concrete bridge girders: An example problem”, PCI JOURNAL, Prestressed Concrete Institute, Chicago, IL, Vol. 54, No. 4, Fall 2009, pp.73-111


4. PCI (Precast/Prestressed Concrete Institute). 2016. *Recommended Practice for Lateral Stability of Precast, Prestressed Concrete Bridge Girders*. CB-02-16-E. Chicago, IL: PCI


8. WSDOT, *Bridge Design Manual*, Washington State Department of Transportation