

Research Report
Research Project T9903, Task 53
Mobility Improvement

**ANALYSIS OF THE INITIAL APPLICATION
OF THE STATE OF WASHINGTON HIGHWAY
MOBILITY PROJECT RANKING PROCEDURE
AND RECOMMENDED REVISIONS
FOR THE UPCOMING BIENNIUM**

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LIST OF ABBREVIATIONS

B/C	-	Benefit-Cost Ratio
CALTRANS	-	California Department of Transportation
DOT	-	Department of Transportation
FTA	-	Federal Transit Administration
GP	-	Goal Programming (extension of linear programming)
HOV	-	High Occupancy Vehicle
ISTEA	-	Federal <i>Intermodal Surface Transportation Efficiency Act</i> (1991)
LOS	-	Level-of-Service
LP	-	Linear Programming
MPO	-	Metropolitan Planning Organization
MTC	-	Metropolitan Transportation Commission (San Francisco area)
NCHRP	-	National Cooperative Highway Research Program
SAW	-	Simple Additive Weighting (ranking method)
SR	-	State Road
TAC	-	Technical Advisory Committee
TIB	-	Transportation Improvement Board (Washington State)
TRAC	-	Washington State Transportation Research Center
TDM	-	Transportation Demand Management
TSM	-	Transportation System Management
TOPSIS	-	Technique for Order Preference by Similarity to Ideal Solution
UW	-	University of Washington
WSDOT	-	Washington State Department of Transportation
WSTC	-	Washington State Transportation Commission

EXECUTIVE SUMMARY

This study was prompted by the impending prioritization of the 1997-1999 biennium highway mobility projects for the Washington State Department of Transportation (WSDOT). The ranking procedure analyzed and refined in this report is based on a methodology called *Technique for Order Preference by Similarity to Ideal Solution* (Hwang and Yoon, 1981), more commonly known as TOPSIS, which is the final step in the current prioritization process for proposed Washington State highway mobility projects.

The prioritization process that was developed for state highway mobility projects, documented in the *Capacity Improvements Extension Task Report* (Reed et al. [2], 1995), was initially implemented to prioritize highway mobility projects for the 1995-1997 biennium. The major steps of the process are as follows:

1. Projects are submitted for statewide prioritization by each of the six regions in the State of Washington
2. Projects are screened—only those that are in the State System Plan, meet air quality conformity requirements, and address existing deficiencies are considered for prioritization.
3. Project evaluations are reviewed—each project submitted should be accompanied by technical evaluations and must have criteria worksheets completed for cost efficiency, community support, environmental considerations, modal integration, and land use.
4. Statewide project prioritization based on the criteria scores is performed with TOPSIS

The criteria and their weights are shown in Table 1. Examples of the scoring worksheets for the benefit-cost criterion, as established for the 1995-1997 biennium, are

included in Appendix A. The worksheets used to score the six non-monetary criteria are included in Appendix B.

Table E.1
Criteria Categories and Weight Distribution
Washington State Highway Mobility Prioritization

Criteria Category	Criteria Weight
Benefit-Cost Ratio (B/C)	0.65
Community Support	0.14
Wetlands Assessment	0.0267
Water Quality	0.0267
Noise Assessment	0.0266
Mode Integration	0.07
Land Use	0.06
Total	1.0000

LITERATURE REVIEW

The extensive background research that was conducted before the formulation of the current Washington State priority programming procedure confirmed that few good examples exist nationwide of multimodal planning and programming. The WSDOT priority programming process represents one attempt by an agency to develop an objective prioritization process that is based on both monetized and other quantifiable non-monetary criteria.

Most of the ranking models reviewed were more complex than the TOPSIS ranking model that was selected. Many of the methods require all of the same input data as TOPSIS, as well as additional data. However, these models not only rely to a greater degree on subjective parameters, but they employ mathematics that are probably even less intuitively understandable than those of TOPSIS. The stated advantage of the TOPSIS model is that it is straightforward and easier to understand than most of the presented models. Given the varied technical and non-technical backgrounds of the people within the state who must accept the process, this is a crucial consideration.

The procedure that utilizes TOPSIS can be expanded to many of the more sophisticated models that exist, which could be the 'next step' in prioritization at some future time. In the meantime, TOPSIS provides a rational means by which projects can be prioritized and selected, and it gives decision-makers a transition into the concept of optimization ranking models.

OBJECTIVES AND METHODOLOGY

All of the revisions to the highway mobility prioritization procedure are based on feedback from WSDOT personnel on the 1995-1997 programming cycle. Their requests can be summarized into four major objectives.

1. Increase user friendliness to minimize user errors and reduce production time
2. Increase the consistency between relative rankings of the same projects on different lists, regardless of the characteristics of the other projects on the list
3. Reconcile the mathematical results of the program with the general "intuitive" understanding of how the program works, particularly with regard to the influence of the benefit-cost ratio (B/C) over the final rankings
4. Create a "seamless" transition for the regions from the previous version of the ranking procedure to the new version

The study proceeded under two major assumptions. First, because the criteria definitions and weight distribution are the result of extensive coordination with Washington State transportation decision makers and have been adopted as state policy, it was not the intent of this project to second-guess those priorities. Instead, all analysis and proposed revisions were targeted toward the actual scoring and ranking methodology.

Second, because the current complete "database" for the highway mobility ranking process consisted only of the project lists for 1995-1997, this study assumed that the 1995-1997 biennium project lists were typical of future lists.

Below is a summary of the project's methodology.

1. Solicit feedback on the 1995-1997 biennium ranking process and results from WSDOT regions and headquarters, and summarize the issues.
2. Analyze the results of the 1995-1997 biennium prioritization in light of the identified issues.
3. Make revisions to the TOPSIS procedure that would enhance user friendliness and speed up the final output production.
4. Identify the mathematical explanation for the issues raised.
5. Devise refinements to the methodology to increase consistency between statewide and regional lists.
6. Examine the correlation between TOPSIS-6 rankings and B/C rankings, and make methodology adjustments to meet project objectives.
7. Refine TOPSIS-6 (version utilized for the 1995-1997 biennium) according to analysis results to create the new version, TOPSIS-8.
8. Rank the 1995-1997 biennium project lists with TOPSIS-8 and compare the results to those before revision.
9. Test TOPSIS-8 on the preliminary project lists for the 1997-1999 biennium, as they are submitted.
10. Examine a Simple Additive Weighting ranking procedure and compare its results to those of TOPSIS-6 and TOPSIS-8.
11. On the basis of the results of testing the 1995-1997 lists, recommend revisions that best meet the project objectives and that are confirmed by the results of testing the preliminary 1997-1999 lists.
12. Recommend areas for further study

TOPSIS-6 AND THE 1995-1997 BIENNIUM PRIORITIZATION

The algorithm used to rank projects for the State of Washington is called TOPSIS because it is based on the methodology of the same name. This method allows elements

with disparate units (in this case, projects with disparate characteristics) to be easily evaluated. The premise of TOPSIS is that it

1. normalizes the scores in an evaluation matrix into dimensionless units
2. multiplies each of the scores by their relative assigned weights
3. formulates a theoretical "ideal-best" project and a theoretical "ideal-worst" project
4. prioritizes proposed projects by calculating their relative distances between the ideal solutions.

Table 2 shows the seven criteria that were considered in the prioritization process for the 1995-1997 biennium, as well as the relative weights and the possible range of scores for each of the criteria based on the established scoring procedure. The table shows that three of the seven criteria have no upper limit to their scoring ranges

Table E.2
Criteria Categories, Weights and Scoring Ranges
1995-1997 Biennium

	Criteria						
	Benefit-Cost Ratio	Community Support	Wetlands	Water Quality	Noise	Mode Integration	Land Use
Criteria Weight	0.65	0.14	0.0267	0.0267	0.0266	0.07	0.06
Most favorable score	no upper limit [†]	0	0.5	0	0	0	19
Least favorable score	0	17	no upper limit	41	no upper limit	10	0

[†] Although no upper limit is defined, projects with B/C scores exceeding 70 were excluded from the TOPSIS ranking procedure for the 1995-1997 biennium

The B/C scores for certain projects in the 1995-1997 biennium were radically higher than the typical range of 0.5 to 3.0 because of financial partnerships between WSDOT and other public agencies on these projects. Numerous tests showed that because of the strong weight of the B/C category (65 percent), projects with exceedingly high B/Cs

would be ranked by TOPSIS at the top of the list in order of descending B/C value (in other words, the non-monetary criteria would have little to no influence on the project rankings when B/Cs were very high), and could also have a significant impact on the relative order of projects with B/Cs in a lower, albeit more typical, range.

For this reason, the researchers decided that projects with extremely high B/C scores should be excluded from the ranking algorithm, and those projects could automatically be ranked at the top of the list in order of B/C (as TOPSIS would rank them anyway). Then the remainder of the projects could be ranked on the basis of less extreme distances from the “ideal-best” B/C. A B/C threshold of 70 was determined to be reasonable for the 1995-1997 biennium. Nonetheless, the definition of the upper B/C threshold remained a strong issue for 1997-1999 prioritization.

Once the weights and logic have been input for each of the seven criteria and criteria scores have been input for all of the projects, the prioritization can be performed utilizing the TOPSIS algorithm. The ranked lists of urban projects and rural projects for the 1995-1997 biennium are included in Appendix C.

Distribution of 1995-1997 Scores

Appendix D contains histograms of the distributions of scores for each of the criteria for the 1995-1997 statewide lists of urban and rural projects. The *B/C*, *Wetlands* and *Noise* scoring distributions were used to formulate part of the revisions for TOPSIS-8.

In addition, general observations were made about trends in the project scores in each of the seven criteria categories. The scoring distributions for the 1997-1999 priority programming can also be examined, once they have been finalized. This will begin to create a cumulative database that can be used for two types of analysis. First, it can be used to evaluate how well the scoring procedures reflect the range of considerations called required by state policy. Second, it can be used to assess the degree to which submitted projects reflect the state priorities established through the criteria categories and weightings.

Issues Raised by the 1995-1997 Biennium Project Ranking

Consistency between Statewide and Regional Rankings

The State of Washington is divided into six transportation regions. Every biennium, each of the six regions submits a list of urban projects and a list of rural projects to WSDOT headquarters for priority programming. Once projects have been submitted, they are compiled into comprehensive statewide urban and rural lists, and the ranking procedure is performed on each of the two lists. Each project then has a statewide ranking, either on the urban or rural list.

During the 1995-1997 biennium, before submitting their lists of projects for statewide ranking, many of the regions ran the ranking algorithm on their own lists of projects to see how they would fare against each other. Because the results of TOPSIS-6 were dependent on the characteristics of the projects being ranked, and because the characteristics of a single region's list of projects were most likely different from those of projects on the statewide list, quite often the regional rankings were different from the statewide rankings

Two measures were formulated to assess the degree of correlation between the statewide and regional rankings.

- The first measure consisted of *the number of projects that move in ranking* between the statewide and regional scenarios. This indicated the quantity of projects moved. Because the numbers of proposed projects varied tremendously from region to region, the number of projects moved for each region was presented along with the total number of projects on the region's list.
- The second measure consisted of a *correlation coefficient* between the statewide and regional rankings. This provided an indication of the magnitude of the difference between the statewide and regional ranked lists. (A mathematical explanation of the calculation of correlation coefficients is included in Appendix E.)

Although the two measures helped put the consistency between the statewide and regional rankings into some perspective, their main advantage was that they provided a baseline to which revised ranking procedures could be compared. In addition to the two numerical measures being calculated, the statewide and regional rankings for the 1995-1997 biennium lists of projects were compared graphically. Appendix F contains the correlation graphs of the statewide and regional rankings for the urban and rural lists for each of the six regions. In these graphs, the statewide ranking of each of the projects on the list is plotted along the x-axis against its regional ranking along the y-axis. A perfect correlation between the two scenarios is indicated by a straight line.

A summary of the numerical measures for the 1995-1997 biennium projects (ranked with TOPSIS-6) is presented in Table 3. The table shows much variation in the consistency between the two ranking scenarios, ranging from perfect correlation between the statewide and regional rankings (i.e., Region 6—Urban) to a correlation that approaches an inverse linear relationship between the two (i.e., Region 4—Rural).

Table E.3
Summary of Correlation Between the Statewide and Regional Rankings
1995-1997 Biennium Project Lists

Region #	Urban Project List			Rural Project List		
	# Projects Moved	# Projects on List	Correlation Coefficient	# Projects Moved	# Projects on List	Correlation Coefficient
1	30	41	0.93	9	13	0.68
2	2	3	0.50	0	2	1.00
3	10	30	0.99	11	22	0.97
4	6	10	0.85	6	6	-0.71
5	4	10	0.93	0	14	1.00
6	0	9	1.00	7	7	0.32

Correlation between TOPSIS-6 and B/C Rankings

The second issue raised after the 1995-1997 biennium project ranking was a desire for the ranking results to be intuitively more understandable. Benefit-cost analysis is an

integral part of traditional engineering analysis, so the calculated monetary values of a project's benefits and costs that make up the B/C score are tangible to most engineers and planners. The heavy weight of the B/C criterion, together with the more abstract nature of the non-monetary criteria, contributes to an overall perception that the projects should be "65 percent in order of B/C value." However, this is not necessarily how the rankings work out.

To allow quantitative evaluation of this rather qualitative issue, two measures were devised to compare the TOPSIS project rankings to the rankings that would have resulted if the projects had been prioritized simply by descending B/C value.

- The first measure was the *percentage of projects in B/C order*. This quantity measure consisted of the percentage of projects on a TOPSIS-ranked list that had a B/C value greater than the B/C value of the project ranked immediately following them.
- The second measure was a *B/C correlation coefficient*, which measured the linear relationship between the TOPSIS ranking and the B/C ranking (or the magnitude of variation). The calculation of the correlation coefficient was similar to that described for measuring consistency between the statewide and regional rankings. However, instead of the two data sets being the TOPSIS statewide rankings and the TOPSIS regional rankings, the two data sets were the statewide TOPSIS ranking and the statewide B/C ranking.

The B/C correlation measures for the 1995-1997 biennium ranked project lists is shown in Table 4. Although the two measures helped put the correlation between the TOPSIS and B/C rankings into some perspective, their main function was to provide a baseline to which revised ranking procedures could be compared.

Note also that, unlike in the case of the consistency measures, it is *not* the goal of the prioritization process for the B/C correlation coefficient to be 1.0. A correlation

coefficient of 1.0 would indicate no difference between the TOPSIS rankings and the B/C rankings. This would mean that the other six criteria had no influence over the project rankings, which is contradictory to state transportation policy and legislation.

Table E.4
Correlation of TOPSIS-6 and B/C Rankings
1995-1997 Biennium Statewide Lists

	Urban List	Rural List
Percentage of Projects in B/C Order	61 % (63 / 103 projects)	59 % (38 / 64 projects)
B/C Correlation Coefficient	0.86	0.81

Conclusions from Review of the 1995-1997 Prioritization Procedure

Evaluation of the ranked project lists for the 1995-1997 biennium provided (1) quantitative measures of the identified issues, and (2) a baseline to which revisions could be compared. Specifically, the following conclusions can be drawn.

1. There is no guarantee of consistency between the statewide and regional rankings under the TOPSIS-6 ranking procedure as it was designed for the previous biennium. Because the statewide ranking is the official ranking, and because the regional ranking is an unofficial screening measure, it would be most desirable if revisions to the methodology allowed the regions to approach replicating the conditions of the statewide list.
2. It is possible to calculate measures that will put the correlation between the TOPSIS order and the B/C order into a more objective perspective. These measures can be used to gauge the relative increase or decrease in B/C correlation that results from proposed refinements to the ranking methodology.
3. The issues that have been raised regarding the TOPSIS ranked order versus the B/C order (or in other words, the influence of the six non-monetary criteria on the final

project rankings) also indicates a need for clear mathematical explanation regarding how the TOPSIS rankings are determined.

4. The distributions of the criteria scores from the 1995-1997 biennium are useful (1) for defining parameters for revisions to the algorithm, (2) for future review of the project scoring procedures, and (3) for future assessment of the degree to which submitted projects reflect state policy.

TOPSIS MATHEMATICAL ISSUES

Feedback from numerous WSDOT personnel indicated misunderstanding regarding the role that the TOPSIS algorithm plays in producing a final ranked project list.

Although the TOPSIS algorithm does have characteristics that can somewhat influence project rankings, it really is only a *tool* whose function is to calculate geometric distances. It is important to clarify that TOPSIS, by itself, is not the prioritization process; it is only a step in the prioritization process, and its outcome is wholly dependent on the variables and assumptions that are determined in the steps that precede it. The variables that must be established *before* the utilization of TOPSIS in the prioritization process are

- the relative weights for each of the criteria that are to be considered
- the scoring procedure and numerical scale for each of the criteria
- the actual scores in each of the criteria for all of the projects being prioritized.

Once these parameters have been established, TOPSIS determines a preference order by calculating a composite value for each of the projects. This value represents the distance of each of the project's weighted scores from the ideal score in the respective criteria categories. Appendix G contains the mathematical explanation of the steps of TOPSIS procedure, as presented by Hwang and Yoon (1981).

Issues

The issues that result from the mathematical procedure that is employed by the TOPSIS ranking method are as follows.

1. Normalized scores will vary not only according to the value of the scores being normalized, but also according to the number of projects on the list.
2. Definitions of the ideal-positive and ideal-negative solutions depend on the scores of the projects on the list being ranked.

Conclusions

In light of the identified issues, the following conclusions can be drawn regarding the TOPSIS methodology.

1. Variation in the possible range of scores on a list of projects is the major source of the dynamic nature of the final relative rankings from list to list. Scoring ranges affect both the normalized values of the criteria scores and the definition of the ideal-positive and ideal-negative solutions.
2. Establishing constrained scores and using those constraints to establish standard ideals should greatly stabilize the predicted relative ranking orders. This will stabilize the normalization process and will allow the distances from ideal solutions to be uniform for a given alternative, independent of the characteristics of other projects.
3. Although constraining the possible range of scores will stabilize the normalized scores, small discrepancies between differing lists will remain, since the normalization is also a function of the number of projects on the list. Vector normalization is a necessary step in the prioritization process, since the seven criteria reflect different scales and units, and since TOPSIS utilizes vector calculations to rank projects. There is no way to avoid this without dictating the number of projects that must be included on a list.

RECOMMENDED FORMATTING CHANGES TO TOPSIS

The formatting revisions to TOPSIS-6 were made to the program *before* methodology revisions had been analyzed for the priority programming process. Because the revised program is so much simpler to run, these revisions also expedited the subsequent analysis of possible revisions to the methodology, in addition to responding to Objective #1 (*increase user friendliness*) of this study.

Formatting revisions reflected in TOPSIS-8 are intended to achieve the following goals:

- reduce efforts in producing final output
- make the program more “user friendly”—thus minimizing potential for user error
- allow a smooth transition from TOPSIS-6 to TOPSIS-8 for users at WSDOT.

Appendix H contains sample input files and output files both before revision (TOPSIS-6) and after revision (TOPSIS-8). Appendix I contains the complete input workbook file for TOPSIS-8. The formatting revisions that were incorporated into TOPSIS-8 are listed in their entirety in the final section of this summary, *Summary of TOPSIS-8 Features*.

REVISIONS TO TOPSIS METHODOLOGY

Recommended Revisions

Specific revisions to the TOPSIS methodology were based on the results of testing revision alternatives on the 1995-1997 urban and rural lists of projects. Consequently, in addition to the formatting changes to TOPSIS to improve output and ease of use, the following revisions to the TOPSIS methodology are recommended:

- The upper B/C threshold should be defined internally, and any projects that have a B/C score that exceeds the threshold should be automatically excluded from the ranking process and re-inserted at the top of the final ranked list. Tentatively, the

upper threshold is recommended to be 30, given the results of testing the 1995-1997 lists. However, this value should be confirmed with the 1997-1999 lists.

- The raw wetlands scores should be converted from their previous “limitless” scale to a constrained scale of 0 to 40, and the raw noise scores should be converted from their previous “limitless” scale to a constrained scale of 0 to 200. These conversions would be performed internally for the TOPSIS calculations, but the actual raw scores would appear on the input and final output files. Appendix J contains graphs that illustrate the scoring range of the wetlands and noise criteria before and after the scoring constraints were applied. In these graphs, the distribution curves of the constrained scores maintain roughly the same shape as the curve for unconstrained scores for both criteria categories.
- Standard theoretical positive-ideal and negative-ideal project scores should be appended to the list of projects so that the distances of ranked projects from the ideal solution will not depend on the scores on the project list. These theoretical projects would not appear on the final output file.

Table 5 illustrates the revised scoring ranges for TOPSIS-8 ranking calculations.

The table shows that the upper (least favorable) wetlands and noise scores now have limits, and while the B/C score still has no upper limit, its upper threshold for TOPSIS ranking has been redefined to be 30.

Appendix K contains the complete listing of TOPSIS-8, which is written in the EXCEL Macro 4.0 language.

Table E.5
Criteria Categories and Weights with Revised Scoring Ranges
1995-1997 Biennium

	Criteria						
	Benefit-Cost Ratio	Community Support	Wetlands	Water Quality	Noise	Mode Integration	Land Use
Criteria Weight	0.65	0.14	0.0267	0.0267	0.0266	0.07	0.06
Most favorable score	no upper limit [†]	0	0.5	0	0	0	19
Least favorable score	0	17	40	41	200	10	0

[†] Although no upper limit is defined, projects with B/C scores exceeding 30 are recommended to be excluded from the TOPSIS ranking procedure for 1997-1999 biennium

Results of Revisions

Consistency between Statewide and Regional Rankings

Appendix L contains the correlation graphs that illustrate the consistency between the statewide and regional rankings produced with the revised methodology of TOPSIS-8. The graphs also illustrate the TOPSIS-6 results for comparison to the results of the refined procedure.

Tables 6 and 7 show the numerical summary of these comparisons. The tables suggest that by constraining the scores and defining standard ideals, the consistency between the statewide and regional rankings can be vastly improved. Even in the urban list of projects for Region 1, where the greatest degree of variation between the two scenarios still exists, the magnitude of the changes has been drastically reduced. The remaining inconsistencies (due to variation between the normalization of the differing lists) can be put further into perspective by examining the priority index (PI) values of the projects that continue to move between scenarios, as will be discussed in the following section.

Table E.6
Comparison of Consistency Between the Statewide and Regional Rankings
1995-1997 Biennium Urban Project List

Region #	Urban List before revision (TOPSIS-6)			Urban List after revision (TOPSIS-8)		
	# Projects Moved	# Projects on List	Correlation Coefficient	# Projects Moved	# Projects on List	Correlation Coefficient
1	30	41	0.93	25	41	0.99
2	2	3	0.50	0	3	1.00
3	10	30	0.99	11	30	0.99
4	6	10	0.85	2	10	0.99
5	4	10	0.93	0	10	1.00
6	0	9	1.00	0	9	1.00

Table E.7
Comparison of Consistency Between the Statewide and Regional Rankings
1995-1997 Biennium Rural Project List

Region #	Rural List before revision (TOPSIS-6)			Rural List after revision (TOPSIS-8)		
	# Projects Moved	# Projects on List	Correlation Coefficient	# Projects Moved	# Projects on List	Correlation Coefficient
1	9	13	0.68	0	13	1.00
2	0	2	1.00	0	2	1.00
3	11	22	0.97	2	22	0.99
4	6	6	-0.71	0	6	1.00
5	0	14	1.00	2	14	0.99
6	7	7	0.32	0	7	1.00

Priority Index Number Analysis

Appendix M shows a side by side comparison of the statewide and regional rankings and their respective priority index numbers. Within these listings, the projects that still change rankings between the two scenarios are highlighted in gray. In every case where rankings switch places, the PI values are very close to one another. This indicates that these projects are similar in attractiveness with respect to the ideal solution. The similarity in the priority index numbers provides a rational basis by which to facilitate tradeoff decisions. Decisions can be made on a case by case basis, with the PI values serving as evidence that the projects under question are similar in their attractiveness as

preferred solutions. In addition to providing a rational basis for settling ranking disputes, similar PIs could also provide insight if one expensive project slightly outranked two or more less expensive projects. In this case, it would be possible to justify funding the two projects instead of the one, if they were all determined to be similarly attractive.

B/C Correlation

Table 8 summarizes the correlation with B/C rankings that results from the revised TOPSIS, given a defined upper B/C threshold of 30.

The table shows that in both the urban and rural lists of projects, TOPSIS-8 rankings result in a higher correlation between the TOPSIS rankings and the B/C rankings. In other words, the B/C has a stronger influence over the project rankings. The reasons for this are that (1) constraining the wetlands and noise criteria slightly decreases their negative influence over rankings, particularly when a project receives an extreme unfavorable score in either of these categories, and that (2) lowering the upper B/C threshold increases the influence of the B/C criterion because the proportional differences of B/C scores from the ideal solution become less pronounced.

Table E.8
Correlation of TOPSIS-8 and B/C Rankings
1995-1997 Biennium Statewide Lists

	Urban List		Rural List	
	TOPSIS-6	TOPSIS-8	TOPSIS-6	TOPSIS-8
% of Projects in B/C Order	61 % (63 / 103 projects)	71 % (73 / 103 projects)	59 % (38 / 64 projects)	66 % (42 / 64 projects)
B/C Correlation Coefficient	0.86	0.91	0.81	0.87

Preliminary Results of the 1997-1999 Prioritization

At the time analysis had been completed, it was too early to present the ranking results of the 1997-1999 biennium lists produced by these revisions because the lists had not yet been finalized. However, preliminary tests of the 1997-1999 biennium lists

indicated that the results are consistent with those presented in this report. As a matter of fact, the upper B/C threshold of 30 looks as though it will be very appropriate. So far, of the projects submitted, one has a B/C of over 200, but the remainder of the projects range from 27 down to less than 1. The project with a B/C of 200 would obviously be excluded from the ranking algorithm, and 30 would be an ideal B/C score in line with the actual projects.

COMPARISON OF TOPSIS TO SAW MODEL RESULTS

In the Simple Additive Weighting (SAW) model, project scores would be normalized on a linear scale, multiplied by their respective weights, and added across criteria to determine a final ranking index number. This method is intuitively more straightforward than the TOPSIS method and thus could be potentially more attractive to users. For this reason, the SAW Model was applied to the 1995-1997 urban and rural lists, and its results were compared to those of TOPSIS.

Analysis showed that many of the issues concerning TOPSIS are similar for the additive model, except that they are more exaggerated for the additive model.

- Scaling issues similar to those for TOPSIS normalization exist for the SAW method. They are actually more exaggerated under linear normalization than under vector normalization.
- As a result, the SAW model produces project rankings that are less correlated with B/C rankings than does the TOPSIS model. TOPSIS better meets the correlation objectives defined in this study under the same model parameters.
- The constrained scenario does guarantee complete consistency between statewide and regional rankings under the SAW method, but the comparison also showed that the constrained model of TOPSIS provides close to complete consistency, and the remaining inconsistencies are inconsequential.

Appendix N contains the ranked list of projects produced by the SAW model under both “unconstrained score” and “constrained score” scenarios. Under identical parameters, the SAW model does slightly better than TOPSIS in consistency between differing lists, but TOPSIS is more predictable and provides B/C correlation much more in line with the project objectives. Thus the SAW model was not shown to be an improvement over TOPSIS. In fact, the final conclusion is that TOPSIS is the better model and remains the preferred ranking model.

SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. Inconsistency can be greatly minimized by defining the ideal projects

To establish the **ideal-negative** project,

- Five out of seven criteria already have established ‘worst’ scores.
- Worst scores must be determined for the remaining two criteria, *wetlands* and *noise*.

To establish the **ideal-positive** project,

- Six out of seven criteria already have established ‘best’ scores.
- A best score must be established for benefit-cost ratio, at least for the purposes of the ranking algorithm.
- Conversion of benefit-cost to a constrained scale is not recommended.
 - Its influential weight will be more sensitive to an “arbitrary” scale.
 - It is the most “tangible” criterion, so there is little advantage in changing it to something less tangible.
 - Numerous test runs showed that even if they are included in the ranking algorithm, projects with extremely high B/Cs will rank at the top of the list anyway.

- Other numerous test runs showed that selection of a “cut-off B/C” can be somewhat arbitrary, without compromising the extent of ranking consistency.
2. Normalization cannot be made entirely consistent within given constraints.
Scaling issues remain, no matter what method of normalization is used. However, normalization of scores is essential for inter-attribute comparisons, and the drawbacks of the process are slight and tolerable.
 3. Even with variance caused by normalization, remaining inconsistencies are within reason.
Controversy surrounding remaining ranking inconsistencies should be minimal and can be addressed through policy decision. Projects that continue to switch rankings between lists have priority index numbers that are similar, meaning that they have similar attractiveness as a preferred alternative. Similar PIs can be used as the basis on which to make tradeoff decisions.
 4. The refinements recommended in this report to the TOPSIS methodology will not change the results of the statewide rankings as much as they will change the results of the regional rankings to better predict what will happen to a project on the statewide list.

Summary of Recommended Revisions to TOPSIS Methodology

1. Convert existing “unconstrained” scores to a “constrained” scale. Convert wetlands to a scale of 0 - 40, and convert noise to a scale of 0 - 200.
2. Establish an upper B/C threshold (a “cut-off B/C”) of 30. Projects with B/C scores higher than the threshold will be excluded from the ranking algorithm, but later will be automatically be ranked at the top of the list in order of their B/Cs. The definition of this threshold should be reexamined each biennium, and redefined if the submitted B/C scores so warrant.

3. Once an upper and lower limit have been defined for each of the seven criteria, the ideal-positive and ideal-negative solutions can be defined based on those limits. They should be defined internally within the TOPSIS model so that all lists of projects are consistently compared to standard ideals.

Complete Summary of New TOPSIS-8 Features

Below is a summary of all of the changes to TOPSIS recommended in this report, both to the mechanics of the program and to the methodology.

1. Reduce the efforts required to produce final output.
 - a) 'SR' and 'Region' have their own separate columns in the project identification.
 - b) Columns have been added to include the beginning and ending milepost numbers in the project identification.
 - c) Criteria scores and costs are carried with the project names and index numbers into the new output file.
 - d) Formatting of the output file is built into the program.
 - e) An additional blank column is inserted into the first column of the output file, to be filled at the user's discretion.
2. Make the program more "user friendly," thus minimizing potential for user error.
 - a) Non-monetary criteria worksheets are contained in an EXCEL workbook.
 - b) High B/C projects are automatically sorted out of the ranking algorithm, but they are still included at the top of the final ranked project list.
 - c) Scores equaling zero are internally converted to 0.001 (to avoid potential normalization errors).
 - d) Additional 'alerts' are built into the program that warn the user of input error.

- e) TOPSIS instructions and a sample input sheet are appended to the input file template.
 - f) The cells in the TOPSIS-8 macro are protected to prohibit a user from inadvertently changing the program coding.
3. Minimize ranking inconsistencies.
- a) *Wetlands* and *noise* are internally converted to a constrained scale.
 - b) 'Ideal-positive' and 'ideal-negative' projects are internally defined.
 - c) High B/C projects are internally excluded from the ranking algorithm but included in the final ranked list.
4. Allow a smooth transition from old version to the new version.
- a) Aside from the revised format of the input file, all other changes are built into the program and will occur automatically.
 - b) Scoring procedures required for the regions will remain the same as those in the previous biennium.
 - c) Detailed comments have been added to the TOPSIS macro.

CHAPTER I: INTRODUCTION

This study was prompted by the impending prioritization of the 1997-1999 biennium highway mobility projects for the Washington State Department of Transportation (WSDOT). The ranking procedure analyzed and refined in this report is based on a methodology called *Technique for Order Preference by Similarity to Ideal Solution* (Hwang and Yoon, 1981), more commonly known as TOPSIS. Ranking by TOPSIS is the final step in the current prioritization process for proposed Washington State highway mobility projects, the initial application of which occurred last biennium (1995-1997). Once the prioritization process had undergone one complete programming cycle, it was appropriate to evaluate and refine the process in time for the upcoming cycle.

The study found that whereas the ranking process is mathematically sound, as it was initially determined to be, some adjustments are necessary to allow this strictly mathematical model to better reflect the intentions of its practical application.

This report covers the following topics:

- a description of the previous highway prioritization process and the events that led to its replacement
- a literature review of the state-of-the-art in multi-criteria and multimodal transportation analysis and of the research that led to the establishment of the current ranking model within the State of Washington
- a summary and evaluation of the initial application of the TOPSIS ranking procedure
- a simplified mathematical explanation of the TOPSIS methodology and current issues
- an analysis of possible revisions to the TOPSIS methodology
- recommended revisions to TOPSIS for the 1997-1999 biennium prioritization process.

PREVIOUS "CATEGORY C" PROCESS

The use of TOPSIS for Washington State priority programming is a result of the “Prioritization of Capacity Improvements Study” (Reed et al., 1995), which began in 1990 and was completed for WSDOT in time for the 1995-1997 programming cycle. The highway mobility prioritization study was initiated as one component of a series of measures adopted by WSDOT to improve its approach to priority programming. Before these changes, highway improvements had been categorized by facility type, as follows:

Category A: Non-interstate preservation, safety, and minor operational improvements

Category B: Interstate capacity improvements, completion, and preservation

Category C: Non-interstate highway capacity improvements

Category H: Non-interstate bridge work

The current highway mobility category evolved from Category C. Projects in this category tend to consist of larger and more expensive non-interstate projects. Competition among these projects is stiff, and prioritizing them has historically been fairly complex.

The former Category C prioritization procedure consisted of two major components: (1) a screening process and (2) a ranking process, which comprised the following steps (Reed et al., 1995):

I. Screening process

The screening process consisted of two statutory conditions:

1. Projects must address existing highway deficiencies (such as congestion)
2. Community support must exist for addressing these deficiencies

II. Ranking process

Projects that met these two conditions then underwent a ranking process that required six sets of calculations. The first four calculations were

1. Accident Reduction Factor (ARF)—indicated a relative reduction in accidents

2. Level-of-Service Factor (LOS)— indicated a relative improvement in LOS
3. Volume Factor (VRF)—indicated a relative volume of traffic served on a facility
4. Engineering Factor (EF) = ARF + LOS + VRF

The three factors ARF, LOS, and VRF all were determined by assigning points to each of the projects on the list on the basis of the projects' relative comparisons. For example, the project with the highest projected accident reduction would receive the maximum number of ARF points, and the project with the lowest accident reduction would receive the lowest number of ARF points. All other projects would receive points pro-rated on the basis of their accident reduction relative to the best and the worst. The other two factors, LOS and VRF, were scored by a similar process. The EF was then calculated by summing the three factors for each project.

The next calculation was

5. Cost Factor (CA)—consisted of annualized capital and maintenance costs minus the annualized savings in operating costs and accident reduction. Note that travel time savings benefits were not included in this calculation.
6. Finally, the relative ranking was determined with the following formula:

$$\text{Relative ranking} = \frac{EF}{\sqrt{\frac{CA}{10000}}}$$

Thus the ranking was determined by dividing the number of *points* that reflected a project's projected improvements relative to the other projects by a scaled down *cost* factor. This approach had many drawbacks. The scaling factors in the denominator were intended to reduce the bias against more expensive projects, but a review determined that in actual application this goal was often not achieved (Reed et al., 1995). Also, the process was not flexible enough to include additional criteria. CA could not accommodate additional

monetized benefits (such as travel time savings) that would cause benefits to exceed costs, since this would produce a negative value under the square root sign in the denominator. Finally, this procedure double counted projected accident reduction by including it in both the EF and the CA calculations.

Nevertheless, the Category C prioritization procedure was determined to be acceptable for ranking highway projects that were similar in nature and scale and whose primary purpose was to move vehicles. However, as priorities shifted away from a strict focus on vehicle capacities, the inflexibility of the Category C process became more obvious, increasing the need for a more comprehensive process.

WSDOT SHIFT TO COMPREHENSIVE PLANNING

Beginning in 1990, a series of measures implemented by the State of Washington reflected a shift toward more comprehensive statewide priority programming. This shift was first reflected at the federal level in 1991 by the passage of the Intermodal Surface Transportation Efficiency Act (ISTEA), which mandated the consideration of multimodal transportation alternatives for projects to qualify for federal funding. Although all of these measures were interrelated and were a direct result of the priority shift at WSDOT, they were adopted independently of one another.

- Washington State priority programming law
- WSDOT Multimodal Transportation Plan
- Washington State Transportation Policy Plan
- Prioritization of Capacity Improvements Study

Each of the measures is discussed in the following sections.

Washington State Priority Programming Law

The *Washington State Transportation Priority Programming for Highway Development Law* (RCW 47.05) governs the selection process for the WSDOT highway construction program. It was revised in 1993 to reflect shifting priorities in the

programming process. Following are some of the pertinent highlights from RCW 47.05 (source: WSDOT, 1995):

- It is the intent of the legislature “that the investment of state funds to address deficiencies on the state highway system be based on a policy of priority programming having as its basis the rational selection of projects...”
- The transportation improvement program “shall consist of investments needed to address identified deficiencies on the state highway system to improve mobility, safety, support for the economy, and protection of the environment...”
- The law mandates that a six-year investment program be adopted that shall “identify projects for two years and major deficiencies proposed to be addressed in the six year period giving consideration to relative benefits and life cycle costing...”

Some of the criteria specified in RCW 47.05 to be included in priority programming are as follows:

- a) cost-effective movement of people and goods
- b) accident and accident risk reduction
- c) protection of state’s natural environment
- d) continuity of the highway transportation network
- e) consistency with local comprehensive plans and regional transportation plans
- f) public views concerning proposed improvements
- g) conservation of energy resources
- h) relative benefits and costs of candidate programs.

WSDOT Multimodal Transportation Plan

Also in 1993, WSDOT restructured state programs into a Multimodal Transportation Plan, in which the State Highway Program is only one of many components

(WSDOT, 1994). One of the outcomes of the restructuring was to convert the state highway categories A, B, C, and H into the following categories:

Capital Programs

- Preservation (P)
- Improvements (I)

Operating Programs

- Maintenance (M)
- Transportation System Management (Q)

The revised system purposely shifts the category distinction away from facility type. A further breakdown of the Capital Construction Program, which consists of Preservation (P) and Improvement (I), is illustrated in Figure I.1.

The former Category C projects became the I-1 Mobility projects under this revised structure. I-1 Mobility projects are sub-categorized into *urban*, *rural*, *bicycle*, and *HOV* projects, and resources are allocated separately among these sub-categories. Separating the projects into the sub-categories is a state policy decision meant to encourage more even dispersal of funded projects throughout the state. The prioritization procedure presented in this report is applied to the I-1 Mobility *urban* and *rural* sub-categories of projects.

Washington State Transportation Policy Plan

The Washington State Policy Plan was established by the Transportation Commission (WSTC) to promote the development of policies and strategies that would produce a balanced multimodal transportation system, consistent with state legislation. Each year, the State Transportation Policy Plan Steering Committee recommends policy and strategy proposals to the WSTC, which in turn transmits them to the state legislature.

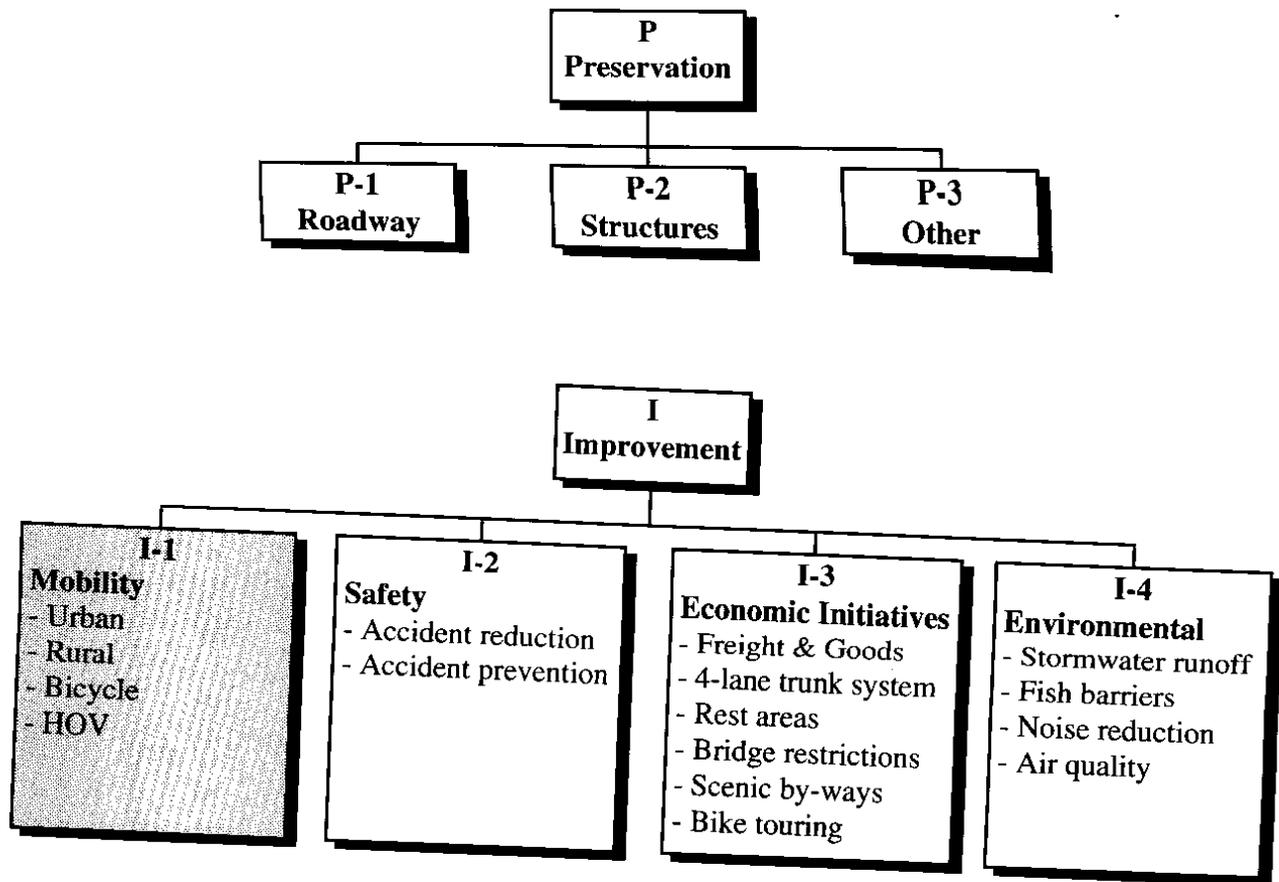


Figure I.1
Washington State Department of Transportation
Capital Construction Program Structure
(WSDOT, 1995)

The first Policy Plan was adopted in 1990, and new policies have been developed and added to the plan each year since. These policies form the foundation on which program funding and direction are based. The policies established from 1990 to 1995 are categorized into six areas: (1) protecting our investments, (2) personal mobility, (3) transportation support for economic opportunity, (4) environmental protection and energy conservation, (5) working together, and (6) transportation finance

Some of the state transportation goals, as stated in the 1995 annual report to the legislature, include the following:

- provide personal mobility choices for urban, rural, and intercity travel that are safe, reliable, affordable, and convenient

- make transferring between modes efficient and effective
- reduce the impacts of congestion on personal mobility
- design transportation systems that avoid disrupting and degrading the natural environment and heritage resources and that are aesthetically pleasing and energy efficient
- reduce pollutants from transportation systems
- integrate land-use planning and transportation planning
- ensure funding to responsibly meet the state's transportation goals.

In addition, policies include the promotion of social equity in transportation, multimodal choices, telecommunications, and transportation efficiency through transportation demand management (TDM) and transportation system management (TSM).

Prioritization of Capacity Improvements Study

The *Prioritization of Capacity Improvements Study* (Reed et al., 1995), performed by researchers at the Washington State Transportation Research Center (TRAC) at the University of Washington (UW), was initiated in 1990 as a review of the Category C rating system. The restructuring of the state highway program in the Multimodal Transportation Plan occurred in the midst of the prioritization study, so although it began as a Category C review, it ended in 1995 as a review of the Category I-1 ranking system.

The current prioritization process, initially applied for the 1995-1997 biennium, is the product of the Capacity Improvements study. In addition to developing a more mathematically reliable ranking algorithm, the Capacity Improvements study recommended a process that reflected the new state policy emphasis on comprehensive funding strategies and multimodal considerations. As a result, the current ranking procedure, established in that study, includes the following features that had not previously been included in the Category C procedure:

- emphasis on the movement of people and goods, rather than the movement of vehicles

- flexibility to accommodate any type or number of evaluation criteria
- ranking potential that is less sensitive to the magnitude of the project
- applicability across different modes
- broader range of criteria than those traditionally used.

Proposed state highway projects are now evaluated and prioritized according to a more comprehensive range of criteria than was formerly used. The flexible nature of this methodology encourages updates and refinements, based on changing priorities and feedback, such as those that form the basis of this current study. Priority programming is just one step in the complete process required to implement transportation projects within the state. Figure I.2 shows the implementation process for state transportation projects, from state policy development to specific project selection, and where priority programming fits into that process.

SUMMARY OF STATE HIGHWAY MOBILITY PRIORITY PROGRAMMING

The prioritization process that was developed for state highway mobility projects, as presented under the *Priority Programming* step in Figure I.2 and as documented in the *Capacity Improvements Extension Task Report* (Reed et al. [2], 1995), is detailed below.

1. Projects are submitted for statewide prioritization by each of the six regions in the State of Washington
 - A. The mobility process assumes that each project submitted has already been deemed the best alternative in an alternatives analysis process - the prioritization process is not intended to be an alternatives analysis.
 - B. RCW 47.05 mandates that all projects submitted be part of a six-year comprehensive plan.
2. Screening process - only projects that meet the following criteria are included in the prioritization process

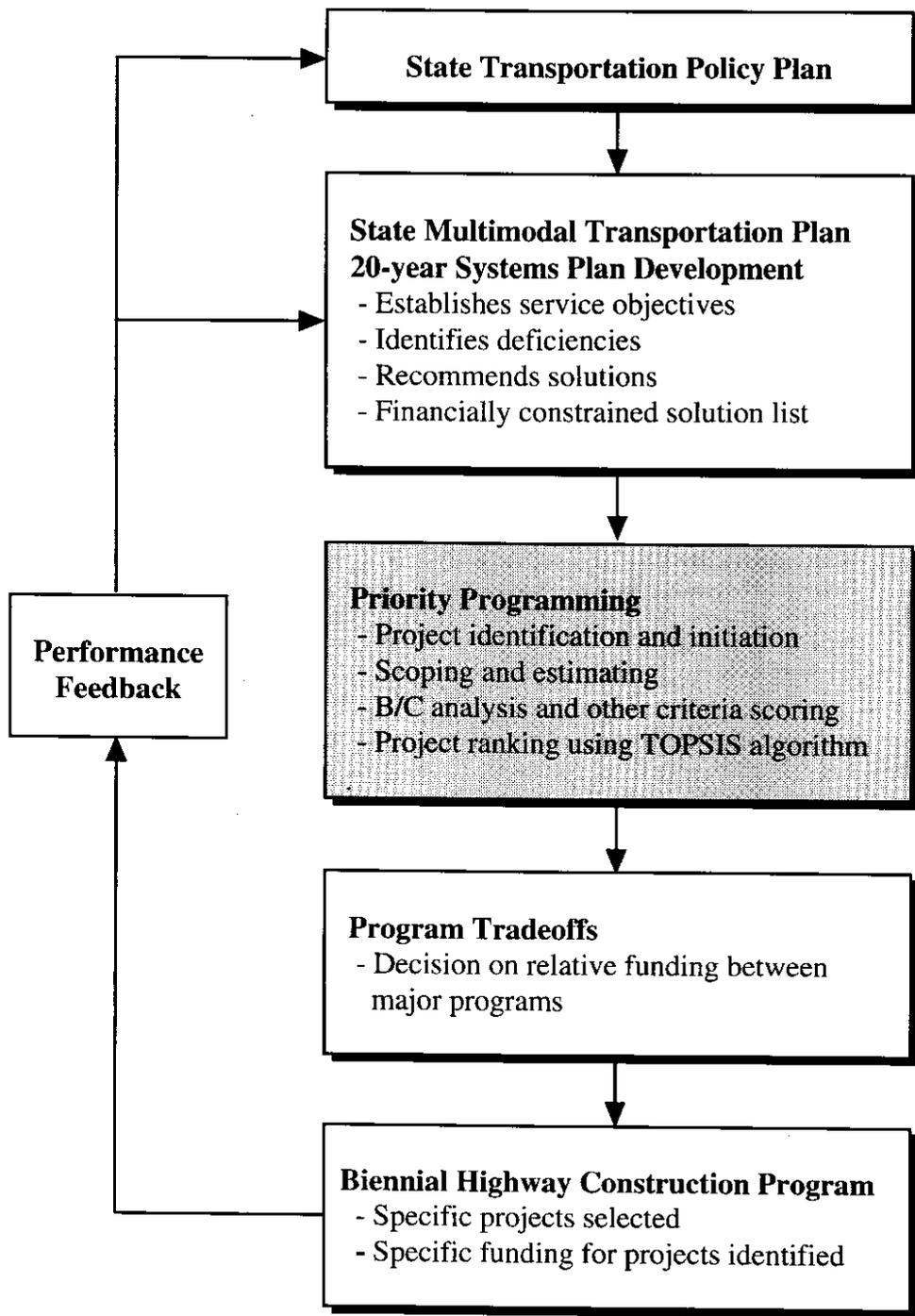


Figure I.2
Budget Development Process for Transportation Mobility Projects
State of Washington (WSDOT, 1995)

- A. Projects must be contained in the financially constrained State Systems Plan.
 - B. Projects must meet air quality conformity requirements.
 - C. Project submittals should concentrate only on existing deficiencies, unless otherwise requested by metropolitan planning or transportation planning organizations.
3. Review of project evaluation scores - each project submitted should be accompanied by detailed technical evaluations and must have criteria worksheets completed for cost-efficiency, community support, environmental considerations, modal integration, and land use.
 4. Mathematical ranking and prioritization of projects statewide
 - A. Criteria values are input into an EXCEL spreadsheet.
 - B. The TOPSIS algorithm macro is executed within EXCEL.
 - C. TOPSIS creates a new spreadsheet in which the priority index (ranking) is calculated for each project, and the projects are listed in ranked order.

The following elements must be established for the TOPSIS ranking methodology to be implemented:

- criteria categories
- criteria weights
- scoring procedures for each of the criteria.

To ensure that the new prioritization process provides for the full scope of possible project types, and to ensure that the criteria allow the most accurate evaluation of projects, TRAC researchers solicited extensive feedback and consensus from state transportation personnel and decision makers. The establishment of the procedure was also reviewed by an independent consultant hired by WSDOT.

A technical advisory committee (TAC) was established; it included regional planners, programmers, and traffic engineers, as well as representatives from WSDOT

Headquarters and the Transportation Data Office. The TAC established all of the parameters for cost-efficiency analysis. This served dual purposes of (1) providing technical consistency among the regions and (2) providing a means by which improvements to the cost-efficiency criteria could be recommended to programming management. In addition to producing a series of benefit calculation worksheets for specific types of projects, the TAC made technical decisions regarding such issues as vehicle occupancy assumptions, volume and growth assumptions and projections, and discount rate selection (Reed et al., 1995).

Numerous criteria review sessions were also held with WSDOT personnel. In these sessions the non-monetary criteria worksheets and scoring procedures were scrutinized, debated, and refined. In addition to providing feedback on specific components of the non-monetary criteria worksheets, WSDOT staff suggested simplifying assumptions to reduce required evaluation time for each of the projects. The seven criteria categories that resulted from these consensus-building exercises are described below:

1. Benefit-Cost Ratio (B/C)

The cost-efficiency of a project is measured by the B/C, which is the present value of the monetized project benefits divided by the project costs. Monetary benefits and costs projected over a 20-year period are converted into present value (today's dollars) using a discount rate of 4 percent.

The benefit categories consist of

- travel time savings for passenger and freight movement
- user operating savings
- accident reduction.

The cost categories consist of

- construction
- environmental retrofit
- preliminary engineering

- annual operating and maintenance.

2. Community Support

The community support category consists primarily of yes/no questions that assess financial participation, endorsement, and opposition by local governments, local organizations, and private groups or individuals. This criterion also addresses potential disruption of neighborhoods and displacement of homes, businesses, or farm land.

3. Wetlands Assessment

This category assesses the intrusion of proposed projects upon classified wetlands and associated buffer areas, in accordance with federal, state, and local regulations. Since mitigation costs are already included in the construction cost estimate, this category seeks to reflect the magnitude of public resistance to wetland impacts (Niemeier et al., 1996). It considers the acreage of any wetlands within 300 feet of proposed projects and assigns penalty points weighted according to the classification of the encroached wetlands.

4. Water Quality and Permitting

This category assesses potential impacts on the acreage of impervious surface area within 2000 feet of any body of water. Analysis consists of yes/no questions primarily regarding the number and nature of permitting requirements for a proposed project. The sub-total score reflects the magnitude of permitting requirements and is divided in half if no foreseeable permitting conflicts exist.

5. Noise Assessment

This category assesses the potential noise impacts for a proposed project. Points are accrued on the basis of a calculated "risk factor," which is based on the number of lanes of the proposed project, as well as the number of noise receptors and their proximity to that project. Risk factor points are weighed twice as heavily for new projects as they are for improvements to existing projects.

6. Modal Integration

The purpose of this criterion is to assess the level of modal integration supported by a proposed project, in accordance with Washington State policy goals. This category consists of yes/no questions concerning efficient use of existing capacity, connectivity between existing systems, integration of alternative modes such as bicycling and walking, and “multimodally” packaged projects.

7. Land Use

This category assesses the support that proposed projects provide for Washington State mobility and land-use management objectives. Land use criteria consist of yes/no questions concerning coordination between WSDOT engineers and planners, provision of convenient accessibility to transit, connectivity between urban activity centers, and consistency with regional and local comprehensive and/or transportation plans.

Examples of the scoring worksheets for the benefit-cost criterion, as established for the 1995-1997 biennium, are included in Appendix A. The worksheets used to score the six non-monetary criteria are included in Appendix B.

Once the evaluation criteria categories were established, their relative importance was determined by weighting the criteria so that together the weights would total 1.0. Sensitivity analysis was performed to provide information regarding the influence various weighting distributions would have over the final ranking order of projects. Six different weighting scenarios were developed. Results showed that if one criterion (such as cost-efficiency) held 50 to 70 percent of the total weight, the remaining criteria would have a minimal effect on the resulting ranked order, unless they combined to produce a *composite* negative score of 30 percent weight or more. In this case, it was determined that negative scores in remaining criteria could have noticeable influence over a project’s final ranking (Niemeier et al., 1996).

Because the criteria weight distribution would critically influence project rankings, a Delphi technique was used to establish the weights. The Delphi process is accepted as an

effective method by which consensus can be established among a group of people with conflicting or divergent interests (Niemeier et al., 1996). A wide range of the state's transportation decision makers were included in the Delphi establishment of criteria weights, including representatives from planning, project development, local programs, the Washington State Transportation Commission (WSTC), the Transportation Improvement Board (TIB), and district administrators. This exercise first solicited individual assignments of criteria weight distributions that were based on each individual's interpretation of state policy and the sensitivity analysis results. Individual assessments of the weight distributions and their effect on priorities of different project types were summarized and redistributed to the participants. Then, through a group summary and feedback process, the opportunity was provided to discuss and revise individual views. The feedback and revision process continued until satisfactory consensus was achieved and individuals were comfortable with the results. The weights that were established using the Delphi process are presented in Table I.1. These weights have been adopted as state policy, and any changes to the weight distribution would require a similar exercise.

Projects scored in each of these seven weighted categories could then be evaluated and prioritized with the TOPSIS methodology. TOPSIS creates a preference order by calculating each project's relative closeness to an ideal solution (explained in detail throughout the following chapters of this report).

Table I.1
Criteria Categories and Weight Distribution
Washington State Highway Mobility Prioritization

Criteria Category	Criteria Weight
Benefit-Cost Ratio (B/C)	0.65
Community Support	0.14
Wetlands Assessment	0.0267
Water Quality	0.0267
Noise Assessment	0.0266
Mode Integration	0.07
Land Use	0.06
Total	1.0000

CHAPTER II : LITERATURE REVIEW

The express purpose of this project was to refine the existing scoring and ranking process for project prioritization in the State of Washington. To do that effectively, the research behind the existing process had to be fully understood. In addition to providing an understanding of the fine points of the current ranking model, full comprehension of previous work allowed the current researchers to avoid redundancy when examining the implications of possible revisions to the process.

BACKGROUND RESEARCH

The state-of-the-art of multimodal and multicriteria analysis in the U.S. and Canada was recently assessed in National Cooperative Highway Research Program (NCHRP) Synthesis 201, "Multimodal Evaluation in Passenger Transportation" (Rutherford, 1994). A survey of all state departments of transportation (DOTs), metropolitan planning organizations (MPOs), and most Canadian provinces produced 18 examples of multimodal planning evaluation. Additional case studies presented in detail represent the state of practice in five specific applications of multimodal evaluation: (1) intercity corridors, (2) regional studies, (3) regional screening, (4) urban corridors, and (5) regional programming. The fifth application, regional programming, is closest to the type of decision-making that occurs in WSDOT priority programming. The report cites the San Francisco Area Metropolitan Transportation Commission (MTC) programming process as the state of practice in this application.

The NCHRP synthesis lists 16 criteria categories that represent a comprehensive range of evaluation criteria. These were collected and grouped from the studies reviewed and are generally known to the planning profession. The studies were cross-checked against the 16 criteria, and the following observations were made:

- Few of the studies employed a broad range of evaluation criteria.
- System coordination and integration are more often considered in regional programming studies than in the planning studies.
- The following criteria were excluded from evaluation more often than they were included: mobility, system coordination and integration, land use, freight, energy, safety, cost-effectiveness, equity, financial arrangements, and institutional factors.
- Few mobility measures were used, and no multimodal measures of mobility were identified or used.

Additionally, the report observed that some of the studies did not follow a traditional systems analysis method, thus leaving out important steps, and that projects tied with federal procedures tended to use traditional criteria more extensively. The report concluded that there is little agreement within the transportation profession regarding how evaluation criteria should be utilized, and it made the following recommendations:

- Comprehensive guidance is needed at the national level regarding criteria and criteria measurements, evaluation methods, and impact estimation, the basis of which could be the Federal Transit Administration's (FTA) alternatives analysis guidelines.
- A multimodal measure of mobility should be developed to measure effectiveness across modes.
- Multimodal planning could be further aided both by additional documentation and training at the federal level and by renewed efforts for information exchange.

The findings of the NCHRP synthesis provided the foundation for the "Prioritization of Capacity Improvements Study" (Reed et al., 1995). The Capacity Improvements Study documented the formulation of the Washington State Highway Mobility Priority Programming method as it currently exists. This study also reiterated the NCHRP synthesis assertion that the San Francisco Area MTC methodology represents the state of practice for multimodal regional programming. The MTC process consists of three

major steps, each of which utilizes a different set of criteria: (1) screening criteria, which are applied to all projects regardless of mode, (2) scoring criteria, which assign points based on technical merit and external impacts and whose technique varies with the mode being evaluated, (3) programming criteria and principles, which ensure that projects meet objectives such as mobility, clean air, and equity. The MTC methodology is a good example of a process that can be applied to a wide range of project types, but two drawbacks were also identified. First, the process requires a subjective assignment of points, and second, the process does not explicitly compare the movement of people and goods across modes (Niemeier et al., 1996).

The Capacity Improvements Study recognized two additional methodologies in which multimodal evaluation is practiced. The California Department of Transportation (CALTRANS) utilizes a method for the Los Angeles area in which projects are screened and then evaluated by both qualitative and quantitative criteria. The process stresses cost effectiveness, but it also includes community and environmental factors. The identified drawbacks are that the process depends on a subjective assignment of points and places a disproportionate emphasis on highway projects over other modes.

The study also presented the Washington State TIB methodology, which stresses multi-agency cooperation and mode integration. The identified drawbacks of the TIB process are that it utilizes a very narrow range of criteria and that its point assignment is subjective. However, the TIB process is recognized in the Capacity Improvements Study as “a good first step toward a multimodal selection process.”

The background research that was conducted for the Capacity Improvements Study reinforced the findings of the NCHRP synthesis. It concluded that few good examples exist of multimodal planning and programming, and that this is due both to the inflexible funding process that pre-dates ISTEA and to the slow nature of change within transportation decision-making.

The WSDOT priority programming process, as documented in the Capacity Improvements study, represents one of the few attempts by agencies to develop an objective prioritization process based on monetized benefits and costs, as well as other quantified criteria (Niemeier et al., 1996). The two major considerations that needed to be confirmed for the prioritization process were the following:

1. **the criteria and weights that would best reflect state transportation policy and priorities**—As has already been described, the criteria and weights were established through extensive coordination with WSDOT and other Washington State decision makers and were adopted as state policy.
2. **the mathematical mechanism (ranking model) to be used for project evaluation and prioritization**—TOPSIS was chosen as the preferred ranking method. Some alternative ranking models that were examined prior to the verification of this process are presented in the following section.

OPTIMIZATION AND RANKING MODELS

Several ranking models that could potentially have been used within the WSDOT priority programming process are presented in a paper titled “Optimization Models for Transportation Project Programming Process” (Niemeier et al., 1995). This paper identified many of the issues that complicate the selection of a statewide prioritization method. These are as follows:

- Diverse regions with differing system objectives compete for the same dollars.
- Explicit ties between program implementation and policy goals are lacking.
- Financial constraints are combined with an overburdened transportation system.
- A wide range of factors must be considered.

The Optimization Model paper presented three categories of traditional evaluation methods and the disadvantages of each of the techniques. These can be summarized as follows:

1. Cost-effectiveness evaluation—The main disadvantage of this method is that it frequently cannot provide a bottom line.
2. Point score rankings—This method is used extensively, particularly for multimodal evaluation. The main drawbacks of this method are that it allows for subjectivity and it often results in scored projects being clumped together.
3. Cost-benefit analysis—This is the preferred technique for ranking projects. Its primary disadvantage is that in practice it is limited to the inclusion of only the benefits and costs that are traditionally used and well-documented.

Given the drawbacks of these established techniques, the paper presented possible methods for evaluating projects by a traditional economic indicator, as well as additional non-monetary criteria. The major objective of the paper was to illustrate how linear programming (LP) and goal programming (GP) models can be used to evaluate projects with a diversity of evaluation criteria.

The paper demonstrated five optimization models, which are based on the following assumptions. These assumptions are consistent with the parameters that apply to Washington State priority programming:

- The purpose of project selection is to allocate resources within a budget constraint.
- Decision-maker program implementation objectives are defined.
- Each project is considered independent of other projects.
- Each budget period is considered independent of other budget periods.
- Costs and benefits are discounted to the current budget year.

Allocation of available funds includes both the consideration of budget constraints and the achievement of the decision-maker's objectives. The five optimization models can be summarized as follows:

Model 1—Priority Index

This model utilizes the empirical priority index technique of TOPSIS, which is actually the method that was adopted for use by Washington State priority programming.

The model creates a preference order of solutions by calculating projects' relative closeness index to an ideal solution. The advantages of this method are that it is straightforward and that it provides means by which trade-offs between priority indices can be determined. The drawbacks of the model are that trade-off decisions must be performed manually and can therefore be subjective. However, note that the parameters defined in the ranking model, as with any model, are intrinsically subjective. Also, this method provides no explicit technique for selecting the best combination of projects to meet objective achievement levels.

Model 2—Linear Programming

Linear programming provides a means of maximizing or minimizing an objective function to specified constraints (Niemeier et al., 1995). This LP model optimizes the priority index to a budget constraint. The drawback of this model is that it does not focus on the degree to which achievement objectives are met.

Model 3—Goal Programming

Goal programming is an extension of linear programming. This GP model seeks to maximize specified achievement objectives. Its major drawback is that it does not consider budget constraints.

Model 4—GP with Budget Constraint

This model is an extension of Model 3, with an added achievement objective of a budget constraint. This maintains the advantages of Model 3, while also ensuring that the selected project list will not exceed available funding.

Model 5 - LP with Achievement Objectives

This model is an extension of Model 2, with the added consideration of objectives to better distribute achievement objectives. This maintains the budgetary advantage of Model 2, while forcing the project solution set to attain minimum levels of the achievement objectives.

The use of priority indices in Models 1 and 2 does not provide explicit consideration of policy objectives. Instead, ranked projects are compared relative to each other. Unlike Model 1, Model 2 employs a formal approach to making trade-offs between project rank and cost, by funding more projects of similar priority. The study determined that neither Models 2 nor 3 will satisfy all decision-maker objectives. Models 4 and 5, on the other hand, result in solutions that much more consistently attain achievement objectives. Model 5 was recommended as best in the paper because it can be used to achieve the maximum improvement in a transportation system within given budgetary constraints.

Although TOPSIS was the least sophisticated model presented in this paper and was not the technique recommended for achieving maximum optimization, it was chosen to be used for Washington State priority programming. The reasons for this choice were stated both explicitly and implicitly in the Optimization Model paper. The paper concluded by stating that the choice of model will depend on the assumptions used and that if “decision makers are primarily interested only in funding a set of projects that do not exceed budgetary constraints, Model 1 may be the appropriate choice.” This appears to be the case within the State of Washington.

The TOPSIS methodology employed by Model 1 was presented in Multiple Attribute Decision Making—Methods and Applications (Hwang and Yoon, 1981). In addition to presenting TOPSIS, which is based on the principle that a selected alternative should be closest to an ideal solution, this report presented various other methods for prioritizing alternatives. Hwang and Yoon presented other ranking methods to be used when cardinal preference of attributes is given; these are described below.

Simple Additive Weighting (SAW) Method

This simple method is based on the principle that a selected alternative should have the largest utility. It can be applied to a list of projects whose varying independent attributes have been scored, and it requires only two major steps. First, relative weights

must be assigned to each of the attributes, which are usually normalized so that their sum equals 1.0. Second, the intra-attribute values are normalized. This means that whatever the scale of the different attributes, they should each be converted to a comparable scale within the decision matrix. For example, if one attribute's scores range from 0 to 10 and another's range from 0 to 500, they both must be converted to the same scale. It does not matter what the scale is (could be 0 to 1, or 0 to 100) as long as it is consistent between each of the attributes. Once the attribute scores cover comparable scales, they can be multiplied by their respective attribute weights. The utility for each project is then determined simply by adding the scaled weighted scores across the attributes. Projects are ranked in order of descending utility.

For this method to work, the attributes must be numerical and comparable, which means that a high value for one attribute must receive approximately the same numerical value as a high value for another attribute. In addition, a reasonable basis must be established for determining the attribute weights. Although this method evokes some of the same issues as the TOPSIS method, its simplicity and intuitive appeal may cause it to be an attractive choice. For this reason, researchers for the current project applied the SAW Model to rank a Washington State priority programming list, and its results were compared to those of TOPSIS. This exercise was meant to illustrate the drawbacks and scaling issues of the SAW method.

Hierarchical Additive Weighting Method

This model vertically expands on the SAW Model by establishing a hierarchy of attributes in which a given level dominates or covers some of all of the elements in the level immediately below. Each of the levels is weighted successively. A "pairwise comparison matrix approach" is then used, in which elements within a single level are compared with respect to the purpose of the adjacent higher level, and the process is repeated up the hierarchy. The resulting priorities need to be composed in such a way that one overall

priority vector reflects the impact of the lowest elements on the top element and represents the overall utility of the project.

This method is deemed more desirable if a large number of attributes is considered, since it is easier to assess weights using a hierarchical structure of objectives. However, because the attributes considered in Washington State priority programming are completely independent of each other, this method could not be reasonably applied. It is possible that the Hierarchical Model could be considered if the screening criteria were to be included in the prioritization model, rather than as a precursor to it. However, this would still result in a model more complicated than the TOPSIS model, and because it would most likely utilize additive weighting, it also would have the same inherent disadvantages of that method.

ELECTRE Method

The ELECTRE Method (Elimination and Choice Translating Reality) is based on the principle that overall preference rankings be arranged to best meet given concordance measures. This model uses the concept of “outranking relationships,” in which successive pairwise comparisons are made between alternatives, and the outranked (or dominated) alternatives are eliminated. ELECTRE examines pairwise dominance relationships, as well as the degree to which weighted evaluations differ from each other, on the basis of defined “concordance” and “discordance” sets (hence the method may also be called concordance analysis).

The mathematics of ELECTRE first require the formulation of a weighted, normalized decision matrix. Then the concordance and discordance sets must be defined. The concordance set is composed of all of the criteria by which one alternative would be preferred to another alternative. Inversely, the discordance set is composed of the complementary criteria that would cause an alternative to be less preferable to another alternative. On the basis of these definitions, ELECTRE creates concordance and discordance dominance matrices, whose intersection determines an aggregate dominance matrix, which is used to eliminate less favorable alternatives.

Hwang and Yoon considered ELECTRE to be one of the best methods for alternative selection because it fully utilizes the information contained in the decision matrix and consists of simple logic and a refined computational procedure. Its primary weakness, however, is the use of the threshold values that make up the concordance and discordance sets. These values are somewhat arbitrary, yet they may have a significant impact on the final solution. From the practical standpoint of priority programming, ELECTRE requires consensus to be built on all of the same factors as TOPSIS (criteria definition and criteria weights), but it then requires consensus on the additional (and highly subjective) factor of concordance thresholds. Also, this method provides no mechanism for the establishment of project tradeoffs because rather than defining the preference relationships of alternatives, it simply determines the overranking relationships and eliminates alternatives that are dominated.

CONCLUSION

The extensive background research that was conducted prior to the formulation of the current Washington State priority programming procedure confirmed that few good examples exist nationwide of multimodal planning and programming. The WSDOT priority programming process represents one attempt by an agency to develop an objective prioritization process based both on monetized and other quantifiable non-monetary criteria. Once the definition of the criteria, as well as their relative weights, were established by Washington State decision-makers, a number of models could potentially be used for project ranking and/or alternative selection.

Most of the ranking models reviewed here are more complex than the TOPSIS ranking model that was selected. Many of the methods, such as the linear programming models or ELECTRE, require all of the same input data as TOPSIS, as well as additional data. The more sophisticated input result in more sophisticated output, such as defined project tradeoffs or an assessment of the degree to which a group of selected alternatives achieves defined system objectives. However, these models not only rely to a greater

degree on subjective parameters, but they employ mathematics that are probably even less intuitively understandable than TOPSIS.

The one exception to this is the SAW method, which requires the same criteria definition as TOPSIS, yet employs even simpler mathematics, so it is intuitively more straightforward than TOPSIS. Although these qualities make this method potentially attractive to decision makers, it does have a major drawback with regard to scaling of the criteria scores. The simplest way to illustrate this point is to actually apply the SAW Model to the priority programming list of projects and compare the results of the prioritization to those of TOPSIS. This was done and is described later in this report.

Note that all of the models examined here employ a linear weighting scheme, which in itself contains certain drawbacks. First, a linear weighting scheme does not recognize potential synergistic benefits that could result from the combination of two or more criteria. Also, since a linear weighting scheme assumes criteria are independent, it is important that criteria scoring avoid double counting of benefits, which is often difficult.

The stated advantage of the TOPSIS model is that it is straightforward and easier to understand than most of the presented models. Given the varied technical and non-technical backgrounds of the decision makers within the state who must accept the process, this is a crucial consideration. Indeed, more sophisticated models require far more sophisticated input. This would require a colossal amount of consensus-building - particularly if system achievement objectives were to be defined internally in the model. TOPSIS does provide a numerical basis by which preference order trade-offs can be rationally determined. Although this process must be done manually and can be subjective, this is more in keeping with the climate in which decisions have traditionally been made within WSDOT.

The process that was initially implemented last biennium departed greatly from the previous process, and use of one of the more sophisticated models would most likely have been perceived as too radical a departure. Recent experience with the varied reactions to

even the TOPSIS model has confirmed that this most definitely would have been the case. The Optimization Model paper recognized the need for a gradual transition to these more complex models with a final assertion that “as decision makers and transportation specialists become more familiar with these models, greater acceptability and use will naturally follow.”

The procedure utilizing TOPSIS can be expanded to many of the more sophisticated models that exist, which could be the ‘next step’ in prioritization at some future time. In the meantime, TOPSIS provides a rational means by which projects can be prioritized and selected, and it gives decision-makers a transition into the concept of optimization ranking models.

CHAPTER III: OBJECTIVES AND METHODOLOGY OF STUDY

OBJECTIVES OF STUDY

All of the revisions to the highway mobility prioritization procedure are based on feedback from WSDOT personnel on the 1995-1997 programming cycle. Their requests can be summarized into four major objectives.

1. Increase user friendliness to minimize user errors and reduce production time

Experience during the 1995-1997 biennium showed that the ranking procedure could be fairly arduous, particularly if a user was inexperienced with the program or simply had not run it in some time. In addition, much editing was required to transform the program output into the form desired for final presentation.

2. Increase consistency between relative rankings of the same projects on different lists, regardless of the characteristics of the other projects on the list

There are two scenarios under which projects may be ranked in each region within the State of Washington:

- 1) All of the projects within the state are ranked from an aggregate list, and then results are sorted by region. This is officially how the projects are ranked by WSDOT and will hereafter be referred to as the *statewide* ranking.
- 2) The projects proposed within a region are ranked from a list of only that region's projects. The regions have been unofficially running the ranking algorithm on their lists of projects as they compile them for submittal to WSDOT. This will hereafter be referred to as the *regional* ranking.

Since the characteristics of the statewide and regional lists are different, the ranking orders of a region's projects under the two scenarios can quite often be substantially

different from one another. It was not originally anticipated that the regions would use the program as a screening tool. However, since the reason behind the regional rankings is to “test” how projects will fare in the statewide ranking, the inconsistency between the two scenarios has caused some controversy.

3. Reconcile the mathematical results of the program with the general “intuitive” understanding of how the program works

The purpose of TOPSIS is to objectively evaluate and rank projects according to predetermined weighted criteria scores. Improved accessibility to the ranking algorithm has allowed its results to be much more readily scrutinized than results from the past. Some questions have been raised regarding the intuitive sense of some of the resulting ranking decisions, particularly with regard to the influence of the benefit-cost ratio (B/C) criterion versus the other criteria. In addition to responding to questions from the regions, this objective is also driven by the need for a wide variety of players with non-technical backgrounds (such as Washington State legislators, Transportation Commissioners, and private citizens) to be able to understand and accept the results of the ranking procedure.

4. Create a “seamless” transition for the regions from the previous version of the ranking procedure to the new version

At the onset of this study the regions were already compiling and scoring projects for the 1997-1999 biennium ranking process, so a revised process needed to utilize the work that had already been done to the greatest degree possible, and the required input needed to be as similar to that of the established procedure as possible.

METHODOLOGY OF STUDY

Two major assumptions were made to complete this project:

1. Since the criteria definitions and weight distribution are the result of extensive coordination with Washington State transportation decision makers and have been adopted as state policy, they were treated as “untouchable” for the purposes of this project. In other words, all analysis and proposed revisions were targeted toward

the actual scoring and ranking procedures. It was not the intent of this project to second-guess the priorities reflected by the criteria categories and their relative weights.

2. Since the current complete “database” for the highway mobility ranking process consisted only of the project lists for 1995-1997, this study assumed that the 1995-1997 biennium project lists were typical of future lists. All revisions were tested on the 1995-1997 lists to predict whether the project’s objectives would be met for upcoming bienniums’ lists. The revisions were additionally tested on the 1997-1999 biennium project lists to the greatest extent possible, given their preliminary nature.

With these assumptions in place, the following methodology was used to achieve the objectives of this project.

1. Solicit feedback on the 1995-1997 biennium ranking process and results from WSDOT regions and headquarters, and summarize the issues.

The issues, as touched upon in the project objectives, were as follows:

- a. Increase the user friendliness of the program.
 - b. Increase the consistency of relative rankings of the same projects on differing lists.
 - c. Define parameters such that the mathematical results are intuitively understandable.
2. Analyze the results of the 1995-1997 biennium prioritization in light of the identified issues.
 3. Make revisions to TOPSIS that would enhance user friendliness and speed up the final output production.

These revisions responded to issue (a) and also expedited the analysis required for subsequent revisions.

4. Identify the mathematical explanation for the existence of issues (b) and (c).

TOPSIS ranks projects by establishing theoretical “ideal-best” and “ideal-worst” projects. Projects on the list are ranked in order of their relative position between these two ideals. Prior to recent revisions, these ideals were based on the scores unique to the list of projects it was ranking. Since scores vary from list to list, as does the number of projects on the list, relative rankings can also vary. In addition, the scaling of the project scores (particularly the B/C value) also influences the final ranking orders.

5. Devise refinements in the methodology to increase consistency between statewide and regional lists.

The project found that in order to best establish consistency between the statewide and regional rankings, the two ideal projects must be defined. These ideals would always be the same, so all projects would be measured by the same standards, regardless of the other projects on the lists.

Establishing ideal projects created an additional issue of what to do with criteria whose ranges have no upper limit. Three of the seven criteria previously had no upper limit (and thus no defined maximum), so most of the revisions required to achieve consistency were aimed toward developing constraints for these criteria.

6. Examine parameters that affect the correlation between the TOPSIS and B/C rankings and make methodology adjustments to meet project objectives.

The definition of criteria scoring ranges has an impact on the final ranked order of a list of projects. As a result, it was desired that parameters be defined so that the ranked order be intuitively understandable. Since the B/C criterion is weighed most heavily, and is the most tangible score, it forms the basis of that intuitive understanding. To assess this fairly subjective measure, objective measures were established that would assess the correlation between the TOPSIS mathematically ranked order and the B/C order alone.

7. Refine TOPSIS-6 according to analysis results to create the new version, TOPSIS-8.

Note that TOPSIS-6 was the version of TOPSIS utilized for the 1995-1997 prioritization. Some interim improvements were included in TOPSIS-7, a version (developed by Reed, 1995) that was never released. Thus the version of TOPSIS with revisions recommended in this report is TOPSIS-8.

8. Rank the 1995-1997 biennium project lists with TOPSIS-8 and compare the results to those before revisions

Comparisons were made to illustrate both the consistency between statewide and regional rankings, and the correlation between the TOPSIS and the B/C rankings.

9. Test TOPSIS-8 on the preliminary projects lists for the 1997-1999 biennium, as they are submitted.

Since all testing was performed on the 1995-1997 list of projects, this would confirm that it was appropriate to use them to predict the results of ranking future lists and that the projects objectives would be achieved for the upcoming biennium.

10. Examine a Simple Additive Weighting ranking procedure and compare its results to those of TOPSIS-6 and TOPSIS-8.

Given the complexities involved with refining TOPSIS to meet the defined WSDOT objectives, it seemed logical to ponder whether TOPSIS was an appropriate model for this type of application. Therefore, a simpler additive model was examined, and its ranking results and issues were compared to those of TOPSIS.

11. On the basis of the results of testing the 1995-1997 lists and confirmation by the results of testing the preliminary 1997-1999 lists, recommend revisions that best meet the project objectives.

12. Recommend areas for further study

CHAPTER IV: TOPSIS-6 AND THE 1995-1997 BIENNIUM PRIORITIZATION

SUMMARY OF TOPSIS

The algorithm used to rank projects for the State of Washington is called TOPSIS because it is based on the methodology of the same name. This method allows elements with disparate units (in this case, projects with disparate criteria) to be easily evaluated.

The premise of TOPSIS is that it

1. normalizes the scores in an evaluation matrix into dimensionless units
2. multiplies each of the scores by their relative assigned weights
3. formulates a theoretical "ideal-best" project and a theoretical "ideal-worst" project
4. prioritizes proposed projects by calculating their relative distances between the ideal solutions.

The theoretical "ideal-best" project is determined by combining all of the best scores in each of the criteria categories. The "ideal-worst" project is determined by combining all of the worst scores in each of the separate criteria categories. TOPSIS is based on the concept that the chosen alternative should be closest to the ideal-best solution and farthest from the ideal-worst solution. It is important to note that under this method, the definition of ideal-best and ideal-worst solutions is unique to the actual scores that appear on the list of projects being ranked. Since calculation of positive and negative ideals varies, the calculation of the relative distances of projects being ranked to those ideals varies also. Any changes in input has an impact on the model results.

Table IV.1 shows the seven criteria that were considered in the prioritization process for the 1995-1997 biennium, as well as the relative weights and the possible range of scores for each of the criteria based on the established scoring procedure.

Table IV.1
Criteria Categories, Weights and Scoring Ranges
1995-1997 Biennium

	Criteria						
	Benefit-Cost Ratio	Community Support	Wetlands	Water Quality	Noise	Mode Integration	Land Use
Criteria Weight	0.65	0.14	0.0267	0.0267	0.0266	0.07	0.06
Most favorable score	no upper limit [†]	0	0.5	0	0	0	19
Least favorable score	0	17	no upper limit	41	no upper limit	10	0

[†] Although no upper limit is defined, projects with B/C scores exceeding 70 were excluded from the TOPSIS ranking procedure for the 1995-1997 biennium

Two items should be noted from Table IV.1:

1. For two criteria (*B/C* and *Land Use*), higher scores are more favorable. In the cases of the other five criteria, lower scores are more favorable. This is defined as the “logic.” The logic is indicated for each criterion in the TOPSIS input sheet (“1” indicates that higher is better, and “0” indicates that lower is better). This input allows TOPSIS to accommodate different logic in its calculations.
2. Three of the seven criteria have no upper limit. In the case of the *B/C* criterion, this will affect the definition of the ideal-best project. In the case of the *Wetlands* and *Noise* criteria, it will affect the definition of the ideal-worst project.

Note that the limitless nature of the *B/C* criterion was addressed as an issue for the 1995-1997 prioritization. *B/C* values typically range from perhaps 0.5 to 3.0. Inspection of the project lists from last biennium showed that in some cases the *B/C* values were radically higher than this range. It was not uncommon for a project’s *B/C* to exceed 10 or 20, and in the most extreme cases the *B/C* exceeded 200. The primary reason for this range stems from the fact that many projects are financially partnered by other government agencies, either local or federal. When this is the case, WSDOT’s *B/C* methodology allows for only the WSDOT share of the cost to be included in the *B/C* calculation, whereas

the entire range of benefits are included. Say, for example, that WSDOT's share of a \$2,000,000 project is 10 percent, or \$200,000, and that the project's monetized benefits are calculated to be \$2,400,000. Although the B/C of the project overall would be 1.2, by WSDOT calculations it would be 12.0 because the total project benefits of \$2,400,000 would be divided only by the WSDOT share of the cost of \$200,000.

These extreme B/C values were not anticipated when the prioritization process was initially formulated. It became clear that inclusion of the few projects with exceptionally high B/C scores resulted in little differentiation between the distances of the *remainder* of the projects from the "ideal-best" B/C score. This dramatically reduced the overall impact of B/C on the resulting rankings of the vast majority of projects that had mid- to low-range (albeit more typical) B/C scores.

Numerous tests showed that because of the strong weight of the B/C category (65 percent), projects with exceedingly high B/Cs would be ranked by TOPSIS at the top of the list in order of descending B/C value (in other words, the other non-monetary criteria would have little to no influence on the project rankings when B/Cs were very high). For this reason, the researchers decided that projects with extremely high B/C scores should be excluded from the ranking algorithm but that those projects could automatically be ranked at the top of the list in order of B/C (as TOPSIS would rank them anyway). Then the remainder of the projects could be ranked on the basis of less extreme distances from the "ideal-best" B/C. A B/C threshold of 70 was determined to be reasonable for the 1995-1997 biennium. Nonetheless, the definition of the upper B/C threshold remained a strong and current issue for 1997-1999 prioritization.

Steps to Execute the Program

Once the weights and logic are input for each of the seven criteria, and criteria scores are input for all of the projects, the prioritization can be performed. Four steps are required to run the prioritization program:

1. Open the EXCEL spreadsheet that contains the TOPSIS macro.

2. Open the EXCEL input file that contains the list of projects, their scores in the seven criteria, and their WSDOT costs.
3. Highlight a list of projects and scores in the input spreadsheet.
4. Execute the TOPSIS macro.

Major Steps of the TOPSIS-6 Ranking Algorithm (1995-1997 Biennium)

Once the TOPSIS program has been correctly executed, the ranking procedure is performed automatically on the selected list of projects and scores. The major steps of the TOPSIS-6 ranking algorithm are described below.

1. Calculate the sum of squares value for each of the criteria categories.
2. Create the normalized matrix by dividing each criterion score by the sum of squares value in its respective criterion category.
3. Create the weighted normalized matrix by multiplying each normalized score by its respective criterion category weight.
4. Identify the scores for each criterion that make up the positive-ideal and negative-ideal solutions.
5. For each project, calculate the composite separation value from the positive-ideal solution [positive separation = S_{i+}].
6. For each project, calculate the composite separation value from the negative-ideal solution [negative separation = S_{i-}].
7. For each project, calculate the priority index number, which represents its relative closeness to the ideal solution [Priority Index = $PI = S_{i-} / (S_{i+} + S_{i-})$].
8. Sort projects in descending order of PI value—this is the ranked preference order.
9. Create an output file that includes the ranked list of project names and their PI values.

The ranked lists of urban projects and rural projects for the 1995-1997 biennium are included in Appendix C. A more detailed explanation of the mathematics involved with these steps will be presented in Chapter V of this report.

DISTRIBUTION OF 1995-1997 SCORES

Appendix D contains histograms of the actual distributions of scores for each of the criteria for the 1995-1997 statewide lists of urban and rural projects. Some interesting observations can be made from these histograms.

Out of 64 rural projects,

- 28 projects (44 percent of the total) have *B/C* scores of less than 1.0.
- The highest *B/C* score is in the 50-60 range.
- All *Community Support* scores are favorable at 10 or less, even though the possible (poorest) score is 17.
- 53 projects (84 percent of the total) have favorable *Wetlands* scores of 5 or less, but the worst score falls in the 40-60 range.
- All of the *Water Quality* scores are 30 or less.
- 32 projects (50 percent of the total) have favorable *Noise* scores of 0, but the highest score falls in the 501-600 range.
- 51 projects (81 percent of the total) have unfavorable *Mode Integration* scores of 8 or more.
- 34 projects (53 percent of the total) have *Land Use* scores of either 11 or 14 out of a possible most favorable score of 19.

Out of 103 urban projects,

- 23 projects (22 percent of the total) have *B/C* scores of less than 1.0.
- Three projects have *B/C* scores that are off the chart (292, 233 and 86). The highest urban *B/C* score included in the analysis is in the 60-70 range.
- All *Community Support* scores are favorable at 8 or less, even though the possible (poorest) score is 17.
- 87 projects (85 percent of the total) have favorable *Wetlands* scores of 5 or less, but the highest urban *Wetlands* score falls in the >90 range.
- All of the *Water Quality* scores are 30 or less.

- 26 projects (25 percent of the total) have *Noise* scores of 0, and 57 projects (55 percent of the total) score below 50, but the highest score falls in the >1000 range.
- 58 projects (56 percent of the total) have unfavorable *Mode Integration* scores of 8 or more.
- 77 projects (75 percent of the total) have *Land Use* scores of either 11 or 14 out of a possible most favorable score of 19.

The *B/C*, *Wetlands* and *Noise* scoring distributions were considered when revisions for the current biennium were formulated. In addition, examination of these distributions will be helpful for future review of the process. For example, none of the projects proposed for last biennium scored poorly in *Community Support*, and a majority of the projects scored poorly in *Mode Integration*. The scoring similarities in these two criteria (together worth 21 percent of the total weight) will decrease the overall differentiation between projects. Additionally, flags are raised regarding the noise scores for urban projects, in which a quarter of the projects are listed as having *no* noise receptors, and over half have very few receptors, which seems unrealistic. It might also be interesting to assess why the scores 11 and 14 predominate the land-use category, in both the urban and rural lists.

The scoring distributions for the 1997-1999 priority programming can likewise be examined, once they have been finalized. This will begin to create a cumulative database that can be used for two types of analysis. First, it can be used to evaluate how well the scoring procedures reflect the range of considerations called for in state policy. Second, it can be used to assess the degree to which submitted projects reflect state priorities established through the criteria categories and weightings.

ISSUES RAISED BY THE 1995-1997 BIENNIUM PROJECT RANKING

The primary purpose of this study was to assess the results of the 1995-1997 priority programming in light of the issues that were raised by various WSDOT personnel and to make appropriate adjustments for the upcoming biennium. The following sections present the assessment of (1) consistency between the statewide and regional rankings, and (2) the correlation of the TOPSIS rankings with the B/C rankings.

Consistency between Statewide and Regional Rankings

The State of Washington is divided into six transportation regions. The regions, shown in Figure IV.1, are primarily identified by number in this report, but they are named according to geographic location within the state, as follows:

- Region 1: Northwest Region
- Region 2: North Central Region
- Region 3: Olympic Region
- Region 4: Southwest Region
- Region 5: South Central Region
- Region 6: Eastern Region

Every biennium, each of the six regions submits a list of urban projects and a list of rural projects to WSDOT headquarters for priority programming. As of the 1995-1997 biennium, the submitted projects must include calculated scores in each of the seven criteria mentioned in the previous section. Once projects have been submitted, they are compiled into comprehensive statewide urban and rural lists, and the ranking procedure is performed on each of the two lists. Each project then has a statewide ranking, either on the urban or rural list. Finally, the urban and rural ranked lists are sorted by region in order of the statewide rankings. The end result is 12 lists, which reflect how highway mobility resources are allocated throughout the state: a ranked urban project list and a ranked rural project list for each of the six regions.

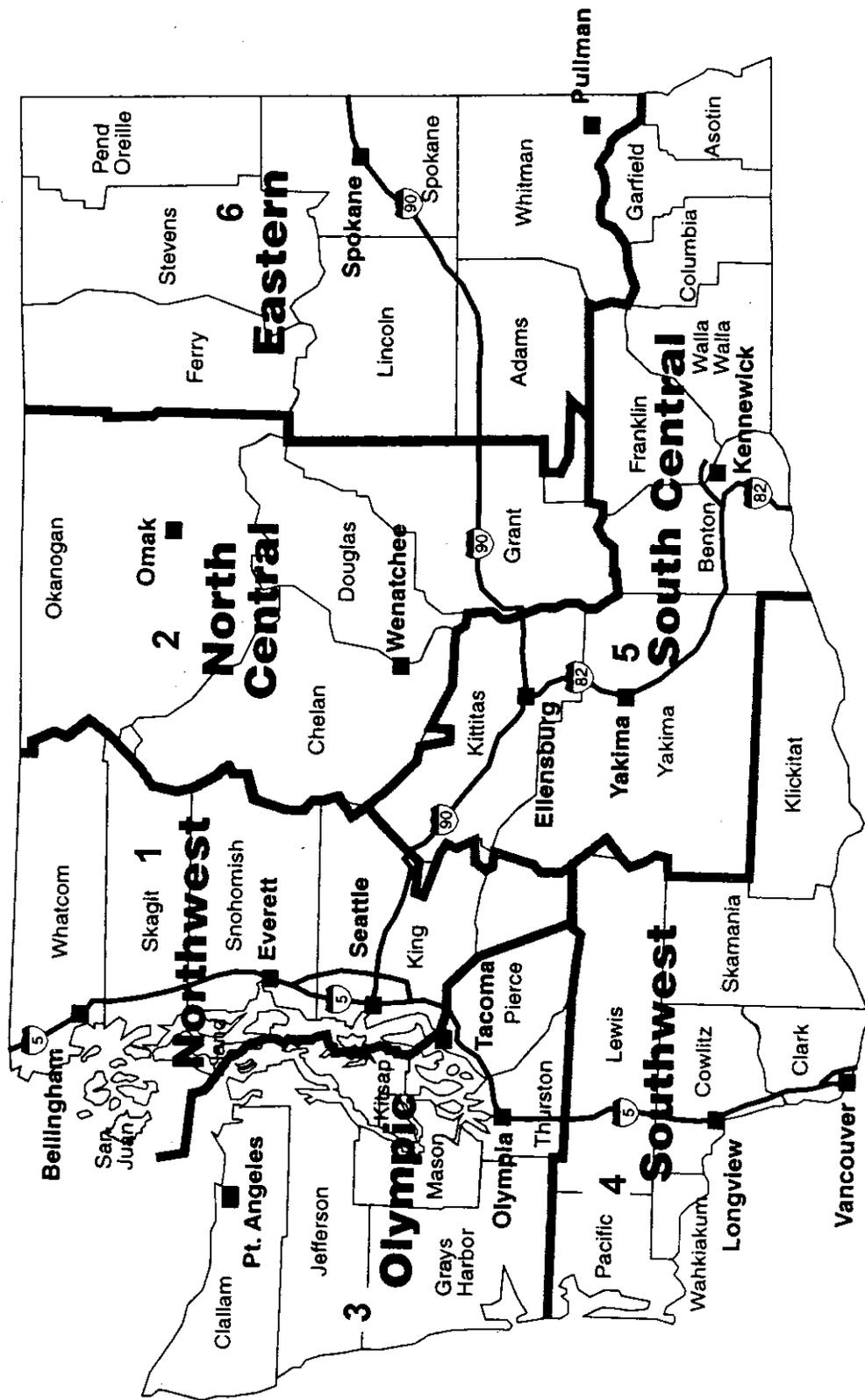


Figure IV.1
 Map of Washington Transportation Regions
 (source: WSDOT, 1996)

During the 1995-1997 biennium, prior to submitting their lists of projects for statewide ranking, many of the regions ran the ranking algorithm on their own lists of projects to see how they would fare against each other. Since the results of TOPSIS-6 were dependent on the characteristics of the projects being ranked, and since the characteristics of a single region's list of projects were most likely different than the statewide list, quite often the regional rankings were different than the statewide rankings. The mathematical reasons for this will be discussed in greater detail in Chapter V of this report.

Two measures were formulated to assess the degree of correlation between the statewide and regional rankings. The first measure consisted of *the number of projects that move in ranking* between the two scenarios. This indicated the quantity of projects moved. Since the numbers of proposed projects varied tremendously from region to region, the number of projects moved for each region was presented along with the total number of projects on the region's list.

The second measure consisted of a *correlation coefficient* between the statewide and regional rankings. This provided an indication of the magnitude of the difference between the statewide and regional ranked lists. For example, a one-place movement in ranking is much less extreme than a ten-place movement in ranking. The value of a correlation coefficient will always be between -1.0 and +1.0. A correlation of +1.0 indicates a perfect linear relationship between two data sets (i.e., no difference between the statewide and regional rankings), and a correlation coefficient of 0 indicates no linear relation at all. A correlation coefficient approaching -1.0 indicates a close inverse linear relationship. (A mathematical explanation of the calculation of correlation coefficients is included in Appendix E.) Example IV.1 illustrates hypothetically how the two measures were used to assess the consistency between the statewide and regional rankings for a region's project lists.

Although the two measures allowed the consistency between the statewide and regional rankings to be put into some perspective, their main advantage was that they provided a baseline to which revised ranking procedures could be compared. Both measures were dependent on the number of projects on a list, so they were not intended to be used for comparison between project lists. The number of projects on the 12 lists from the 1995-1997 biennium varied between two and 41. Although the number of projects moved obviously reflected varying impact, depending on the total number of projects on the list (two out of four projects moved is a much more substantial difference than two out of 40), the correlation coefficient did as well. The fewer the number of projects on a list, the more sensitive the correlation coefficient was to ranking differences between the two scenarios.

Example IV.1:

Below are two lists of projects, one for Region X and one for Region Y. The projects are listed in the order of their TOPSIS-6 statewide rankings. In the column next to the statewide rankings are the rankings produced when the region ran the ranking algorithm on its own list of projects. Each of the two lists of ranked projects are followed by the two consistency measures, (1) the number of projects moved in ranking and (2) the correlation coefficient between the statewide and regional rankings.

Region Y has only two projects that moved (Project S and Project T switch places), and the correlation coefficient of 0.94 indicates that although the rankings are not perfectly correlated, they have a fairly strong linear relationship. Region X, on the other hand, not only has more projects, but has more projects that moved in ranking between the two scenarios. The correlation between the two ranking scenarios, although still fairly linear, is less linear (farther from 1.0) than that of Region Y. The reason is in part because more projects moved in ranking, but also because the ranking differences are more extreme (for example, Project D ranks #4 on the statewide list, but ranks #8 on the regional list).

Region X			Region Y		
Project Name	Statewide Ranking	Regional Ranking	Project Name	Statewide Ranking	Regional Ranking
A	1	1	Q	1	1
B	2	2	R	2	2
C	3	3	S*	3	4
D*	4	8	T*	4	3
E	5	5	U	5	5
F*	6	4	V	6	6
G*	7	6			
H*	8	7			
I	9	9			
J	10	10			

*An asterisk indicates that the statewide and regional rankings for the project are different

projects moved in ranking: 4 / 10 # projects moved in ranking: 2 / 6

Correlation coefficient = 0.867 Correlation coefficient = 0.943

In addition to calculating the two numerical measures, the statewide and regional rankings for the 1995-1997 biennium lists of projects were compared graphically. Appendix F contains the correlation graphs of the statewide and regional rankings for the urban and rural lists for each of the six regions. In these graphs, the statewide ranking of each of the projects on the list is plotted along the x-axis against its regional ranking along the y-axis. A perfect correlation between the two scenarios is indicated by a straight line. These graphs provide an immediate visual assessment of how consistent the statewide and regional rankings were for a particular list of projects. The number of projects moved and the correlation coefficients are included with each graph, so that numerical measures may be examined in conjunction with the graphical illustrations.

A summary of the numerical measures for the 1995-1997 biennium projects is presented in Table IV.2. The table shows much variation in the consistency between the two ranking scenarios. For three out of the 12 lists, the statewide and regional rankings are identical, indicated by a correlation coefficient of 1.0. In the Region 1 urban list, 30 out of 41 projects are ranked differently in the statewide and regional scenarios. However, the

correlation coefficient of 0.93 indicates that a fairly strong linear relationship exists between the two ranking scenarios. This means that most projects moved few places between the two scenarios. Extreme differences between the two ranking scenarios can be seen in the rural lists for Region 4 and Region 6. In both of these cases, every project on the list moved in ranking between the two scenarios. In addition, the correlation coefficients for these two lists indicate little to no linear correlation between the statewide and regional rankings. All of these numerical assessments are further illustrated by the correlation graphs in Appendix F.

Table IV.2
Summary of Correlation Between the Statewide and Regional Rankings
1995-1997 Biennium Project Lists

Region #	Urban Project List			Rural Project List		
	# Projects Moved	# Projects on List	Correlation Coefficient	# Projects Moved	# Projects on List	Correlation Coefficient
1	30	41	0.93	9	13	0.68
2	2	3	0.50	0	2	1.00
3	10	30	0.99	11	22	0.97
4	6	10	0.85	6	6	-0.71
5	4	10	0.93	0	14	1.00
6	0	9	1.00	7	7	0.32

Because of the multidimensional nature of the analysis, it was virtually impossible to identify trends in the data that explained why in some cases the statewide and regional rankings were identical and in other cases there was no correlation between them whatsoever. However, the mathematical explanation in Chapter V will address the reasons for the inconsistency.

Correlation between TOPSIS-6 and B/C Rankings

The second issue raised after the 1995-1997 biennium project ranking was a desire for the ranking results to be intuitively more understandable. Benefit-cost analysis is an integral part of traditional engineering analysis, so the calculated monetary values of a project's benefits and costs that make up the B/C score are very tangible to most engineers and planners. The other six "non-monetary" criteria were established to allow projects to be scored in categories that have been identified as significant but traditionally have not been included in analysis because of no established way to quantify them. Under the current prioritization process that utilizes TOPSIS, the B/C criterion carries 65 percent of the criteria weight, and the other six criteria together carry 35 percent. The heavy weight of the B/C criterion, together with the more abstract nature of the non-monetary criteria, contributes to an overall perception that the projects should be "65 percent in order of B/C value." However, this is not necessarily how the rankings work out.

To allow quantitative evaluation of this rather qualitative issue, measures were devised to compare the TOPSIS project rankings to the rankings that would have resulted if the projects had been prioritized simply by descending B/C value. The first measure was the *percentage of projects in B/C order*. This quantity measure consisted of the percentage of projects on a TOPSIS-ranked list that had a B/C value greater than the B/C value of the project ranked immediately following them. The second measure was a *B/C correlation coefficient*, which measured the linear relationship between the TOPSIS ranking and the B/C ranking (or the magnitude of variation). The calculation of the correlation coefficient was very similar to that described for measuring consistency between the statewide and regional rankings. However, instead of the two data sets being the TOPSIS statewide rankings and the TOPSIS regional rankings, the two data sets were the statewide TOPSIS ranking and the statewide B/C ranking. Example IV.2 illustrates how these measures were applied to a ranked list of projects.

Example IV.2:

Below is a list of projects in the order of statewide TOPSIS rankings. In the column next to each statewide ranking is the B/C score for that project (the TOPSIS ranking also reflects the six non-monetary scores, which are not shown). Next to the B/C score is the ranking that would have resulted if the list of projects had been ranked simply by descending B/C score. The list of ranked projects is followed by the two measures, (1) the percentage of projects in B/C order, and (2) the correlation coefficient between the TOPSIS-6 and the B/C rankings.

Project Name	TOPSIS-6 Ranking	B/C Score*	B/C Ranking
A	1	10.3*	1
B	2	5.2*	2
C	3	3.2	4
D	4	4.5*	3
E	5	2.7	6
F	6	2.8*	5
G	7	1.1	9
H	8	1.4*	7
I	9	1.2*	8
J	10	0.8	10

* An asterisk indicates a B/C score that is higher than the B/C immediately following it

Percentage of projects in B/C order = 60% (6 out of 10 projects)

B/C correlation coefficient = 0.94

By the definition of the first measure, 60 percent of these projects rank in order of B/C. The B/C correlation coefficient of 0.94 indicates that the relationship between the TOPSIS-6 rankings and the B/C rankings is fairly linear. In other words, projects that are out of B/C order do not deviate far from the B/C ranking by very many places (in this case, by one or two places).

Just as in the case of the consistency measures, these B/C correlation measures were only marginally useful as stand-alone numbers. The *percentage of projects in B/C*

order was more illustrative than the *B/C correlation coefficient*. The percentage measure provided a rudimentary indication of how much influence the non-monetary criteria had over the B/C criterion in project rankings. The correlation coefficient alone did not provide much insight into the question at hand. Although the two measures allowed the correlation between the TOPSIS and B/C rankings to be put into some perspective, their main function was to provide a baseline to which revised ranking procedures could be compared.

Note also that, unlike in the case of the consistency measures, it is *not* the goal of the prioritization process for the B/C correlation coefficient to be 1.0. A correlation coefficient of 1.0 would indicate no difference between the TOPSIS rankings and the B/C rankings. This would mean that the other six criteria had no influence over the project rankings, which is contradictory to state transportation policy and legislation.

Table IV.3 presents the B/C correlation measures for the 1995-1997 biennium ranked project lists. The table shows that 61 percent of the urban projects and 59 percent of the rural projects rank in order of B/C. The urban and rural B/C correlation coefficients are 0.86 and 0.81, respectively. These coefficients indicate that although the correlation is roughly linear, some projects that ranked out of B/C order most likely did so by a substantial number of places. This supposition can be verified by visual inspection of the

Table IV.3
Correlation of TOPSIS-6 and B/C Rankings
1995-1997 Biennium Statewide Lists

	Urban List	Rural List
Percentage of Projects in B/C Order	61 % (63 / 103 projects)	59 % (38 / 64 projects)
B/C Correlation Coefficient	0.86	0.81

ranked project lists in Appendix C. Although these measures did provide some perspective regarding the existing correlation between the TOPSIS and B/C rankings, they were most useful in comparing the measures before and after the ranking process had been refined.

CONCLUSIONS

Evaluation of the ranked project lists for the 1995-1997 biennium provided (1) quantitative measures of the identified issues, and (2) a baseline to which revisions could be compared. Specifically, the following conclusions can be drawn.

1. There is no guarantee of consistency between the statewide and regional rankings under the TOPSIS-6 ranking procedure as it was designed for the previous biennium. Since the statewide ranking is the official ranking, and since the regional ranking is an unofficial screening measure, it would be most desirable for revisions to the methodology to allow the regions to approach replication of the conditions of the statewide list.
2. It is possible to calculate measures that will put the correlation of TOPSIS rankings to B/C rankings into a more objective perspective. These measures can be used to gauge the relative increase or decrease in B/C correlation that results from proposed refinements to the ranking methodology.
3. The issues that have been raised regarding the TOPSIS ranked order versus the B/C order (or in other words, the influence of the six non-monetary criteria on the final project rankings) also indicates a need for clear mathematical explanation regarding how the TOPSIS rankings are determined.
4. The distributions of the criteria scores from the 1995-1997 biennium will be useful (1) for defining parameters for revisions to the algorithm, (2) for future review of the project scoring procedures and, (3) for future assessment of the degree to which submitted projects reflect state policy.

CHAPTER V: MATHEMATICAL EXPLANATION OF TOPSIS ISSUES

Feedback from numerous WSDOT personnel indicated predominant misunderstanding regarding the role that the TOPSIS algorithm plays in the production of a final ranked project list. The goals of this chapter are to explain the concepts behind TOPSIS in terms that can be readily understood and to differentiate between issues that result from TOPSIS directly and those that are merely a result of the assumptions and priorities that drive the prioritization process.

Although the TOPSIS algorithm does retain characteristics that can have some influence over project rankings, it really is only a *tool* whose function is to calculate geometric distances. It is important to clarify that TOPSIS by itself is not the prioritization process—it is only a step in the prioritization process, and its outcome is wholly dependent on the variables and assumptions that are determined in the steps that precede it. The variables that must be established *prior to* the utilization of TOPSIS in the prioritization process are

- the relative weights for each of the criteria that are to be considered
- the scoring procedure and numerical scale for each of the criteria
- the actual scores in each of the criteria for all of the projects being prioritized.

Once these parameters have been established, TOPSIS determines a preference order by calculating a composite value for each of the projects. This value represents the distance of each of the project's weighted scores from the ideal score in the respective criteria categories. This is accomplished through the following six steps:

- Step 1: Construct a normalized decision matrix.
- Step 2: Weight the normalized decision matrix.
- Step 3: Determine the positive-ideal and negative-ideal solutions.
- Step 4: Calculate the separation measures from the ideal solutions.

Step 5: Calculate the relative closeness to the ideal solution.

Step 6: Rank the preference order.

THE MATHEMATICAL PROCEDURE USED BY TOPSIS

Appendix G contains the mathematical explanation of the steps in the TOPSIS procedure, as presented by Hwang and Yoon. The sections below further explain the following concepts, understanding of which is crucial to understanding the role TOPSIS plays in prioritization:

- composition of a decision matrix
- normalization and weighting of a decision matrix
- determination of positive-ideal and negative-ideal solutions
- calculation of relative distance between ideal solutions.

Composition of a Decision Matrix

A problem in which multiple projects are to be evaluated by multiple attributes is most concisely expressed by a decision matrix in which the “j” number of columns indicates the attributes (criteria) to be considered, and the “i” number of rows indicates the competing alternatives (Yoon and Hwang, 1995). Thus, a typical element, x_{ij} , indicates the score for the “ith” alternative with respect to the “jth” attribute. Figure V.1 illustrates a decision matrix that would be used to express a problem in which four (“i” number of) projects (A) are to be evaluated by six (“j” number of) attributes (X).

$$D = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \\ x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}$$

Figure V.1
Example of a Decision Matrix

Normalization and Weighting of a Decision Matrix

Normalization is the process of transforming disparate attributes (criteria scores) in a decision matrix to comparable scales so that they can be combined. One of the advantages of TOPSIS is that it produces mathematically reliable results even if the numerical criteria categories have very different characteristics. The reason this is achievable, however, is that the first step of TOPSIS is to transfigure the attributes so that they are comparable.

TOPSIS utilizes a *vector normalization* procedure, which means that raw scores are transformed so that all columns (attributes) have the same unit length vector. Thus, each normalized score (r_{ij}) is calculated by dividing the raw criterion score (x_{ij}) by the sum of the squared scores of all the projects in its criterion category (which represents the “norm” of the total outcome for that criterion). Mathematically, this process is expressed as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

The advantage of vector normalization is that all criteria are transferred to dimensionless units, thus allowing inter-attribute comparison. A drawback of this method is that the establishment of uniform vector lengths between attributes results in a non-linear transformation. Thus the maximum and minimum values may not be similar between the separate normalized criteria, so the transformation is hard to visualize. In addition, whereas vector normalization is conducive to the non-linear calculation employed by TOPSIS, it does not lend itself to linear comparison methods.

Linear comparison models (such as the Simple Additive Weighting Method) require a *linear scale normalization*. This procedure divides the actual criterion score by the

maximum possible value in that category. The transformation of the raw value (x_{ij}) to the normalized value (r_{ij}) is calculated by:

$$r_{ij} = \frac{x_{ij}}{\max_j x_j}$$

The result is a transformation in which all values in a category fall between 0 and 1, with the more favorable outcome approaching 1. All scores are transformed proportionally, so that their relative order and magnitudes remain equal. Linear normalized scores can be scaled up simply by multiplying them by a uniform factor. For example, to multiply all of the terms by 50 would allow them to be scaled from 0 to 50.

The following example presents a hypothetical decision matrix with disparate categories of attributes, normalized under both the vector and linear scale methods. Each of the examples in this chapter is based on examples presented by Hwang and Yoon (1981).

Example V.1—Step 1 (Normalize Criteria Scores)

Below is a decision matrix that contains the scores for four alternatives, A_1 , A_2 , A_3 and A_4 in six criteria categories, X_1 , X_2 , X_3 , X_4 , X_5 and X_6 .

$$D = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \\ 2.0 & 1,500 & 20,000 & 5.5 & 5 & 9 \\ 2.5 & 2,700 & 18,000 & 6.5 & 3 & 5 \\ 1.8 & 2,000 & 21,000 & 4.5 & 7 & 7 \\ 2.2 & 1,800 & 20,000 & 5.0 & 5 & 5 \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}$$

In all criteria categories except X_4 , a higher score is more favorable. In category X_4 (which could represent a criterion such as cost) a lower score is more favorable. Note that the scoring ranges are vastly different for the different criteria categories. To allow inter-attribute as well as intra-attribute comparisons, this decision matrix must be normalized by either vector or linear normalization.

Matrix D transformed by vector normalization:

$$R = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \\ .4671 & .3662 & .5056 & .5063 & .4811 & .6708 \\ .5839 & .6591 & .4550 & .5983 & .2887 & .3727 \\ .4204 & .4882 & .5308 & .4143 & .6336 & .5217 \\ .5139 & .4392 & .5056 & .4603 & .4811 & .3727 \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}$$

where all of the X_1 values equal the X_1 values from decision matrix D divided by $[(2.0)^2 + (2.5)^2 + (1.8)^2 + (2.2)^2]^{1/2}$, and all of the X_2 values equal the X_2 values from decision matrix D divided by $[(1,500)^2 + (2,700)^2 + (2,000)^2 + (1,800)^2]^{1/2}$, and so on.

Matrix D transformed by linear scale normalization:

$$R' = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \\ 0.80 & 0.56 & 0.95 & 0.82 & 0.71 & 1.00 \\ 1.00 & 1.00 & 0.86 & 0.69 & 0.43 & 0.56 \\ 0.72 & 0.74 & 1.00 & 1.0 & 1.00 & 0.78 \\ 0.88 & 0.67 & 0.95 & 0.90 & 0.71 & 0.36 \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}$$

where all of the X_1 values equal the values from matrix D divided by the highest X_1 value of 2.5, and all of the X_2 values equal the values from matrix D divided by the highest X_2 value of 2,700, and so on. Also, since lower values are more favorable for column X_4 , a reciprocal method is required for normalization of those values - the minimum X_4 value of 4.5 must be divided by each of the values from matrix D.

Both of these methods transform the original scores into scales that are comparable across the criteria. The choice of normalization method depends on whether the ranking procedure is linear (which requires linear scale normalization) or non-linear (which requires vector normalization).

Note that the normalized values that result from linear scale normalization are a function only of the maximum (or minimum) value for each criterion category on the list.

The normalized values of vector normalization, however, are both a function of all of values of the scores for each criterion *and* the number of projects on the list. Say, for example, that a fifth project, A_5 , was added to the decision matrix in Example V.1- Step 1. In the linear scale normalized matrix, the only way this would affect the normalized scores of the other projects would be if one or more of the scores in project A_5 determined a new maximum (or minimum for criterion X_4). If the scores of project A_5 fell “in-between” the scores already shown in the example, the normalized values of projects A_1 through A_4 would remain exactly the same. Consequently, the addition of a fifth alternative might or might not affect the normalized values of the scores of the other projects.

This is not the case with vector normalization, in which the addition of another project is *guaranteed* to affect the normalized values of the scores of the other alternatives. This is because the addition of another project would add another “squared” term to the calculation of the sum of squares value for each criterion category, thus increasing all of the sum of squares values. Since the sum of squares value would increase for each of the criterion categories, all of the normalized scores for all of the projects would decrease proportionately. This is a key concept because it is one of the factors that affects the consistency between lists of projects with differing characteristics.

Once the decision matrix has been normalized, it is in proper form for the criteria weights to be applied. The following example illustrates criteria weighting of the normalized matrix in Example V.1 - Step 1. **Since the TOPSIS procedure utilizes the vector normalization method, the ensuing examples will build on the vector normalized matrix, R** (leaving the linear normalized matrix, R' , simply to be used for comparison to R).

Example V.1—Step 2 (Weight the Normalized Criteria Scores)

Below are the criteria weights that reflect the relative importance of the six criteria categories, X_1 , X_2 , X_3 , X_4 , X_5 and X_6 .

Criteria	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆
Weight	0.2	0.1	0.1	0.1	0.2	0.3

The normalized weighted decision matrix, V, is constructed by applying these weights to the normalized matrix, R, from Example V.1—Step 1, as follows:

$$V = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \\ .0934 & .0366 & .0506 & .0506 & .0962 & .2012 \\ .1168 & .0659 & .0455 & .0598 & .0577 & .1118 \\ .0841 & .0488 & .0531 & .0414 & .1347 & .1565 \\ .1028 & .0439 & .0506 & .0460 & .0962 & .1118 \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}$$

where all of the X₁ values equal the normalized X₁ values from matrix R multiplied by the X₁ criteria weight of 0.2, and all of the X₂ values equal the X₂ values from matrix R multiplied by the X₂ criteria weight of 0.1, and so on.

Determination of Ideal Solutions

Once the normalized weighted matrix has been constructed, the single best score and the single worst score is identified for each of the criteria categories (columns). The best scores are combined to define the ideal-positive solution, and the worst scores are combined to define the ideal-negative solution. This exercise is illustrated in the following example.

Example V.1—Step 3 (Determine Ideal Solutions)

Below is the normalized weighted decision matrix that was constructed in Example V.1—Step 2. For each of the seven criteria categories, the best score is underscored with a double line, and the worst score is underscored with a single line.

Note that for category X₄, in which a lower score is more favorable, the lowest score is indicated as best and the highest score is indicated as worst. In the other five

categories, where higher scores are more favorable, the highest score is indicated as best and the lowest score is indicated as worst.

$$V = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \\ .0934 & \underline{.0366} & .0506 & .0506 & .0962 & \underline{.2012} \\ \underline{.1168} & \underline{.0659} & \underline{.0455} & \underline{.0598} & \underline{.0577} & \underline{.1118} \\ \underline{.0841} & .0488 & \underline{.0531} & \underline{.0414} & \underline{.1347} & .1565 \\ .1028 & .0439 & .0506 & .0460 & .0962 & .1118 \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}$$

The resulting definitions of ideal solutions are as follows:

	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆
Ideal-positive solution	.1168	.0659	.0531	.0414	.1347	.2012
Ideal-negative solution	.0841	.0366	.0455	.0598	.0577	.1118

These definitions of ideal solutions are based entirely on the characteristics of the actual projects being ranked.

Calculation of Relative Distance between Ideal Solutions

Once the normalized weighted criteria scores have been calculated and the ideal solutions have been identified, the distances of the actual scores from the ideal solutions can be calculated. This can be simply illustrated by using two projects, A₁ and A₂, with two attributes, X₁ and X₂, as is shown in Figure V.2. In this figure, the ideal-positive solution and the ideal-negative solution are shown as A* and A-, respectively.

The figure shows that the distances (also known as separation measures) from the ideal solutions can be defined as follows:

- S_{A₁}* : the separation of Alternative A₁ from the ideal-positive solution A*
- S_{A₁}- : the separation of Alternative A₁ from the ideal-negative solution A-
- S_{A₂}* : the separation of Alternative A₂ from the ideal-positive solution A*

S_{A2}^- : the separation of Alternative A_2 from the ideal-negative solution A^-

These distances are calculated simply through Pythagorean's theorem, which states that the hypotenuse of a right triangle is calculated by the square root of the sum of each of its two other sides squared.

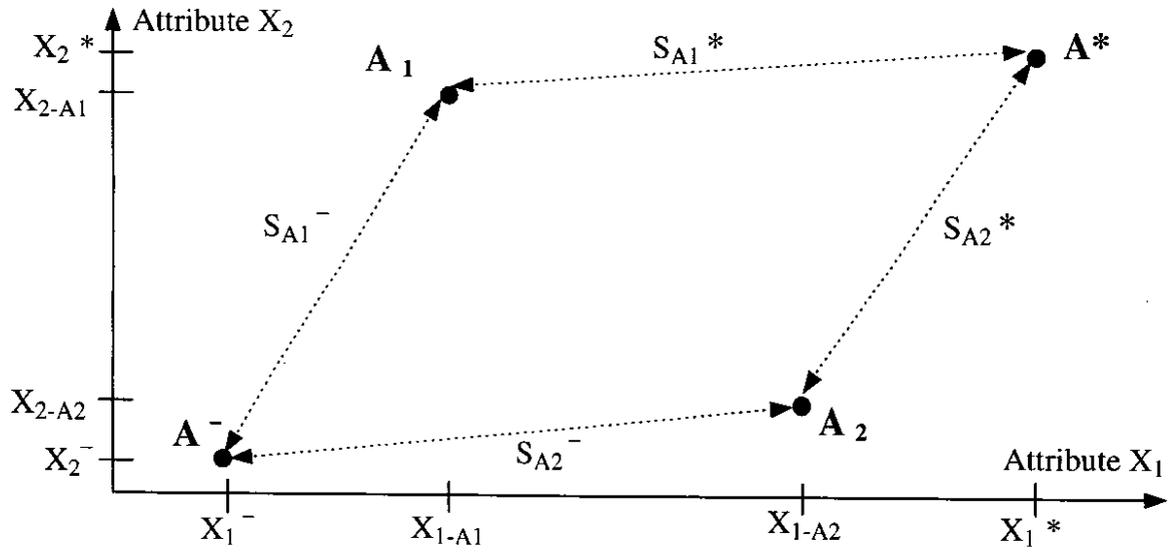


Figure V.2
Separation of Two Alternatives from Ideal Solutions

This can be illustrated for Alternative A_1 as follows:

$$S_{A1}^* = [(X_2^* - X_{2-A1})^2 + (X_1^* - X_{1-A1})^2]^{1/2}$$

$$S_{A1}^- = [(X_2^- - X_{2-A1})^2 + (X_1^- - X_{1-A1})^2]^{1/2}$$

Likewise for Alternative A_2 :

$$S_{A2}^* = [(X_2^* - X_{2-A2})^2 + (X_1^* - X_{1-A2})^2]^{1/2}$$

$$S_{A2}^- = [(X_2^- - X_{2-A2})^2 + (X_1^- - X_{1-A2})^2]^{1/2}$$

The operation inside each of the parentheses is simply the ideal score (X^* is the positive-ideal and X^- is the negative ideal) minus the actual score for each criterion. Since the distances are squared, they can be either positive or negative - it makes no difference.

Although this concept can only be drawn in two dimensions (thus for two criteria), the separation can be calculated in this manner for any number of criteria. Every additional criterion adds another “squared” term to the separation measure calculation. If, for example, a third criterion, X_3 , were to be considered, the separation measures for Alternative A_1 would be as follows:

$$S_{A1}^* = [(X_3^* - X_{3-A1})^2 + (X_2^* - X_{2-A1})^2 + (X_1^* - X_{1-A1})^2]^{1/2}$$

$$S_{A1}^- = [(X_3^- - X_{3-A1})^2 + (X_2^- - X_{2-A1})^2 + (X_1^- - X_{1-A1})^2]^{1/2}$$

If a fourth criterion, X_4 , were to be considered, the separation measures would be

$$S_{A1}^* = [(X_4^* - X_{4-A1})^2 + (X_3^* - X_{3-A1})^2 + (X_2^* - X_{2-A1})^2 + (X_1^* - X_{1-A1})^2]^{1/2}$$

$$S_{A1}^- = [(X_4^- - X_{4-A1})^2 + (X_3^- - X_{3-A1})^2 + (X_2^- - X_{2-A1})^2 + (X_1^- - X_{1-A1})^2]^{1/2}$$

and so on.

The following example illustrates the calculation of the separation measures.

Example V.1—Step 4 (Calculate Separation from Ideal Solutions)

Below are the normalized weighted decision matrix V that was constructed in Example V.1—Step 2 and the ideal solutions defined in Example V.1—Step 3.

$$V = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \\ .0934 & .0366 & .0506 & .0506 & .0962 & .2012 \\ .1168 & .0659 & .0455 & .0598 & .0577 & .1118 \\ .0841 & .0488 & .0531 & .0414 & .1347 & .1565 \\ .1028 & .0439 & .0506 & .0460 & .0962 & .1118 \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}$$

	X_1	X_2	X_3	X_4	X_5	X_6
Ideal-positive solution	.1168	.0659	.0531	.0414	.1347	.2012
Ideal-negative solution	.0841	.0366	.0455	.0598	.0577	.1118

The separation measures for the four alternatives can be summarized as follows:

Alternative	Positive	Negative
	Separation	Separation
	S*	S ⁻
A ₁	.0545	.0983
A ₂	.1197	.0439
A ₃	.0580	.0920
A ₄	.1009	.0458

where,

$$S_{A1}^* = .0545 = [(.1168 - .0934)^2 + (.0659 - .0366)^2 + (.0531 - .0506)^2 + (.0414 - .0506)^2 + (.1347 - .0962)^2 + (.2012 - .2012)^2]^{1/2}, \text{ and}$$

$$S_{A1}^- = .0983 = [(.0841 - .0934)^2 + (.0366 - .0366)^2 + (.0455 - .0506)^2 + (.0598 - .0506)^2 + (.0577 - .0962)^2 + (.1118 - .2012)^2]^{1/2}$$

and so on.

Once the separation measures have been calculated, the relative closeness to the ideal solution is determined as follows:

$$PI = \frac{S^-}{S^* + S^-}$$

The value PI is known as the *priority index* value. The PI represents the proportional distance of the alternative (A) between the positive-ideal (A*) and negative-ideal (A⁻) solutions. If the A = A* (which would result in an S* of 0), C would equal 1. If, on the other hand, A = A⁻ (which would result in an S⁻ of 0), C would equal 0. Alternatives located in-between would have a PI value somewhere between 0 and 1, with more favorable projects having a PI closer to 1. Once the PI has been calculated for each of the alternatives, they all can be ranked by their descending PI values. The calculation of

relative closeness of alternatives to the ideal solutions and their subsequent rankings is illustrated in the following example.

Example V.1—Step 5 (Calculate PI Values and Rank Projects)

Below are the separation measures for the four alternatives that were calculated in Example V.1—Step 4:

Alternative	Positive Separation S^*	Negative Separation S^-
A_1	.0545	.0983
A_2	.1197	.0439
A_3	.0580	.0920
A_4	.1009	.0458

These values can be used to determine the relative closeness of each alternative to the ideal solution in that set of alternatives. The relative closeness, otherwise known as the priority index number, for each of the alternatives can be summarized as follows:

Alternative	Priority Index (PI)
A_1	.643
A_2	.268
A_3	.613
A_4	.312

where $PI_{A_1} = 0.643 = 0.0983 / (.0545 + .0983)$, and $PI_{A_2} = 0.268 = .0439 / (.1197 + .0439)$, and so on.

These calculated values can be appended back to the original list of projects and scores, and then sorted according to descending PI value.

Alternative	PI	Criteria Scores					
		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆
A ₁	.643	2.0	1,500	20,000	5.5	5	9
A ₃	.613	1.8	2,000	21,000	4.5	7	7
A ₄	.312	2.2	1,800	20,000	5.0	5	5
A ₂	.268	2.5	2,700	18,000	6.5	3	5

The preference order is A₁, A₃, A₄ and then A₂.

Summary of TOPSIS Mathematical Issues

The issues that result from the mathematical procedure that is employed by the TOPSIS ranking method are as follows.

- Normalized scores will vary not only according to the value of the scores being normalized, but also according to the number of projects on the list.
- Definitions of the ideal-positive and ideal-negative solutions depend on the actual scores of the projects on the list being ranked.

CONCLUSIONS

In light of the identified issues, the following conclusions can be drawn regarding the TOPSIS methodology.

1. Variation in the possible range of scores on a list of projects is the major source of the dynamic nature of the final relative rankings from list to list. Scoring ranges affect both the normalized values of the criteria scores and the definition of the ideal-positive and ideal-negative solutions.
2. Establishing constrained scores and using those constraints to establish standard ideals should greatly stabilize the predicted relative ranking orders. This will stabilize the normalization process and allow the distances from ideal solutions to be uniform for a given alternative, independent of the characteristics of other projects.

3. Although constraining the possible range of scores will stabilize the normalized scores, small discrepancies between differing lists will remain, since the normalization is also a function of the number of projects on the list. Vector normalization is a necessary step in the prioritization process, since the seven criteria reflect different scales and units, and since TOPSIS utilizes vector calculations to rank projects. There is no way to avoid this without dictating the number of projects that must be included on a list.

CHAPTER VI: FORMATTING CHANGES TO TOPSIS

TOPSIS-6 was the version of the TOPSIS algorithm that was utilized for the 1995-1997 project prioritization. TOPSIS-6 was revised in response to varying requests by WSDOT personnel. In addition to the methodology issues that were identified in the previous chapter, WSDOT personnel also asked that use of the program be made more straightforward and that the opportunities for user error be minimized.

After the 1995-1997 prioritization, some interim formatting revisions were made to the program and its name was updated to TOPSIS-7. However, this version was never released. The final version of TOPSIS that will result from the revisions presented in this report is TOPSIS-8. Therefore, the program prior to revisions will be referred to as TOPSIS-6, and the revised program will hereafter be called TOPSIS-8.

The formatting revisions to TOPSIS-6 were made to the program *before* methodology revisions had been analyzed for the priority programming process. Because the revised program is so much simpler to run, these revisions also expedited the subsequent analysis of possible revisions to the methodology, in addition to responding to Objective #1 of this study.

It became evident during the 1995-1997 process that the program was prone to user error, mainly for two reasons:

1. Extensive sorting, copying, and pasting were required to transform the TOPSIS output into the format desired by WSDOT.

The ranked output of TOPSIS-6 included only the project descriptions and priority index numbers. To produce the final formatted lists of ranked projects that included the projects' criteria scores, the following steps had to be taken:

- a) Sort the input file of project names and criteria scores alphabetically by project description.

- b) Sort the output file of project names and priority index numbers alphabetically by project description (the output file is produced in ranked order - in the order of descending priority index value).
- c) Copy and paste the sorted project descriptions and criteria scores together with the sorted priority index numbers into a final output file.
- d) Resort the composite list of projects names, criteria scores, and priority index numbers back into the order of descending priority index value (ranked order).
- e) Format the final output file column widths and fonts to fit onto one landscape page width.

Done correctly, these steps would result in a formatted output file that included the project descriptions, criteria scores, and priority index numbers. However, the numerous steps also provided numerous opportunities for errors either in sorting or in “matching up lists” for the final output file

In addition, the project descriptions for TOPSIS-6 included each project’s region number, State Road (SR) number, and the project’s name, all of which were contained in one column. This made the final step of sorting the ranked list by region somewhat cumbersome, since the region number was simply a part of the text in the project description.

2. Although the ranking algorithm was easy to use, its simplicity allowed the program to be run incorrectly, which could in turn produce an incorrectly ranked output file. The only steps required to rank a spreadsheet of compiled project descriptions and criteria scores are to highlight the spreadsheet containing the projects and scores, and to run the TOPSIS macro. Although instructions were provided with TOPSIS-6, they often became separated from the actual program, especially when substantial time had elapsed between applications of the program. A common error was to

incorrectly highlight the input spreadsheet. This often still resulted in a ranked output file, although the rankings were incorrect.

In addition, the exclusion of the projects with extreme B/C values from the ranking process and their inclusion on the final ranked list presented potential errors. In some cases, the projects with the extreme B/C values were inadvertently included in the ranking process, which resulted in the calculation of incorrect priority index numbers and project rankings. In other cases, the extreme B/C projects were correctly cut from the ranking algorithm, but then “lost” and inadvertently left off the final ranked output file.

In response to these issues, the formatting revisions reflected in TOPSIS-8 are intended to achieve the following goals:

1. reduce efforts in producing final output
2. make the program more “user friendly”—thus minimizing potential for user error
3. allow a smooth transition from TOPSIS-6 to TOPSIS-8.

REVISIONS TO REDUCE EFFORTS IN PRODUCING FINAL OUTPUT

Appendix H contains sample input files and output files both before revision (TOPSIS-6) and after revision (TOPSIS-8). The TOPSIS-8 files include the following features that are not included in TOPSIS-6:

- ‘SR’ and ‘Region’ have their own separate columns in the project identification.
- Columns have been added to include the beginning and ending milepost numbers in the project identification.
- Criteria scores and costs are automatically carried with the project names and priority index numbers into the new output file (this feature was actually added for TOPSIS-7 [Reed, 1995] and carried over to TOPSIS-8).
- Formatting of the output file is automatically done within the program.

- An additional blank column is inserted into the first column of the output file, to be used at the user's discretion.

The main external change to the input file is that the one-column project description has been expanded into five columns (region number, SR number, project name, beginning milepost number, and ending milepost number). These revisions should allow for easier sorting and manipulation of the files, as well as eliminate some the steps in which match-up errors can be made.

REVISIONS TO MAKE TOPSIS MORE "USER FRIENDLY"

In addition to the physical revisions to the input file, revisions were made to the internal ranking process that would further minimize the chances for user error. These features include the following:

- High B/C projects are automatically sorted out of the ranking algorithm, but are still included at the top of the final ranked project list.
- Scores equaling zero are internally converted to 0.001 (to avoid potential normalization errors—if all of the project scores in one criterion category are zero, the normalization step would require division by zero, which would result in a math error and terminate the program).
- Additional 'alerts' are built into the program that warn the user of input error.
- TOPSIS instructions and a sample input sheet are appended to the input file template.
- The cells in the TOPSIS-8 macro are protected to prohibit a user from inadvertently changing the program coding when the macro is open (which is required to run the macro).

Appendix I contains the complete input workbook file for TOPSIS-8. The workbook contains three separate worksheets: (1) the actual input sheet that should be completed and highlighted for the application of the TOPSIS-8 macro, (2) instructions for running TOPSIS-8, and (3) a sample input sheet.

REVISIONS TO ALLOW A SMOOTH TRANSITION FROM TOPSIS-6 TO TOPSIS-8

Finally, since projects were already being compiled for prioritization for the upcoming biennium, the researchers tried to make the revised process consistent with the previous process. For this reason, TOPSIS-8 includes the following highlights:

- Aside from the revised format of the input file, all other changes are built into the program and will occur automatically. These include both the “user friendliness” revisions presented in this chapter and revisions to the methodology that are presented later.
- Scoring procedures required for the regions remain the same as those in the previous biennium.
- Detailed comments have been added to the TOPSIS macro.

CHAPTER VII: REVISIONS TO TOPSIS METHODOLOGY

ANALYSIS OF REVISIONS

To constrain the criteria scores and establish standard ideals, the following solutions had to be determined:

- how to constrain the upper limits for noise and wetlands scores
- how to constrain the upper limit for B/C score
- how to define the lower limit of B/C, given the intrinsic value of a B/C of 1.

These solutions, addressed below, formed the basis of the recommended methodology revisions to the TOPSIS ranking algorithm.

Analysis of Wetlands and Noise Scoring Constraints

Conversion of wetlands and noise scores to a constrained scale was based on (1) the distribution of the 1995-1997 scores and (2) feedback from the WSDOT programming and transportation data offices.

Distribution of the 1995-1997 scores for these two categories (Appendix D) showed that most of scores tended to be at the lower (positive) end of the scale. For this reason, intervals of original scores for conversion were smaller at the lower end of the scale and were graduated to increasingly larger intervals at the high end of the scale. This allowed constrained scores to be distinguished from each other in generally the same proportion as the original unconstrained scores.

The conversion of the original unconstrained wetlands scores to a constrained scale is illustrated in Table VII.1, and the conversion of the original unconstrained noise scores to a constrained scale is illustrated in Table VII.2.

Table VII.1
Conversion of Wetlands Scores to Constrained Scale

Equivalent # of Acres (Original Score)	Revised Score
0 - 0.5	0.5
0.6 - 1.0	1
1.1 - 3.0	2
3.1 - 5.0	4
5.1 - 10.0	8
10.1 - 20.0	10
20.1 - 30.0	15
30.1 - 40.0	20
40.1 - 60.0	25
60.1 - 90.0	35
> 90.0	40

Table VII.2
Conversion of Noise Scores to Constrained Scale

Equivalent # of Receptors (Original Score)	Revised Score
0	0
0.1 - 10	1
10.1 - 20	3
20.1 - 30	6
30.1 - 40	10
40.1 - 50	15
50.1 - 60	20
60.1 - 80	25
80.1 - 100	30
100.1 - 130	35
130.1 - 160	40
160.1 - 190	50
190.1 - 220	60
220.1 - 260	80
260.1 - 300	100
330.1 - 400	125
400.1 - 500	150
500.1 - 600	175
> 600	200

Since one of the objectives of this study was to facilitate a smooth transition from last biennium's procedure to this biennium, the conversion of the wetlands and noise

scores to a constrained scale was done internally within the TOPSIS program for calculation purposes only. The scoring procedures remain the same for the regions.

Testing of the ranking results showed that revision of the wetlands and noise scores to these scales resulted in minimal change to the relative rankings. The cases in which the converted scores did produce a change in rankings occurred when projects had very negative scores in either one of these two categories and thus originally ranked poorly because the score had been so extreme. Projects still tended to rank poorly, but often they moved up in ranking by one or two places (i.e., instead of ranking last on a list, a project might rank third to last).

Appendix J contains graphs that illustrate the scoring range of the wetlands and noise criteria before and after the scoring constraints were applied. In these graphs, the distribution curves of the constrained scores maintain roughly the same shape as the curve for unconstrained scores for both criteria categories.

Analysis of Upper Benefit-Cost Thresholds

Establishment of an upper B/C constraint was more complex than wetlands and noise. Since the value of benefit-cost is tangible to engineers and planners, there would be little advantage to converting it to something less tangible. In addition, the B/C criterion carries a much more significant weight than the wetlands and noise criteria (65 percent, compared to 2.7 percent), so ranking results would be much more sensitive to changes in the B/C criterion.

For these reasons, it was not practical to convert B/C scores to a constrained scale. Instead, it is recommended that the practice initiated last biennium practice be continued: that projects with B/C scores that exceed a defined upper threshold be excluded from the ranking algorithm.

The determination of a definition for the upper B/C threshold needed to be examined from two perspectives:

1. the influence that the upper B/C threshold has on consistency between statewide and regional rankings
2. the influence that the upper B/C threshold has on the correlation between the TOPSIS rankings and the B/C rankings.

Numerous tests showed that the consistency measures were similar at varying upper B/C thresholds. Thus the definition of B/C threshold did not have notable influence over consistency between statewide and regional rankings. However, the definition of the upper B/C threshold was determined to have a significant impact on the influence of the B/C scores on final project rankings.

There are many reasons that projects rank in order different than descending B/C values, either slightly or dramatically. The reasons that are independent of the mechanics of TOPSIS and are instead a function of the established criteria weights are as follows.

1. In cases where projects with low B/Cs receive high rankings, generally the remaining “non-monetary” scores collectively are more favorable.
2. In cases where projects with high B/Cs receive low rankings, generally one or more of the non-monetary scores are unfavorable.

It has also been determined, however, that the definition of the upper B/C threshold has a rather significant influence on the rankings of the projects with much lower B/Cs. This is an issue that is independent of the criteria weights, and it can be addressed within the TOPSIS algorithm.

Generally speaking, the lower the upper B/C threshold, the more “in line” the rankings are with the B/C ranking. The simplest explanation for this that *the relative distance between two scores will diminish as the range in which they are located increases*. For example, the difference between a B/C of 0.5 and a B/C of 3.0 is proportionately much less within a range of 0-70 than within a range of 0-30 (2.5 / 70 verses 2.5 / 30). This effect is exaggerated further by the heavy weight of the B/C score.

It is important to remember that TOPSIS understands B/C only as a score. Although the difference between a B/C of 0.6 and 2.5 is intrinsically significant, TOPSIS does not recognize that intrinsic quality. It only sees it as a distance, relative to the possible range of distances. B/C values traditionally range from about 0.5 to 3.0, so it was originally not anticipated that the extremely high ranges of B/C scores (resulting mainly from financial partnering) would exist. Since they do exist, an upper B/C threshold must be low enough to allow the B/C score to have reasonable influence.

This leads to a related concern voiced by some of the regions that the non-monetary criteria have too much influence. Ignoring for the moment that one reason for this could simply be a disagreement with the priorities established by the decision makers, to some degree, it could also be the reciprocal effect of the upper B/C issue. Establishing a lower B/C threshold will strengthen the influence of the B/C score, particularly at the lower end of the scale, thus indirectly diminishing the effect of the non-monetary scores.

This study established that “reasonable” influence is fairly subjective, so an attempt was made to view the differences between the TOPSIS rankings and the B/C rankings more objectively. In addition to visually inspecting the ranking order of the project lists, two measures were established:

- Percentage of projects that rank in order of B/C—this quantity measure counts each of the projects that have a B/C higher than the project immediately following it on the ranked list.
- Correlation coefficient—this indicates the magnitude of the correlation between the TOPSIS ranked list and the B/C list. A correlation coefficient of 1.0 indicates that the ranked list is in exact order of the B/C list.

The percentage indicates the *number* of projects that rank in B/C order, whereas the correlation indicates the *magnitude* of the deviation of the projects that are not in B/C order.

Table VII.3 shows the results of varying the thresholds for the 1995-1997 lists of projects. As the upper B/C threshold is lowered, the percentage of projects ranked in B/C

Table VII.3
Effect of Varying Upper Benefit-Cost Thresholds on Project Rankings
1995-1997 Project Lists

Upper B/C Threshold ¹	Urban (total of 103 projects on list)				Rural (total of 64 projects on list)			
	# Projects excluded from ranking algorithm ²	Projects with B/C higher than the project immediately following ³		Corr. Coeff between TOPSIS and straight B/C rankings ⁴	# Projects excluded from ranking algorithm ²	Projects with B/C higher than the project immediately following ³		Corr. Coeff between TOPSIS and straight B/C rankings ⁴
		# projects	% of total			# projects	% of total	
≤ 10	26	79	77 %	.9869	8	47	73 %	.9690
≤ 20	13	74	72 %	.9459	6	43	67 %	.9234
≤ 30	10	73	71 %	.9077	3	42	66 %	.8653
≤ 40	8	70	68 %	.8752	1	37	58 %	.8090
≤ 50	5	63	61 %	.8449	1	35	55 %	.7842
≤ 60	5	63	61 %	.8360	0	36	56 %	.7351
≤ 70	3	68	66 %	.8103	0	37	58 %	.7250

Notes: ¹ Definition of upper B/C "cut-off" - All projects with B/C scores above this ranking are excluded from the TOPSIS ranking process, and are ranked at the top of the list in descending B/C order

² Indicates the number of projects on the list with B/Cs above the defined threshold - treated as described in note (1)

³ Indicates the number and percentage of projects that rank in order of B/C

⁴ Indicates the magnitude of the deviation between the ranked list and the straight B/C list. A higher correlation (closer to 1.0) indicates that rankings deviate by fewer places between the two lists

order generally increases, and the correlation coefficient always increases (meaning the magnitude of the deviation from B/C rankings decreases).

On the basis of these results, it is tentatively recommended that the upper B/C threshold be defined as 30. Table VII.3 shows that at an established upper threshold of 30, both the urban and rural lists of projects meet the request by WSDOT that at least 65 percent of the projects rank in B/C order. The correlation coefficient indicates that although higher thresholds can also result in over 65 percent of the projects ranking in B/C order, the magnitude of the “jumps” in ranking that the remainder of the projects will take will be smaller at the lower threshold.

The percentages that are less than 100 percent and the correlation coefficients less than 1.0 indicate that lowering the threshold, and thus strengthening the influence of the B/C on a project’s ranking, still allows the other non-monetary criteria to influence the rankings. In addition, the reasons that a project ranks higher or lower because of the non-monetary criteria are much more obvious by visual inspection under these revised parameters.

Although this recommendation is based on tests that were performed on the 1995-1997 biennium lists, it should not be confirmed until the 1997-99 list of projects is finalized. If the highest B/C value for the upcoming biennium were for instance, no more than 5, it would be unreasonable to establish 30 as the ideal.

Analysis of Lower Benefit-Cost Thresholds

Since many projects with a B/C of less than 1.0 outranked projects with a B/C of greater than 1.0 in the 1995-1997 biennium, WSDOT personnel wondered what the implications would be if the lower B/C threshold were raised above 0 (for example, if policy were adopted that prohibited projects with a B/C below a defined lower threshold from being submitted). The two primary reasons that projects rank in order different than descending B/C values had already been established:

1. In cases where projects with low B/Cs receive high rankings, generally the remaining “non-monetary” scores collectively are more favorable.
2. In cases where projects with high B/Cs receive low rankings, generally one or more of the non-monetary scores are unfavorable.

Another way to look at the non-monetary criteria is as quantified benefits or costs that would be part of the B/C value if a method existed by which to monetize them. This perspective leads to two scenarios: (1) a project with a low B/C but highly favorable scores in the six other criteria would have a higher B/C if the benefits associated with those criteria were monetized; (2) a project with a higher B/C but very poor scores in one or more of the other criteria would have a lower B/C if the costs associated with those criteria were monetized. Note that eliminating projects with lower B/Cs would disregard the former of these two possibilities. Nevertheless, the implications of doing so were examined.

To assess this issue, cases were run with lower B/C thresholds of 0.5 and 0.8, and compared to a base case in which the lower B/C threshold remained zero.

For each of these two test thresholds, two approaches were tested and compared to the base case.

1. Change the definition of the lower B/C threshold, and eliminate all projects with B/Cs below that threshold from the ranking (assume that they would automatically go to the bottom of the ranked list)

Results: While the eliminated projects obviously dropped out of the ranked list, the remaining projects either maintained the same relative order or actually ranked in an order less similar to the B/C order than the base case.

2. Change the definition of the lower B/C threshold (definition of the ideal-negative score), but include the projects with B/Cs below that threshold in the ranking process

Results: The ranked order was *identical* to the base case. This makes sense since the relative distance from a zero threshold verses a threshold of less than

1.0 would be hardly distinguishable, given an upper range of up to 70, or even 30.

These results were consistent whether the lower B/C threshold was defined as 0.5 or 0.8.

The result of the analysis was the conclusion that slightly raising the lower B/C threshold has zero to negative effect on the ranking order (or in particular the B/C order) of a list of projects, unless of course the projects with B/Cs below the minimum are simply excluded from the ranking. The same effect would be achieved through a policy decision mandating that no projects with a B/C below a specified minimum be allowed to be submitted for ranking. If such a measure were desired, it would be more appropriate as a policy measure than as a component of the ranking algorithm. However, to do so would indicate some disregard for the validity of the non-monetary criteria.

SUMMARY OF RECOMMENDED REVISIONS TO TOPSIS METHODOLOGY

Specific revisions to the TOPSIS methodology were based on the results of testing revision alternatives on the 1995-1997 urban and rural lists of projects. Consequently, in addition to the formatting changes to TOPSIS to improve output and ease of use, the following revisions to the TOPSIS methodology are recommended:

- The upper B/C threshold should be defined internally, and any projects that have a B/C score that exceeds the threshold would be automatically excluded from the ranking process, and re-inserted back at the top of the final ranked list. Tentatively, the upper threshold is recommended to be 30, given the results of testing the 1995-1997 lists. However, this should be reconfirmed with the 1997-1999 lists.
- The raw *wetlands* scores should be converted from their previous “limitless” scale to a constrained scale of 0 to 40, and the raw *noise* scores should be converted from their previous “limitless” scale to a constrained scale of 0 to 200. These conversions would be performed internally for the TOPSIS calculations, but the actual raw scores would appear on the input and final output files.

- Standard theoretical positive-ideal and negative-ideal project scores should be appended to the list of projects so that the distances of ranked projects from the ideal solution will not depend on the scores on the project list. These theoretical projects would not appear on the final output file.

Table VII.4 illustrates the revised scoring ranges for TOPSIS-8 ranking calculations.

Table VII.4
Criteria Categories and Weights with Revised Scoring Ranges
1995-1997 Biennium

	Criteria						
	Benefit-Cost Ratio	Community Support	Wetlands	Water Quality	Noise	Mode Integration	Land Use
Criteria Weight	0.65	0.14	0.0267	0.0267	0.0266	0.07	0.06
Most favorable score	no upper limit [†]	0	0.5	0	0	0	19
Least favorable score	0	17	40	41	200	10	0

[†] Although no upper limit is defined, projects with B/C scores exceeding 30 are recommended to be excluded from the TOPSIS ranking procedure for 1997-1999 biennium

Appendix K contains the complete listing of TOPSIS-8, which is written in the EXCEL Macro 4.0 language. Column 'A' of that file contains the actual program coding, and column 'B' contains detailed comments regarding the function of each of the lines of coding in the program.

RESULTS OF REVISED TOPSIS

Below are the major steps of the TOPSIS-8 ranking algorithm (1997-1999 biennium), given the recommended revisions to the process (steps marked with double asterisks were not included in the 1995-1997 version of TOPSIS-6):

- Sort projects out of the ranking list with B/C scores of greater than the specified maximum.**

2. Convert raw *Wetlands* scores to a constrained scale.**
3. Convert raw *Noise* scores to a constrained scale.**
4. Append pre-defined theoretical positive-ideal and negative-ideal solutions to the list of projects.**
5. Replace all scores of 0 with .001 to avoid potential “divide by zero” math errors.**
6. Calculate the sum of squares value for each of the criteria categories.
7. Create the normalized matrix by dividing each criterion score by the sum of squares value of its respective criterion category.
8. Create the weighted normalized matrix by multiplying each normalized score by its respective criterion category weight.
9. Identify the scores for each criterion that makes up the positive-ideal and negative-ideal solutions.
10. For each project, calculate the composite separation value from the positive-ideal solution [positive separation = S_{i+}].
11. For each project, calculate the composite separation value from the negative-ideal solution [positive separation = S_{i-}].
12. For each project, calculate the priority index number, which represents the closeness to the ideal solution [Priority Index = $PI = S_{i-} / (S_{i+} + S_{i-})$].
13. Sort projects in descending order of PI value - this is the ranked preference order.
14. Insert any high B/C projects excluded from the ranking algorithm back on to the top of the ranked project list.**
15. Create an output file that includes a formatted ranked list of project names and their original scores, as well as their PI values.**

Consistency between Statewide and Regional Rankings

Appendix L contains the correlation graphs that illustrate the consistency between the statewide and regional rankings produced with the revised methodology of TOPSIS-8.

The graphs also illustrate the TOPSIS-6 results for comparison to the results of the refined procedure.

Tables VII.5 and VII.6 show the numerical summary of these comparisons. The tables suggest that by constraining the scores and defining standard ideals, the consistency between the statewide and regional rankings can be vastly improved. Even in the urban list of projects for Region 1, where the greatest degree of variation between the two scenarios still exists, the magnitude of the changes has been drastically reduced. The remaining inconsistencies (due to variation between the normalization of the differing lists) can be put further into perspective by examining the PI values of the projects that continue to move between scenarios, as will be discussed in the following section.

Priority Index Number Analysis

Appendix M shows a side by side comparison of the statewide and regional rankings and their respective priority index numbers. Within these listings, the projects that still change rankings between the two scenarios are highlighted in gray. In every case where rankings switch places, the PI values are very close to one another. This indicates that these projects are similar in attractiveness with respect to the ideal solution. The similarity in the priority index numbers provides a rational basis by which to facilitate tradeoff decisions. Decisions can be made on a case by case basis, using the PI values as evidence that the projects under question are similar in their attractiveness as preferred solutions. In addition to providing a rational basis for settling ranking disputes, similar PIs could also provide insight in a case where one expensive project slightly outranked two or more less expensive projects. In this case, it would be possible to justify funding the two projects instead of the one, if they were all determined to be similarly attractive.

Table VII.5
Comparison of Consistency Between the Statewide and Regional Rankings
1995-1997 Biennium Urban Project List

Region #	Urban List before revision (TOPSIS-6)			Urban List after revision (TOPSIS-8)		
	# Projects Moved	# Projects on List	Correlation Coefficient	# Projects Moved	# Projects on List	Correlation Coefficient
1	30	41	0.93	25	41	0.99
2	2	3	0.50	0	3	1.00
3	10	30	0.99	11	30	0.99
4	6	10	0.85	2	10	0.99
5	4	10	0.93	0	10	1.00
6	0	9	1.00	0	9	1.00

Table VII.6
Comparison of Consistency Between the Statewide and Regional Rankings
1995-1997 Biennium Rural Project List

Region #	Rural List before revision (TOPSIS-6)			Rural List after revision (TOPSIS-8)		
	# Projects Moved	# Projects on List	Correlation Coefficient	# Projects Moved	# Projects on List	Correlation Coefficient
1	9	13	0.68	0	13	1.00
2	0	2	1.00	0	2	1.00
3	11	22	0.97	2	22	0.99
4	6	6	-0.71	0	6	1.00
5	0	14	1.00	2	14	0.99
6	7	7	0.32	0	7	1.00

B/C Correlation

Table VII.7 summarizes the correlation with B/C rankings that results from the revised TOPSIS, given a defined upper B/C threshold of 30.

Table VII.7
Correlation of TOPSIS-8 and B/C Rankings
1995-1997 Biennium Statewide Lists

	Urban List		Rural List	
	TOPSIS-6	TOPSIS-8	TOPSIS-6	TOPSIS-8
Percentage of Projects in B/C Order	61 % (63 / 103 projects)	71 % (73 / 103 projects)	59 % (38 / 64 projects)	66 % (42 / 64 projects)
B/C Correlation Coefficient	0.86	0.91	0.81	0.87

The table shows that in both the urban and rural lists of projects, TOPSIS-8 rankings result in a higher correlation between the TOPSIS rankings and the B/C rankings. In other words, the B/C has a stronger influence over the project rankings. The reasons for this are (1) constraining the wetlands and noise criteria slightly decreases their negative influence over rankings, particularly when a project receives an extreme unfavorable score in either of these categories, and (2) lowering the upper B/C threshold increases the influence of the B/C criterion because the proportional differences of B/C scores from the ideal solution become less pronounced.

Preliminary Results of the 1997-1999 Prioritization

At the time this analysis was completed, it was too early to present the ranking results produced by these revisions of the 1997-1999 biennium lists, since they had not yet been finalized. However, preliminary tests of the 1997-1999 biennium lists indicated that the results are consistent with those presented in this report. As a matter of fact, the upper B/C threshold of 30 looks as though it will be very appropriate. So far, of the projects submitted, one has a B/C of over 200, but the remainder of the projects range from 27 down to less than 1. The project with a B/C of 200 would obviously be excluded from the ranking algorithm, and 30 would be an ideal B/C score in line with the actual projects.

CHAPTER VIII: COMPARISON OF TOPSIS TO SAW MODEL RESULTS

In the Simple Additive Weighting (SAW) model, project scores would be normalized on a linear scale, multiplied by their respective weights, and added across criteria to determine a final ranking index number. This method is intuitively more straightforward than the TOPSIS method and thus could be potentially more attractive to users. For this reason, the SAW Model was applied to the 1995-1997 urban and rural lists, and its results were compared to those of TOPSIS.

METHOD OF ANALYSIS

To rank a project list by the SAW method, the following steps were taken:

Step 1

Normalize the criteria scores to a scale of 0 to 50 with the linear scale normalization technique, given the following two considerations:

- a. For five of the criteria, lower scores are more favorable, and for two of the criteria, higher scores are more favorable.
- b. The minimum values of the criteria range between zero and one (as opposed to simply being zero).

The following formulas were used to transform each individual criterion score, x_{ij} , into a normalized score, r_{ij} . For criteria where a lower score is more favorable:

$$r_{ij} = 50 \times \frac{x_{\max} - x_{ij}}{x_{\max} - x_{\min}}$$

And for criteria where a higher score is more favorable:

$$r_{ij} = 50 \times \frac{x_{ij} - x_{\min}}{x_{\max} - x_{\min}}$$

where, r_{ij} = the normalized score
 x_{ij} = the original score

x_{\min} = the lowest possible score for that particular criterion

x_{\max} = the highest possible score for that particular criterion

The result will be normalized scores in which the most favorable values approach 50, for all seven criteria.

Step 2

Multiply normalized scores by their respective criteria weights.

Step 3

Add the normalized weighted scores across the criteria for each project - this sum for each individual project constitutes the priority index value.

Step 4

Sort the project list by descending priority index values—this is the ranked order.

This procedure was performed on the both the urban and rural project lists under each of the following two scenarios:

1. “Unconstrained scores”—this is comparable to the conditions under which projects were normalized last biennium under TOPSIS-6. The definition of x_{\min} and x_{\max} depend on the actual scores on the lists. Thus, lists with different upper and lower criteria limits could potentially have different minimums and maximums, and in turn could have different values for normalized scores.
2. “Constrained” scores—this is comparable to the conditions under which projects are normalized under the proposed TOPSIS-8 procedure. The definition of x_{\min} and x_{\max} would be standard, based on the predefined upper and lower criteria thresholds. Under these conditions, the normalized values of the scores would be the same, independent of the characteristics of the other projects on the list. For this application, the criteria constraints presented in Table VII.4 were used so that they could be compared to the TOPSIS results under the same constraints. Projects with a B/C higher than

the defined maximum of 30 ranked at the top of the list in order of descending B/C value.

RESULTS OF ANALYSIS

Appendix N contains the ranked list of projects produced by the SAW model under both the “unconstrained score” and “constrained score” scenarios. The correlation of the additive rankings with the B/C scores are presented in Table VIII.1, along with the same measures calculated for the TOPSIS rankings. In the table under the “unconstrained” scenario, the B/C criterion has more influence over project rankings under TOPSIS than it does under the SAW model. Under the “constrained” scenario, the distinction between the two methods is less, but the TOPSIS results still reflect the B/C scores more strongly.

Table VIII.1
Comparison of B/C Consistency
SAW Rankings vs TOPSIS Rankings—1995-1997 Biennium Lists

	Urban List		Rural List	
	% of Projects in B/C order	B/C Correlation Coefficient	% of Projects in B/C order	B/C Correlation Coefficient
SAW Model Results (Unconstrained Scores)	54%	0.51	53%	0.69
TOPSIS-6 Results (Unconstrained Scores)	61%	0.86	59%	0.81
SAW Model Results (Constrained Scores)	58%	0.91	63%	0.82
TOPSIS-8 Results (Constrained Scores)	71%	0.91	66%	0.87

The reason that the B/C has diminishing influence under the SAW model is that under normalization, the high value for one attribute must receive approximately the same normalized value as the high value for another attribute. For instance, take a hypothetical project with a B/C of 1 or 2 and high non-monetary scores. Under normalization, this project would receive points in the B/C category much closer to zero than to 50 (say 5 points). However, it could conceivably receive scores close to 50 in the other six

categories. 300 points weighed at 35 percent would far wield far more influence than 5 points weighed at 65 percent.

Testing different “upper B/C” thresholds also determined that, unlike with TOPSIS, the trend in B/C correlation is not predictable with the additive model. With TOPSIS, lowering the upper threshold always increases the correlation between the TOPSIS and B/C rankings. This is not the case with the SAW method.

Although the B/C correlation would be less favorable under the SAW method, the constrained SAW scenario would provide complete consistency between statewide and regional rankings. Since the maximums and minimums are uniform, the normalized score would always be the same, and thus the final priority index value would be as well. The unconstrained SAW scenario, on the other hand, would produce the same or worse consistency as TOPSIS. Since the maximum values for normalization would be dynamic, the normalized scores and resulting rankings would be dynamic as well.

CONCLUSIONS

Many of the issues concerning TOPSIS are similar for the additive model, except that they are more exaggerated for the additive model.

- Similar scaling issues exist for the SAW method as for TOPSIS normalization. They are actually more exaggerated under linear normalization than under vector normalization.
- As a result, the SAW model produces project rankings that are less correlated with B/C rankings than does the TOPSIS model. TOPSIS better meets the correlation objectives defined in this study under the same model parameters.
- The constrained scenario does guarantee complete consistency between statewide and regional rankings under the SAW method, but it was also shown that the constrained model of TOPSIS provides close to complete consistency, and the remaining inconsistencies are inconsequential.

Under identical parameters, the SAW model does slightly better than TOPSIS in consistency between differing lists, but TOPSIS is more predictable and provides B/C correlation much more in line with the project objectives. Thus the SAW model was not shown to be an improvement over TOPSIS. In fact, the final conclusion is that TOPSIS is the better model and remains the preferred ranking model.

CHAPTER IX: CONCLUSIONS

SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

1. Inconsistency can be greatly minimized by defining the ideal projects

To establish the **ideal-negative** project,

- Five out of seven criteria already have established ‘worst’ scores.
- Worst scores must be determined for the remaining two criteria, *wetlands* and *noise*.

Recommendation: Convert existing “unconstrained” scores to a “constrained” scale. Convert wetlands to a scale of 0 to 40, and convert noise to a scale of 0 to 200.

To establish the **ideal-positive** project,

- Six out of seven criteria already have established ‘best’ scores.
- A best score must be established for benefit-cost ratio, at least for the purposes of the ranking algorithm.
- Conversion of benefit-cost to a constrained scale is not recommended.
 - Its influential weight will be more sensitive to an “arbitrary” scale.
 - It is the most “tangible” criterion, so there is little advantage in changing it to something less tangible.

Recommendation: Establish an upper B/C threshold (a “cut-off B/C”). Projects with B/C scores higher than the threshold will be excluded from the ranking algorithm, but later will be automatically be ranked at the top of the list in order of their B/Cs.

- Numerous test runs showed that even if they are included in the ranking algorithm, projects with extremely high B/Cs will rank at the top of the list anyway.

- Other numerous test runs showed that selection of a “cut-off B/C” can be somewhat arbitrary, without compromising the extent of ranking consistency.

Recommendation: Once an upper and lower limit have been defined for each of the seven criteria, the ideal-positive and ideal-negative solutions can be defined based on those limits. They should be defined internally within the TOPSIS model so that all lists of projects are consistently compared to standard ideals.

2. Normalization cannot be made entirely consistent within given constraints. Scaling issues remain, no matter what method of normalization is used. However, normalization of scores is essential for inter-attribute comparisons, and the drawbacks of the process are slight and tolerable.
3. Even with variance caused by normalization, remaining inconsistencies are within reason. Controversy surrounding remaining ranking inconsistencies should be minimal and can be addressed through policy decision. Projects that continue to switch rankings between lists have priority index numbers that are similar, meaning that they have similar attractiveness as a preferred alternative. Similar PIs can be used as the basis on which to make tradeoff decisions.
4. The refinements recommended in this report to the TOPSIS methodology will not change the results of the statewide rankings as much as they will change the results of the regional rankings to better predict what will happen to a project on the statewide list.

COMPLETE SUMMARY OF TOPSIS-8 FEATURES

Below is a summary of all of the changes to TOPSIS recommended in this report, both to the mechanics of the program and the methodology.

1. Reduce the efforts required to produce final output.
 - a) 'SR' and 'Region' have their own separate columns in the project identification.
 - b) Columns have been added to include the beginning and ending milepost numbers in the project identification.
 - c) Criteria scores and costs are carried with the project names and index numbers into the new output file.
 - d) Formatting of the output file is built into the program.
 - e) An additional blank column is inserted into the first column of the output file, to be filled at the user's discretion.
2. Make the program more "user friendly," thus minimizing potential for user error.
 - a) Non-monetary criteria worksheets are contained in an EXCEL workbook.
 - b) High B/C projects are automatically sorted out of the ranking algorithm, but they are still included at the top of the final ranked project list.
 - c) Scores equaling zero are internally converted to 0.001 (to avoid potential normalization errors).
 - d) Additional 'alerts' are built into the program that warn the user of input error.
 - e) TOPSIS instructions and a sample input sheet are appended to the input file template.
 - f) The cells in the TOPSIS-8 macro are protected to prohibit a user from inadvertently changing the program coding.
3. Minimize ranking inconsistencies.
 - a) *Wetlands* and *noise* are internally converted to a constrained scale.
 - b) 'Ideal-positive' and 'ideal-negative' projects are internally defined.
 - c) High B/C projects are internally excluded from the ranking algorithm but included in the final ranked list.

4. Allow a smooth transition from old version to the new version.
 - a) Aside from the revised format of the input file, all other changes are built into the program and will occur automatically.
 - b) Scoring procedures required for the regions will remain the same as those in the previous biennium.
 - c) Detailed comments have been added to the TOPSIS macro.

RECOMMENDED FURTHER RESEARCH

- It has been proposed that the community support criterion be eliminated from the 1999-2001 biennium rankings. The implications of eliminating this criterion, as well as the subsequent redistribution of the criteria weights, must be studied before the elimination can occur.
- Continued synthesis and evaluation should occur for the non-monetary criteria definitions and scores to assess both the degree to which the worksheets reflect state policy and the degree to which submitted projects reflect state policy. Research is currently being completed at the UW on the sensitivity of TOPSIS rankings to the criteria scores and weights. This research should provide a good statistical basis for continued evaluation of the ranking implications of the criteria.
- Continued efforts should be made to monetize as much of the criteria as possible, particularly for non-traditional benefits. The more criteria that are monetized, the less potential will arise for controversy surrounding project rankings with respect to B/C values.
- Continued efforts should be made to quantify alternative modes for inclusion in this prioritization process. The procedure is intended to be multimodal, and the nature of the projects prioritized for the 1995-1997 ranged more than they ever had. However, alternative modes (such as transit, TDM, or non-motorized) are still not being included for consideration. Quantification methods (particularly for cost-efficiency) must be established for these modes if they are to be included.

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