Shaking Table Tests on Piles

WA-RD 158.1

Final Report
June 1988
SHAKING TABLE TESTS ON PILES

by

John F Stanton, Sunirmal Banerjee
and Izzat Hasayen

Department of Civil Engineering
University of Washington
Seattle, WA 98195

Washington State Transportation Center (TRAC)
4507 University Way N.E., Suite 204
Seattle, Washington 98105

Washington State Department of Transportation
Technical Monitor: Keith Anderson, Materials Engineer

Final Report

Research Project Y-2811
Task 26

Prepared for
Washington State Transportation Commission
Department of Transportation
in cooperation with
US Department of Transportation
Federal Highway Administration

June 1988
SHAKING TABLE TESTS ON PILES

John F. Stanton, Sunirmal Banerjee and Izzat Hasayen

Department of Civil Engineering
University of Washington
Seattle, WA 98195

Washington State Department of Transportation
Transportation Building, KF-01
Olympia, WA 98504

This study was conducted in cooperation with the U.S. Department of Transportation, Federal Highway Administration

This research was undertaken to explore the interaction behavior of soil-pile systems subjected to static and dynamic lateral loads. The principal objective of the study was to assess the applicability and accuracy of one of the prominent methods of analysis by comparing the predicted responses with the measured responses.

Presented in this report are a brief survey of the related literature on the existing analysis techniques and previous experimental studies, the details of the experimental work performed under the current study, and the appraisal of the performance of a finite element program adopted for making theoretical predictions of the experimental responses.

In the present study, both static and dynamic experiments were conducted to obtain experimental data against which the analytical predictions could be verified. The experiments included laboratory simulation of the response of piles subjected to static or dynamic lateral loads applied at the pile-head and of piles embedded in a soil deposit subjected to bedrock motions. Finite element analyses of the model systems were carried out using reasonable estimates of the system parameters. No attempts were made to establish the model parameters through rigorous identification procedures.

It is shown that the agreement between the predicted and measured responses can be excellent even if the properties and parameters of the soil-pile system are only roughly estimated.

Piles, shaking table, lateral load, seismic behavior, earthquake, earthquake simulator, dynamic load test

No restrictions. This document is available to the public through the National Technical Information Service, Springfield, VA 22616

None.

None.

117

None.
DISCLAIMER

The contents of this report reflect the views of the authors, who are responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Washington State Transportation Commission, Department of Transportation or the Federal Highway Administration. This report does not constitute a standard, specification or regulation.
ACKNOWLEDGEMENTS

The research described in this report was funded by the Washington State Department of Transportation in cooperation with the US Department of Transportation, Federal Highway Administration.

The research was conducted by the University of Washington Department of Civil Engineering. John F. Stanton and Sunirmal Banerjee were the Co-principal Investigators. The experimental work was done by former graduate student Izzat Hasayen. Thanks are due to Prof. J.M. Roesset, of the University of Texas at Austin, for permission to use his computer program and for his advice about the analytical part of this research. The assistance of John Frazier and Kenneth Knowlan in making the test apparatus and of Vinh-Phuc Truong, Carole McCutcheon, and Trinh Mai in preparing the report is gratefully acknowledged.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>xiii</td>
</tr>
<tr>
<td>Summary</td>
<td>xv</td>
</tr>
<tr>
<td>Conclusions</td>
<td>xvii</td>
</tr>
<tr>
<td>Recommendations</td>
<td>xix</td>
</tr>
<tr>
<td>CHAPTER 1: Introduction</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 2: Review of Previous Work</td>
<td>3</td>
</tr>
<tr>
<td>2.1 General</td>
<td>3</td>
</tr>
<tr>
<td>2.2 Soil pile-Interaction Models</td>
<td>3</td>
</tr>
<tr>
<td>2.2.1 Discrete Models</td>
<td>3</td>
</tr>
<tr>
<td>2.2.2 Continuum Models</td>
<td>5</td>
</tr>
<tr>
<td>2.2.3 Finite Element Models</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Experimental Verification</td>
<td>7</td>
</tr>
<tr>
<td>CHAPTER 3: Experiments</td>
<td>11</td>
</tr>
<tr>
<td>3.1 General</td>
<td>11</td>
</tr>
<tr>
<td>3.2 Equipment</td>
<td>12</td>
</tr>
<tr>
<td>3.2.1 Shaking Table</td>
<td>12</td>
</tr>
<tr>
<td>3.2.2 Pile and Soil Model</td>
<td>12</td>
</tr>
<tr>
<td>3.2.3 Shear Bag</td>
<td>12</td>
</tr>
<tr>
<td>3.2.4 Loading System</td>
<td>13</td>
</tr>
<tr>
<td>3.3 Instrumentation and Data Acquisition System</td>
<td>14</td>
</tr>
<tr>
<td>3.3.1 Instruments</td>
<td>14</td>
</tr>
<tr>
<td>3.3.2 Data Acquisition System</td>
<td>15</td>
</tr>
<tr>
<td>3.4 Test Program and Procedures</td>
<td>16</td>
</tr>
<tr>
<td>3.4.1 Calibration Tests</td>
<td>16</td>
</tr>
<tr>
<td>3.4.2 Phase I Tests</td>
<td>16</td>
</tr>
<tr>
<td>3.4.3 Phase II Tests</td>
<td>17</td>
</tr>
<tr>
<td>CHAPTER 4: Method of Analysis and Selection of Parameters</td>
<td>21</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>21</td>
</tr>
<tr>
<td>4.2 Theoretical Background</td>
<td>21</td>
</tr>
<tr>
<td>4.3 Selection of Parameters</td>
<td>22</td>
</tr>
<tr>
<td>CHAPTER 5: Comparison of Experimental and Analytical Results</td>
<td>25</td>
</tr>
<tr>
<td>5.1 General</td>
<td>25</td>
</tr>
<tr>
<td>5.2 Phase I - Static Load Test Results</td>
<td>25</td>
</tr>
<tr>
<td>5.3 Phase I - Dynamic Load Test Results</td>
<td>28</td>
</tr>
<tr>
<td>5.4 Phase II Test Results</td>
<td>28</td>
</tr>
</tbody>
</table>
5.4.1 General
5.4.2 Pile Lateral Deformation
  5.4.2.1 Without the Steel Ring
  5.4.2.2 With the Steel Ring
5.4.3 Pile-head displacement
  5.4.3.1 Without the Steel Ring
  5.4.3.2 With the Steel Ring
5.4.4 Soil displacements
5.5 Prediction of full-scale response

CHAPTER 6: Theoretical Prediction of Fixed-head Pile Response

CHAPTER 7: References

LIST OF APPENDICES

A: Input for PILE I program
B: Example Calculations of Beam on Elastic Foundation
LIST OF TABLES

5.1 Free-head pile loaded at the soil surface  
6.1 Fixed-head pile loaded at the soil surface

LIST OF FIGURES

3.1 Soil box with pile - section  
3.2 Soil box with pile - plan  
3.3 Rubber bag - elevation  
3.4 Rubber bag - plan  
3.5 Loading system  
3.6 Load cell  
4.1 Finite element idealization  
5.1 Variations of soil shear modulus with depth used in theoretical predictions.  
5.2 Variation of moment down pile using three different soil shear moduli (P=224 lb., M=335 lb.-in used for theoretical results)  
5.3 Variation of moment down pile for static load of P=224 lb. & M=335 lb.-in  
5.4 Variation of moment down pile for static load of P=185 lb. & M=231 lb.-in.  
5.5 Variation of moment down pile for static load of P=100 lb. & M=125 lb.-in  
5.6 Static load versus pile displacement at soil surface  
5.7 Variation of moment down pile for 10 Hz frequency (P=188 lb., M=250 lb.-in).  
5.8 Variation of moment down pile for 20 Hz frequency (P=178 lb., M=240 lb.-in).  
5.9 Variation of moment down pile for 30 Hz frequency (P=167 lb., M=225 lb.-in).  
5.10 Variation of moment down piles for 40 Hz frequency (P=113 lb., M=180 lb.-in).
5.11 Variation of moment down pile for 50 Hz frequency (P=88 lb., M=140 lb.-in).

5.12 Variation of moment down pile for 60 Hz frequency (P=84 lb., M=135 lb.-in.)

5.13 Variation of moment down pile for 70 Hz frequency (P=66 lb., M=105 lb.-in).

5.14 Variation of moment down pile for 80 Hz frequency (P=38 lb., M=100 lb.-in).

5.15 Variation of moment down pile for 90 Hz frequency (P=36 lb., M=95 lb.-in).

5.16 Variation of moment down pile for 100 Hz frequency (P=32.7 lb., M 85 lb.-in.).

5.17 Variation of moment down pile for 110 Hz frequency (P=27 lb., M=70 lb.-in).

5.18 Pile-head displacement versus frequency, comparison between measured and predicted (PILE1)

5.19 Pile-head lateral stiffness versus frequency

5.20 Dynamic amplification of pile deformation versus frequency for maximum applied nominal motion of 0.05" (without steel ring)

5.21 Dynamic amplification of pile deformation versus frequency for maximum applied nominal motion of 0.1" (without steel ring)

5.22 Dynamic amplification of pile deformation versus frequency for maximum applied nominal motion of 0.25" (without steel ring)

5.23 Dynamic amplification of pile deformation versus frequency for maximum applied nominal motion of 0.05" (with steel ring)

5.24 Dynamic amplification of pile-head displacement versus frequency for maximum applied nominal motion of 0.05" (without steel ring)

5.25 Dynamic amplification of pile-head displacement versus frequency for maximum applied nominal motion of 0.1" (without steel ring)

5.26 Dynamic amplification of pile-head displacement versus frequency for maximum applied nominal motion of 0.25" (without steel ring)
5.27 Dynamic amplification of pile-head displacement versus frequency for maximum applied nominal motion of 0.05" (with steel ring)

5.28 Dynamic amplification of soil displacement at 28" versus frequency for maximum applied nominal motion of 0.05" (without steel ring)

5.29 Dynamic amplification of soil displacement at 28" versus frequency for maximum applied nominal motion of 0.1" (without steel ring)

5.30 Comparison of measured and predicted pile-head load versus moment. (Tests reported by O’Neill and Murchison, 1983).

5.31 Comparison of measured and predicted pile-head load versus displacement (Tests reported by O’Neill and Murchison, 1983)

5.32a Comparison of measured load vs. distance to maximum moment with values predicted by PILE1 (Tests reported by O’Neill and Murchison, 1983)

5.32b Comparison of measured load vs distance to maximum moment with values predicted by various methods (Tests reported by O’Neill and Murchison, 1983)

5.33 Variation of moment down pile (Tests reported by O’Neill and Murchison, 1983)

5.34 Comparison of measured load vs. deflection with values predicted by various methods (Tests reported by O’Neill and Murchison, 1983)

5.35 Comparison of measured load versus maximum moment with values predicted by various methods (Tests reported by O’Neill and Murchison, 1983)

6.1 Variation of moment for a fixed-head pile under static loading using three different soil shear modulus profiles

6.2 Variation of moment for a fixed-head pile under dynamic loading using profile C
ABSTRACT

This research was undertaken to explore the behavior of a soil-pile system subjected to static and dynamic lateral loads. The principal objective of the study was to assess the applicability and accuracy of one of the prominent methods of analysis by comparing the predicted responses with the measured responses.

Presented in this report are a brief survey of the related literature on the existing analysis techniques and previous experimental studies, the details of the experimental work performed under the current study, and the appraisal of the performance of a finite element program adopted for making theoretical predictions of the experimental responses.

In the present study, both static and dynamic experiments were conducted to obtain experimental data against which the analytical predictions could be verified. The experiments included laboratory simulation of the response of piles subjected to static and dynamic lateral loads applied at the pile-head and of piles embedded in a soil deposit subjected to bedrock motions. Finite element analyses of the model systems were carried out using reasonable estimates of the system parameters. No attempts were made to establish the model parameters through rigorous identification procedures.

It is shown that the agreement between the predicted and measured responses can be excellent even if the properties and parameters of the soil-pile system are only roughly estimated.
SUMMARY

Pile foundations must be designed to withstand lateral loads arising from wind action, earthquake forces, wave and water forces and machine vibrations. Careful analysis of the interaction between a pile and the surrounding soil is a crucial part of a satisfactory design, particularly when dynamic loading is involved. Several methods for analyzing the response of piles have been proposed in the past. However, few dynamic experiments have been conducted to substantiate the analyses.

In this research both physical experiments and finite element analyses were performed. The objectives were to obtain information on the dynamic behavior of embedded piles subjected to lateral load and to appraise one of the prominent methods of analysis for its ability to reproduce the experimental results.

The experiments were carried out on the University of Washington shaking table, using scale model piles embedded in Ottawa sand. First, the soil mass was fixed and the pile was loaded horizontally at its head, both statically and dynamically. Then the soil was placed in a special shear-flexible container which allowed it to undergo shear deformations such as would be experienced in the field under seismic loading. The pile was embedded in the container and dynamic base motions were imposed on the whole system. The curvature of the pile and the displacements of the soil mass were recorded.

The curvature (or bending moment) in the pile was the main response quantity used in the correlation between the experimental and the predicted responses. The correlation was excellent for the tests conducted with loads applied at the pile-head. The response was found to depend on the choice of the distribution of shear modulus of the upper soil layers and only mildly on the damping value chosen for the soil. Under base excitation, the agreement between the measured and the predicted responses was reasonable but not as good as that for head-loading. Part of the discrepancy between the responses is attributed to experimental difficulties in the simulation of soil-pile response under base loading. In
addition, adequate representation of the shear modulus and the damping characteristics of the entire profile was necessary in the base loading experiments. This finding suggests that caution should be exercised when extrapolating the results of tests in which the pile is loaded at its head to estimate the seismic response of piles.

In general, the finite element program was found to be capable of reproducing the experimental results remarkably well, and it is recommended that such procedures should be used in design.
CONCLUSIONS

Based on the studies described in this report, the following conclusions can be drawn:

1. A variety of approaches are available to analyze the behavior of soil-pile systems subjected to static or dynamic lateral loads. Among the existing approaches, the finite element formulations proposed by Roesset and his coworkers offer a rational and versatile framework for the computation of the pile response to any kind of loading.

2. The response of piles subjected to lateral loading applied at the pile-head is mainly influenced by the properties of the upper soil layers, whereas response of piles embedded in a soil deposit subjected to base excitation is controlled by the characteristics of the complete soil profile.

3. To predict the response accurately the variation in shear modulus with depth must be accounted for, but a single value of damping for the whole soil mass is adequate.

4. The results of Phase I tests (with loads applied at the pile-head) were adequately predicted using a bilinear variation of soil shear modulus with depth and 6% damping. In view of the difficulties encountered during the base excitation tests, the agreement between the predicted and measured responses was remarkably good.

5. The finite element program was used to predict the response of a full scale pile that had been tested by others. The finite element results correlated much more closely with the measured values than did any of the analytical methods used in the test report.
RECOMMENDATIONS

Encouraging experimental confirmation of the theoretical predictions of soil-pile interaction behavior has been provided in this study. However, additional theoretical and experimental work is needed in order that the analysis technique can be useful to the designer. The following recommendations emerged from the findings of the present study:

1. The finite element analysis approach adopted in this study is capable of reproducing the dynamic response of laterally loaded piles adequately and more realistically than presently available p-y analysis methods. At the present time the approach can serve as a viable and versatile design tool for single piles.

2. Systematic parametric studies using this approach should be carried out and the results should be presented in nondimensional forms for easy use in design calculations.

3. The finite element model should be verified under a wider range of conditions. In particular the response of piles in other soil types (e.g. a purely cohesive soil), and the response of pile groups rather than of a single pile should be investigated in future.

4. In real systems interactions exist between the superstructure and the pile and between the pile and the soil. These interaction effects were treated separately in the present study. Experiments and correlation with theoretical predictions which include both of these interactions together should be conducted.
CHAPTER 1
INTRODUCTION

Pile foundations are used to support important land-based and offshore structures. They must be designed to resist both axial loads (arising from the self-weight of the structure, and live loads), and lateral loads (due to wind action, earthquake forces, soil and water pressures and wave action). In the past, the design of pile foundations to resist such loads has been primarily based on empirical procedures using the results of full-scale load tests. This empirical approach may be acceptable for designing piles for axial loads alone, but it is unlikely to be acceptable for laterally loaded piles because of the paucity of suitable experimental data. Furthermore, all experiments are subject to inherent errors, and full-scale tests are also expensive, so there is a strong need for a reliable method of analysis.

The response of piles to static and dynamic lateral load has been the subject of active research for nearly two decades. As a result, a wide variety of methods of analysis (Matlock and Reese, 1960; Novak, 1974; Kuhlemeyer, 1979, Blaney et al 1976) has been proposed. However, not many studies have been undertaken to verify these theoretical predictions against the performance of either field-scale or laboratory-scale pile foundations. This is especially true for piles subjected to dynamic lateral loads, so the practising engineer is left with little justification for confidence in any of the existing analytical techniques used for the design of laterally loaded piles.

In view of the recognised and unavoidable differences between the ideal conditions assumed in the analytical models and the actual experimental conditions, the applicability and accuracy of the methods of soil-pile interaction analysis may be open to question. The purpose of the present study was to investigate the validity of the computational results of one of the more prominent analytical techniques in order to provide the insight necessary for improving the design methodology. In this paper are presented a brief review of the existing analytical techniques, the results of an experimental verification study on a shaking
table with instrumented model piles, and comparisons of the experimental and theoretical results.
CHAPTER 2
REVIEW OF PREVIOUS WORK

2.1 General

A brief survey of the existing literature was conducted to review relevant information on both theoretical analyses of pile-soil interaction and dynamic testing of both field and model piles. The review presented below is intended to be indicative rather than exhaustive; the practical constraints of time and resources did not permit a more complete review of the considerable body of literature that exists in the published domain. The more prominent works can be grouped into two categories: Theory of soil-pile interaction and Dynamic testing of piles. The former covers the variety of classes of theoretical approaches available and the latter focuses on the well-documented studies of experimental evaluation of the dynamic behavior of piles.

2.2 Soil-Pile Interaction Models

The dynamic response of soil-pile systems has been studied by three general classes of models: discrete models, (using lumped masses, springs and dashpots), continuum models and finite element models. Deliberately omitted from the following summary are those approximate analytical approaches which completely ignore either the surrounding soil medium or the pile itself and attribute the dynamic response to the characteristics of the soil or the pile alone.

2.2.1 Discrete Models

Numerous discrete models have been proposed to account for the response of the soil-pile system for both static and dynamic loadings. Generally, the pile is represented by beam-column elements and the soil is represented by a number of elastic springs at discrete points. This approach was originally applied to static pile analysis by Reese and Matlock (1960), Matlock and Reese (1960) who used the theory of subgrade reaction to solve for
the pile response quantities, in particular the moment at or above the groundline. These researchers and their associates (Reese et al., 1974; Reese and Desai, 1977; Matlock, 1970) have developed the approach over the years to include nonlinear soil behavior, for which the material properties are to be derived from measured p-y curves. This model is currently used by many practising geotechnical engineers for both static and cyclic analysis of piles subjected to lateral load. This type of discrete element approach has even been used to develop dynamic models which are able to describe hysteretic strength degradation, pile-soil separation, strain hardening, etc., by locating at the mass points some appropriate combination of springs, dashpots, and friction blocks. One such model (Bea, 1980) has been used in design of offshore structures subjected to wave and earthquake-induced lateral loading.

Another group of discrete element models is based on Mindlin's method Mindlin, 1936 for determining the spring constants of an equivalent Winkler foundation used to represent the behavior of soils. Originally proposed by Penzien (1970), this approach is able to simulate the inertial and radiation damping effects by using lumped masses at uniform spacing and linking each mass with a bilinear hysteretic spring and a nonlinear dashpot. This type of model has the ability to describe nonlinearity and hysteretic degradation of surrounding soils and can even take into account inhomogeneous, elastoplastic and damping characteristics of the soil by means of minor alterations in the linkages and their properties. Poulos (1973), and several other researchers (Banerjee et al., 1978) have also proposed similar models for seismic analysis of piles.

In spite of their widespread use, these models have some serious drawbacks for dynamic analysis. For example, the analyses based on p-y curves do not include the mass of the soil and are incapable of reproducing inertial and radiation damping effects. For all such models, the major drawback lies in the selection of parameters. Although some agreement can always be obtained with field results by manipulating the properties of the
linkages, a priori selection of parameters is difficult, since their numerical values have no apparent relationship to the commonly measured physical properties of the soil.

2.2.2 Continuum Models

The essence of these analyses is the solution of the three-dimensional wave equation for a viscoelastic or elastic semi-infinite continuum in which an elastic pile is embedded. This approach, first proposed by Tajimi (1969), eliminates the limitations of the discrete models by developing an approximate solution to the wave field propagation along the pile and in the soil due to some dynamic excitation. Many other authors, principally Novak, Nogami and their co-workers (Novak, 1974; Nogami and Novak, 1976, Nogami, 1977; Novak and Abdul-Ella, 1978; Novak and Sheta, 1980) have since then utilized continuum models to evaluate soil-pile interaction and produced some good articles which provide insight into the dynamics of laterally loaded piles. Novak and Abdul-Ella (1978) have shown how the model may work for layered soils and Novak and Sheta (1980) extended the previous work to include effects of soil-pile slippage and separation and soil nonlinearity by using an annular soil region around the pile with special material properties.

The major disadvantage of these continuum models is that they rely on relatively sophisticated mathematics and are expressed in terms which are not used regularly by practicing engineers. In the foregoing papers, complex response of the soil-pile system was evaluated and the results were presented generally in the form of impedance functions which define the complex stiffness of the system. The complex stiffness is frequency-dependent and its real and imaginary parts respectively define the stiffness and damping properties of the system. Most designers are not familiar with the process of deriving from these quantities the values of the moments and shears in the pile that they are interested in.
2.2.3 Finite Element Models

Dynamic response of pile foundations has also been studied by using the finite element technique. Several groups of researchers have developed finite element models of piles embedded in a soil medium to obtain approximate solutions for dynamic lateral behavior of piles. Among these, the more promising models are those developed by Kuhlemeyer and co-workers and Roesset and his co-workers. The two common features of these models are the representation of the near-field soil by axisymmetric or toroidal elements with three degrees of freedom per node and the modelling of radiation damping by a combination of dashpots placed at the boundary. Such a boundary is called a transmitting or energy-absorbing boundary and it can simulate the infinite extent of the surrounding soil.

Ulrich and Kuhlemeyer (1973) developed one of the first such finite element programs utilizing the type of transmitting boundary developed by Lysmer and Kuhlemeyer (1969) and special toroidal elements around the pile. These special elements were used to eliminate the difficulties associated with the high bending stiffness of standard finite elements. Kuhlemeyer (1976, 1979) evaluated both static and dynamic response of an elastic pile embedded in elastic soil and compared the solution with those of Poulos (1973) and Novak (1974) for static and dynamic responses, respectively.

Incorporating the energy-transmitting boundary developed by Kausel (1974), Blaney, Kausel and Roesset (1976) also developed a finite element program to evaluate the response of piles subjected to dynamic lateral loads and seismic excitation. In their study, Blaney et al. (1976) used a combination of linear members and rigid cross links to represent the pile sections and toroidal finite elements for the soil layers. and the entire assembly of elements was connected to an energy-absorbing boundary. Frequency-independent, hysteretic damping in the soil was also included. From the finite element computations, the dynamic stiffness of the pile elements was also computed and was shown to agree reasonably with that reported by Novak (1974). This program was later
modified by Roesset and Angelides (1979) to include the effect of soil modulus degradation with increasing shear strain in order to study nonlinear soil-pile interactions. This program is one of the most versatile tools currently available for modelling dynamic soil-pile interaction. Dobry et al. (1981) used it to perform a systematic study of the dynamic response of single piles and proposed simple formulas for spring and dashpot constants which can be used in discrete Winkler models.

Several other models have been used by researchers for evaluating lateral response of piles (Desai and Kuppusamy, 1979; Randolph, 1981). Unfortunately, most of them attempted to evaluate the response of piles subjected to applied lateral load at the pile head.

A number of research workers have also been working on boundary element models of piles and pile groups subjected to dynamic loading (Sen et al., 1985; Kayania and Kausel, 1982).

2.3 Experimental Verification

In view of the limitations of the presently available methods of analysis it is not surprising that many previous researchers have attempted to study soil-pile interaction effects by experimental modelling and simulation. In fact, a wealth of experimental data from dynamic testing of piles already exists in the literature. Dynamic testing of piles has been performed with full-scale piles (Prakash and Chandrasekharan, 1973; Maxwell et al., 1969; Petrovsky et al., 1975), laboratory scale-model piles including shaking table models (Gaul, 1958; Hayashi et al., 1965) and centrifuge models (Scott, 1979; Prevost and Scanlan, 1983). Unfortunately, however, for many of the previous experimental studies, either sufficient information was not obtained for adequate characterization of the soil and the pile or real soil-pile interaction conditions were not properly simulated. The usefulness of many existing experimental results is, therefore, somewhat questionable.

One of the important full-scale dynamic pile tests was conducted by Novak and Grigg (1976) with small test piles at a site on the campus of the University of Ontario.
Novak and Grigg installed small diameter steel piles into the ground to a depth of 7.5 feet. The shear wave velocity of the ground at the site was estimated by a steady-state vibration technique and from some static tests. Single piles (2.4 and 3.5 inch diameter) and a group of four piles (2.4 inch diameter) were excited either vertically or horizontally with a Lazan harmonic oscillator. The steady-state response amplitudes of the pile were measured with IRD Model 544 electro-dynamic pickups over an excitation frequency range of 6.3 to 62.5 Hz. The measured responses agreed quite well in their general trend with the theoretically predicted responses. But relatively large discrepancies were noted in the estimates of natural frequencies and resonant amplitudes, in some cases by a factor of two. According to Novak and Grigg, the discrepancies were due to difficulties in evaluating adequately the dynamic properties of the near-surface soils (up to a depth of a few pile diameters) from their measurements. In other words, the measured shear wave velocity of the ground was not truly representative of the shear wave velocity of soils at very shallow depths. They also suggested that since the soils up to depths of a few diameters dominated the participation of the soil in the interaction, the discrepancy may be smaller with full-size piles. In that case the measured shear wave velocity may represent the property of the participating soil.

More recently Gle (1981) attempted to correlate experimental results from eleven full-scale dynamic lateral load tests on pile piles embedded in both cohesive and granular soil at three separate sites. The dynamic soil properties were evaluated separately by suitable in-situ and field tests. The experimental response of the piles was compared with theoretical predictions using the Novak Model (Computer Program PILAY) and also from the finite element model developed by Kuhlemeyer (1976). In this study also, the findings of Novak and Grigg were verified. The PILAY results agreed with the experimental response only if reduced stiffness and damping values were used. However, the agreement was much closer for piles of granular soils than piles in cohesive soils, possibly pointing out the importance of nonlinear effects of separation and gapping.
Experiments on laboratory scale model piles in a 10 foot high and 7.5 foot diameter quick sand tank were preformed by Chon (1977). His attempts to correlate the test results with theoretical predictions were not very fruitful and the principal conclusion from his study was that the theoretical models (Kuhlemeyer, 1976; and Baranov, 1967) tend to significantly overestimate the response.

Finally, dynamic responses of a single pile and a group of piles were evaluated in an important recent study (Prevost and Scanlan, 1983) using centrifugal modelling. Although this study was not undertaken to verify any specific theoretical model, some comparisons were made between the observed results and theoretical results from the PILAY program. Again, limitations of the Novak model were pointed out based on the differences between the measured and computed results. In particular, non-dimensional stiffness and damping parameters evaluated from the experimental results were shown to have significant frequency dependence in contrast with Novak's theory.
CHAPTER 3
EXPERIMENTS

3.1 Introduction.

The purpose of the experiments was to generate data on the dynamic behavior of an embedded pile in order to appraise the predictions of the finite element program. This requires a prior knowledge of the soil properties, so Phase I of the testing was conducted to establish them. In it, a pile was embedded in a fixed soil mass and was shaken at its head with a hydraulic actuator. The bending moments were measured at intervals down the pile, from which the soil resistance, and so its properties, could be inferred.

In Phase II of the experiments the soil was placed in a shear-flexible bag on a shaking table. The pile was embedded in the soil and then a sinusoidal displacement was applied to the base. This type of loading resembles true earthquake conditions more closely than does that of Phase I. That is why it was selected, even though most full-scale tests on piles have necessarily involved head-loading.

Tests on small scale models of physical systems face problems of similitude, and this was realized at the outset. However it should be emphasized that the the purpose of the experiments was not to try to simulate at small scale the exact behavior of a full-scale system (which would require at least a centrifuge) but rather to provide experimental data with which to compare the predictions of the finite element program. If the principles on which the program is based are valid, it should be able to predict the dynamic response of any physical system, regardless of its scale.

Essentially the same equipment was used in both phases of the experiments, but in different configurations. In the following sections the equipment is described first and then the test configurations and procedures are outlined.
3.2 Equipment

3.2.1 Shaking Table

The University of Washington shaking table (Fig 3.1and 3.2) is housed in the geotechnical wing of the Structural Engineering Research Laboratory in the Civil Engineering Department. It provides unidirectional horizontal base motions and is driven by an MTS 903.73 servo-controlled hydraulic system, based on displacement rather than force control. The control system is capable of generating internally a variety of periodic motions or of reading arbitrary input from an external source. On the table is mounted a rigid soil box made from steel angles and 1/2 inch thick transparent plexiglass side walls. The soil box is 8 feet long, 6 feet wide and 4 feet deep. A detailed description of the shaking table is available elsewhere (Taga, 1977).

3.2.2 Pile and Soil Model

The model pile used for these experiments was made of ASTM 304 stainless steel tubing with a Young's modulus of 32000 ksi. It was 45" long, 1.24" O.D. and had a wall thickness of 0.19". Air dry Ottawa sand (d10 = 0.176 mm, d50 = 0.251 mm, Cu = 1.5) of known characteristics was used in the experiments. Preliminary tests indicated the pile would float upwards unless it was secured to the table. The model pile was, therefore, attached to the table in a manner to simulate a pinned or fixed connection at the base rock level. The soil was placed around the pile by pluviation in such a way that relatively uniform density could be achieved.

3.2.3 Shear Bag

In order to simulate the motions of a soil deposit subjected to horizontal bedrock motion, a flexible cylindrical shear bag was used to contain the sand in the Phase II experiments. It is shown in Figs 3.3 and 3.4. It was 48" high, 48" in diameter and was made of 0.125 inch thick nylon-reinforced rubber geomembrane. To keep the bag (and the specimen) circular and to prevent it from lateral bulging, several additional attachments
were needed. The top and bottom of the bag were clamped to two steel rings; the upper ring had a hexagonal stiffening frame welded to it and the lower one was welded to a 0.25 inch thick base plate. The base plate was set on the floor of the soil box and horizontal struts bearing against the side walls of the soil box were used to prevent sliding of the specimen inside the soil box. The bag was kept stretched vertically using eight adjustable struts between the flange of the upper ring and the base plate. In addition, hoops made of 0.375 inch pre-stressing strands were placed at 6" centers to prevent lateral bulging of the bag.

3.2.4 Loading System

The loading in both sets of experiments was provided by an MTS actuator Model 244.22 with a capacity of 22 kips and a maximum stroke of ± 3", equipped with a 15 gpm servo valve and driven by a 40 gpm pump. The device is controlled by an MTS 406 controller and an MTS 436 control unit. The control system has a frequency range from 1 to 1100 Hz (with 1-2% accuracy) in three different wave forms; sinusoidal, triangular and square. In the Phase I tests the actuator was mounted on the back wall of the soil box with its axis horizontal, using a heavy steel I-beam as a stiffener, as shown in Figs 3.1 and 3.2. It was supported vertically at the piston end by hanging it from a light cross-beam made from a pair of steel channels.

In the Phase II tests, the same actuator was mounted against a bulkhead in the standard position for driving the shaking table, as shown in Fig 3.5. In this position the system experienced instability problems, apparently associated with the ratio of shaking table weight to actuator capacity and with the control system feedback characteristics. When the drive shaft was connected directly to the table, the box started to shake uncontrollably even when no command signal was applied. Adjustments to the gain and damping in the control system could not correct it. Finally a 1" thick rubber spacing pad was inserted in the drive train, and this eliminated the instability. However it was quite soft in compression and acted as an isolator, so that when a high frequency load signal was applied, the table
displacement was much smaller than that of the actuator. Thus the amplitude of the high
frequency loading which could be applied to the table was severely limited, and for
frequencies above about 10 Hz was almost independent of the amplitude of the command
signal.

3.3 Instrumentation and Data Acquisition System

3.3.1 Instruments

The model pile was equipped with 9 four-arm strain-gage bridges arranged for
temperature compensated reading of curvature. They were located at distances of 2", 15",
24", 27", 29", 31", 33", 36", and 39" above the bottom of the pile so as to provide a fine
mesh of readings where the response was critical and a coarser one elsewhere. Not all
gages were used for all experiments because only 12 channels were available on the data
recorder. The strain gage signals passed through an amplification and conditioning device
before they were fed into the data acquisition system.

A few miniature accelerometers were also employed to record acceleration-time
histories during the experiments. In order to obtain a continuous acceleration profile along
the length of the pile, an accelerometer was attached to a plunger which could be raised or
lowered inside the pile and locked in position against the inner wall at any desired location.
The intention was to obtain acceleration readings from different elevations during the
execution of an experiment using the same accelerometer. The scheme, however, was
rejected since it did not work, particularly when the accelerometer was located near the top
of the pile. The plunger rod vibrated vigorously and appeared to affect the motion of the
pile. A single accelerometer was mounted on the pile head for measuring pile accelerations.
Accelerometers were also embedded in the sand at different elevations to record soil
motions. While for Phase II tests four accelerometers were located at heights of 16", 28",
36" and 40" above the base, only one, located 2" below the soil surface, was used for
Phase I tests. Another accelerometer was fixed to the base of the table to monitor the motion of the shaking table.

Linear Variable Differential Transformers (LVDTs) were used to record displacements. One is built into the actuator for stroke (i.e. displacement) control, and was used for monitoring pile-head movement in Phase I tests. (The LVDT measures the relative displacement between the piston and the body of the actuator. Because the soil box wall was not absolutely rigid, some of the recorded displacement is really attributable to bending of the wall. Tests with accelerometers mounted on the piston and actuator body showed that the fraction of the displacement attributable to the wall motion varied with frequency but was never more than 10% of the total.)

In the Phase II tests, the horizontal displacement of the shear-bag was monitored by four LVDTs attached to the back wall of the soil box. These sensors recorded the relative movements of the bag at elevations of 16", 28", 36", and 40" above the base as shown in Fig 3.5. The table motion was also monitored by an LVDT.

Finally, a specially designed load cell was used to measure the force applied at the pile-head. It is shown in Fig 3.6. It was made of T7075 Aluminum both to reduce the weight (to minimize the inertial loads of the cell) and because its high strength and low modulus lead to high sensitivity. The actuator transferred the load to the pile-head through a linkage containing the load cell. The linkage was necessary to avoid possible damage to the load cell from relative vertical movement between the actuator and the pile. The load cell was wired with the gauges in a four-arm bridge to provide temperature compensation, and was calibrated by a standard dynamometer.

3.3.2 Data Acquisition System

The strain gages, the accelerometers, the LVDTs, and the load cell were fed through the signal conditioning unit into a DEC PDP-11 mini-computer, which collected, stored and processed the data. This system contains 12 channels so that only selected strain gage
readings could be recorded. The computer uses a single analog-to-digital converter which sweeps the 12 channels. The time interval between sensing adjacent channels is approximately 86 microseconds, which is short enough to be considered simultaneous for the present purposes. This speed of reading permits about 1,000 readings of all channels per second so that even at a loading frequency of 100 Hz, 10 readings can be taken from each channel per cycle. This was enough to obtain a reasonable description of the waveform. However, most runs of readings had to be limited to the order of 2-5 seconds due to storage restrictions.

3.4 Test Program and Procedures

3.4.1 Calibration Tests

Static calibration tests were first carried out on the instrumentation. The strain gages on the pile were calibrated prior to embedding the pile in the soil by subjecting the pile to a simple beam bending test. The load cell was calibrated in situ using a standard laboratory dynamometer in series with the load. This method was preferred over the use of a universal testing machine, because it avoided the need to make special end fittings. The LVDT in the actuator had been factory calibrated.

3.4.2 Phase 1 Tests.

To prepare for the Phase I tests, the pile embedded in the sand was shaken at a low frequency (2 Hz) and a small displacement for 30 minutes. This was necessary because earlier attempts to perform these tests showed that otherwise the sand would tend to settle and form a conical depression around the pile, thereby changing the projecting length of the pile during a test. The basic procedure for each of the Phase I tests was to balance the actuator controls, activate the instruments (selected strain gages, accelerometer, and the load cell), activate and balance the data acquisition channels and then apply the load to the pile head. The static tests were displacement-controlled and the dynamic tests were frequency and displacement-controlled.
A total of six static tests were performed using displacements of 0.05", 0.10", and 0.13" in each direction (i.e., push or pull mode). The data were recorded for approximately 1 second so that a time-average of the measurements could be obtained.

The dynamic tests were performed in a similar way. A total of eleven tests was conducted at frequencies ranging from 10 Hz to 110 Hz in increments of 10 Hz.

Although the displacement of the actuator was set at 0.1" for all tests, the actual displacement obtained at the pile-head was smaller and it changed with frequency. This was partly due to the fact that some of the motion was absorbed in flexure of the back wall of the soil box, as already described, and partly due to the fact that, at high frequency, the hydraulic pump and servovalve capacities proved too small to move the necessary amount of oil fast enough. (This type of limitation is common to all hydraulically driven dynamic loading systems.) However the actual displacement of the pile was obtained from the accelerometer data.

The load cell gave unreliable output at frequencies greater than 20 Hz, possibly resulting from slack in the connecting linkages. An attempt was made to correct the problem by using press-fit pins at the joints, but the difficulty persisted, so the load had to be deduced from the pile bending moment profile, as described in Chapter 5.

3.4.3 Phase II Tests

The specimen was prepared by setting up the shear-bag in the center of the soil bin, placing the pile vertically in the middle of it and pouring sand into the bag from a bucket mounted on an overhead crane. The table was then vibrated for approximately half an hour at low frequency to ensure adequate compaction of the sand around the pile.

The natural frequency of the soil mass was first estimated by means of "pull-back" tests. The soil bag was struck a sharp blow with a sledge hammer and the natural frequency was obtained from the response of the accelerometers. The experiment was
repeated several times with impact applied either to the bag itself approximately 40" above the base or at the top stiffening ring.

The regular Phase II tests were then performed by applying sinusoidal base motions to the shaking table. Several experimental difficulties were encountered during the earlier attempts, especially at low frequencies. The upper layers (5" to 10") of sand underwent fluid-like flow, and heaped up in the middle of the bag. There was also a noticeable separation between the upper layers of sand and the edge of the bag. The original intention had been to rely on the LVDTs for the horizontal displacement profile. However, since the LVDTs measured the motion of the bag, but the motion of the soil was the one of greater interest, several tests had to be done to correlate the soil motion (measured by buried accelerometers) with that of the bag.

The struts supporting the upper ring sometimes fell off, but this was remedied by using lock-nuts. Between about 5 Hz and 9 Hz the whole soil bag appeared to undergo some rigid-body rocking motion and the base plate was seen to lift up from the floor of the soil box.

Finally, dynamic tests were carried out using forcing frequencies of 1-20 Hz in 1 Hz increments and 24-40 Hz in 2 Hz increments. At each frequency the table was activated for 2 secs before the 2 secs of readings were taken, in order to achieve conditions as close as possible to steady state during the measurements. Instrument zeros were taken between each run. These experiments were repeated three times at nominal actuator displacements of 0.05", 0.1", and 0.25" respectively. It was noted, however, that the amplitude of the table motion did not change appreciably, regardless of the prescribed nominal displacement at the actuator because the rubber pad in the drive train absorbed much of the actuator movement. Furthermore, an additional set of tests was conducted without the top ring to study the effect of the ring and the supporting struts. These had originally been installed to keep the bag circular and to prevent it from dropping vertically, which might have allowed bulging between the reinforcing hoops and settling of the sand during a test. However the shear bag
held its shape well and no settlement was observed, so the stiffening system was, in retrospect, probably unnecessary. The flow of sand in the upper layers and its separation from the side of the bag were drastically reduced in these experiments.
CHAPTER 4
METHOD OF ANALYSIS AND IDENTIFICATION OF PARAMETERS

4.1 Introduction

Prediction of the response of the soil-pile system in this study was done using a finite element computer program. Program PILE 1, written by Roesset and his co-workers, was selected because it contains special features well suited to the problem. A brief description of the theoretical basis of the program, the method of choosing the material properties and other pertinent issues are discussed in this chapter.

4.2 Theoretical Background

The theoretical background of the program is described in detail by Blaney et al. (1976) and Angelides and Roesset (1980). The program is capable of analyzing the steady-state dynamic response of a single pile embedded in an inhomogeneous soil deposit and subjected to harmonic excitations applied either at the base of the soil deposit or at the pilehead.

The most general form of the analytical model makes use of an annular near-field, or core, region around the pile and a far field region which extends from the edge of the near field to infinity. These are shown in Fig. 4.1. The core is modelled by toroidal finite elements connected to the pile which can reproduce faithfully the interaction between the two. The far-field gives the response that the soil mass would display in the absence of any inclusion such as the pile, and so it cannot reflect interactive effects correctly. It is not modelled explicitly, because it extends to infinity, but its influence on the near field is instead represented by the transmitting boundary. The soil can be treated everywhere as elastic or viscoelastic and nonlinearity can be included approximately by using Seed and Idriss' (1969) equivalent linearization technique.

Two important features of the program deserve mention here. First, although the geometry is axially symmetric (therefore, two-dimensional), the problem is truly three-
dimensional because of the antisymmetric nature of the loading and the deformations. The program uses Wilson's (1965) technique to reduce the problem to a two-dimensional one. Second, in order to simulate outward radiation of energy into the far-field and to avoid reflection of stress waves back towards the pile, the program uses special transmitting boundary conditions at the edge of the core region. Lysmer and Waas (1972) originally developed a consistent or transmitting boundary element for finite element idealization of a semi-infinite two-dimensional layered medium. This work was later extended by Kausel (1974) for the cylindrical problem, and his transmitting boundary was adopted in this program. It has been shown by Kausel, Roesset and Waas (1975) that it is possible to place the transmitting boundary very close to the pile and still achieve near-perfect radiation damping effects. In fact, in the version of the program used for this study, the transmitting boundary is placed directly against the pile, so the near field toroidal elements are omitted, and a linear visco-elastic analysis is performed in the frequency domain. This simplification reduces the fidelity of the response slightly, but is computationally much more efficient.

4.3 Selection of Parameters

To conduct an analysis of the soil-pile interaction, it is necessary to prescribe the geometric and material properties of the system. Those of the pile were established by direct measurement, so it was only necessary to select the soil parameters for the analyses. It is quite common to estimate material parameters such as Young's modulus, shear modulus, Poisson's ratio, and damping on the basis of standard laboratory tests. On the other hand, in the absence of measured values of these parameters, available empirical relationships between the parameters and index properties of the soil can be used for preliminary predictions. In this study, such predictions were used as a starting point, and the values were then adjusted to provide the best match between the predicted and measured response in the Phase I tests.
The sand specimens used in this study were prepared by dry pluviation and subsequent vibration. In order to minimise variation of density from sample to sample, the method of deposition and the duration of vibration were maintained the same. It is well-known that, for the same soil type and method of sample preparation, the soil density varies within a fairly narrow range. The experience of previous researchers (Lee, 1981; Sherif et al., 1984; Adjali, 1988) who prepared samples of the same sand on the shaking table was relied upon to further reduce this range so that the density of the sand could be reasonably estimated.

Hardin and Drnevich (1972) presented empirical equations and curves which have been used extensively in seismic design studies to estimate shear modulus and damping for soils. These relations indicate that shear modulus generally varies with the square root of depth for cohesionless soils and linearly with depth for cohesive soils. In this study, several different soil modulus profiles (linear, bilinear, and proportional to the square root of depth) were tried in order to match the predicted and measured bending moments under static load. Previous studies (Banerjee et al., 1987; Krishnan et al., 1982) have shown that a reasonable average value of damping is sufficient to obtain good predictions of deformation; the distribution of damping with depth has only a secondary influence as long as a reasonable average value is used. The analyses conducted in this study showed that the damping had little importance in the prediction of Phase I experiments, but it significantly influenced the predictions of Phase II experiments.

The overall objective was to appraise the ability of the Finite Element program to reproduce the response of the system to base loading. By first establishing the real soil properties (through the Phase I tests) the correlation of the Phase II results becomes a test of the analytical method rather than of the choice of properties. This is important. It could be argued that the soil properties were obtained using the very program which was to be appraised, and that the argument is circular and therefore invalid. However, this is specious for two reasons. First the responses to base- and head-loading are quite different,
and if there were some error in the program which caused it to identify incorrect soil properties during head-loading, then the error would be compounded in the base-excitation results which would then show very poor correlation. This did not happen. Second, the soil properties which were identified were within the expected range for the material used, and when used with simple approximate analysis techniques, gave rise to reasonable predictions of pile response. These checks thus imbue the assessment procedure with a healthy measure of objectivity.
CHAPTER 5
COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS

5.1 General

In this chapter the results of both the experiments and computer predictions are presented, and the correlation between them is discussed. In those cases where it is appropriate the results are also compared to simpler approximate methods to reinforce confidence in the finite element procedures.

The variation of the shear modulus of soil with depth was not known a priori and was defined by calibration against the Phase I static test results. Three shear modulus profiles were tried: a linear variation with depth, variation with the square root of depth and a bilinear variation. These distributions are shown in Fig. 5.1.

The actual procedure followed was to correlate the measured and predicted results by matching the pile curvatures and bending moments. First the pile bending moments under static head-loading (Phase I) were compared and the soil shear modulus-profile which gave the closest fit was selected. The soil damping was then estimated using these shear moduli and the dynamic head-loading (Phase I) tests. In this way the stiffnesses and damping which gave the closest match were identified separately. Finally, the measured and predicted responses under the (Phase II) base excitation were compared, using the material properties already obtained.

5.2 Phase I - Static Load Test Results

Fig. 5.2 shows the pile bending moments under a static head load. The response predicted by PILE 1 using the three different shear modulus profiles and the measured response are all shown. Of the two traditional profiles, curve B (variation with $d$) gives a closer match than curve A (variation with $\sqrt{d}$). However, even with curve B, the maximum moment still is slightly too high, indicating too little soil resistance in the top
layers. The bilinear curve increases the stiffness in the upper layers and provides a better fit. The pile response is relatively insensitive to the soil stiffness in the lower layers.

Figs 5.3-5.5 show comparisons of measured and predicted bending moments in the pile under three different static loads, and the correlation is generally good. The bilinear soil profile (Curve C) was used.

At the top of the pile the correlation is influenced by the exact location of the load. In the experiments the load was applied at the pilehead through a 1" thick arm with its center 1.5" above the sand surface. It was measured by the load cell which was placed in the middle of that arm (Figs. 3.1, 3.2). Since for head-loading the PILE 1 program can only accommodate loads at the soil surface, a horizontal force and a moment were used together to represent the experimental conditions.

Fig. 5.3 shows a comparison of the measured and predicted bending moment in the pile under a measured horizontal load of 224 lb. This load is replaced by a horizontal force of 224 lb. and a moment of 335 lb.-in. at the soil surface for the PILE 1 program. Fig. 5.3 shows that the top straight portion of the bending moment diagram predicted by the PILE 1 program is slightly higher than that of the experimental curve. This suggests that the lever arm of the applied force in the experiments must have been less than the measured 1.5". Figs. 5.4 and 5.5 show similar comparisons except that the input moment in the PILE 1 program was obtained by multiplying the experimental horizontal force by 1.25" instead of 1.5". This gave a better agreement between the measured moment profile and that obtained from the PILE 1 program.

Despite this discrepancy, the agreement between the measured and the predicted moment along the pile is excellent. The use of a lever arm of 1.25" instead of 1.5" changes the maximum moment by less than 5%.

As discussed in Chapter 4, the bilinear shear modulus distribution, with the exception of the top value, was used to model static loading conditions. Since the static load was measured accurately by the load cell, the use of a 1.5" or 1.25" moment arm for
PILE 1 programs results in minimal differences and only affects the top portion of the curve.

The agreement between the experimental results and those of PILE 1 is excellent for all static tests where different loads using the same shear modulus for the sand were tested. A plot for the applied horizontal load versus the pile's displacement at the sand surface (40" from the bottom) for both the experimental model and PILE 1 is shown in Fig. 5.6.

It is useful to compare these results with the predictions of two existing approximate methods, namely Reese and Matlock's (1960) solution for lateral pile stiffness and classical beam on elastic foundation theory. The former assumes that the modulus of subgrade reaction, k, varies linearly with depth. An example of latter calculation is shown in Appendix B. For a horizontal load of 224 lb and a moment of 335 lb-in, the results are summarized in Table 5.1.

Table 5.1 - Free-head pile loaded at the soil surface

<table>
<thead>
<tr>
<th>Response/Method</th>
<th>Measured</th>
<th>Reese and Matlock</th>
<th>Beam on elastic fndn.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum bending moment</td>
<td>1530</td>
<td>1611</td>
<td>1090</td>
</tr>
<tr>
<td>Pile-head displacement</td>
<td>0.10&quot;</td>
<td>0.11&quot;</td>
<td>0.10&quot;</td>
</tr>
<tr>
<td>Distance from soil surface to point</td>
<td>10.5&quot;</td>
<td>10.5&quot;</td>
<td>10.7&quot;</td>
</tr>
</tbody>
</table>
This static test yielded the following three main results:

1) The soil shear modulus profile which gives the best fit between the measured and predicted moment using the PILE 1 program is shown in Fig. 5.1 as curve C. It consists of two straight lines.

2) The maximum moment occurs approximately 10" below the surface.

3) The agreement between the experimental results and the results of the finite element program is excellent.

5.3 Phase I - Dynamic Load Test Results

Dynamic loads were applied to the pile head at 11 different frequencies ranging from 10 Hz to 110 Hz in 10 Hz increments. The bending moment distributions are shown in Figs. 5.7-5.17. The soil shear modulus profile which was used in the PILE 1 program to predict the moment distribution was the same as that used for static tests except that the top two values of the modulus were changed from 50 and 220 psi to 5 and 90 psi (referred to here as curve D). This change in shear modulus was necessary to fit the experimental data and accounts for the separation of the top 1" of soil from the pile observed during the dynamic test.

It was noted that the load cell gave erratic data, and so it could not be used as an instrument to measure the applied load, especially for frequencies higher than 30 Hz. The reason could not be found. Rattle in the linkage due to loose pins was suspected, but the erratic data persisted even with new press-fit pins. However, the top portion of the bending moment was linear, so that was used to compute the applied load. It was also noted that the calculated distance from the point of action of the load to the sand surface varied with the applied frequency and was variously higher or lower than the measured 1.5".

Three different moment arms were used for the PILE 1 program:

1.33" for frequencies 10 - 30 Hz
1.6" for frequencies 40 - 70 Hz
2.6" for frequencies 80 - 110 Hz

These values were obtained by projecting the top straight portion of the measured moment diagrams to intersect the center line of the pile. The value 2.6" is significantly higher than the point of application of the load, but was needed to correlate with the measured data.

Fig. 5.18 represents the measured and the predicted displacements at the top of the pile. The experimental displacement is that measured by the internal LVDT in the actuator, and represents the sum of the pile and box wall displacements. The true top displacement of the pile is slightly smaller than the one shown in the figure.

Fig. 5.19 shows the experimental and predicted stiffnesses. The measured stiffness is obtained by dividing the peak applied load by the peak pile-head displacement. At frequencies of 40 Hz and below the agreement is very close, but it deteriorates rapidly at higher frequencies. While both measured and predicted stiffnesses increase with frequency, the measured one increases faster. Part of the difference can be attributed to the fact that the displacements became very small at high frequency, leading to reduced accuracy, but this does not completely explain the clear trend shown in the figure.

5.4 Phase II Test Results
5.4.1 General

In phase II the system was loaded by base excitation. Two measures of the pile's response were used, namely the lateral deformation (i.e. the horizontal distance between the centerline of the deformed pile and the chord joining its two ends) and the pile head displacement. The former was obtained by double integrating the flexural strains along the pile, which was achieved here using the conjugate beam method. This represents the deformation of the pile, disregarding any rigid body displacements it might go through, and so is an indicator of the damage to be expected in the pile. The pile head displacement was
obtained by double integrating the accelerations measured by the accelerometer at that location and therefore includes the rigid body motion of the pile as it vibrates back and forth with the soil mass. These two measures were chosen because they show different features of the system’s response.

5.4.2 Pile Lateral Deformation

5.4.2.1 Without the Steel Ring

Figs 5.20-5.22 show the pile lateral deformation divided by the measured base displacement for nominal amplitudes of 0.05" 0.10" and 0.25". PILE 1 was used to predict the response and the theoretical curves are superimposed on the experimental data points. Both show similar trends, but the agreement is not exact. The main differences are that the experimental curves are less peaked than the theoretical ones and that they are less smooth. In general the agreement is slightly better at low input amplitudes.

Several points are worth noting. The shape of the response curves is strongly influenced by the damping value chosen. Larger damping leads to a less peaked curve and lower maximum response. Individual damping values were chosen for each input amplitude so as to achieve the best fit, but, as can be seen, it is not possible to match both the maximum response and the peakedness of the curves. This suggests nonlinear response.

The resonant frequency depends largely on the soil stiffness, since the density can be obtained within close limits. Different stiffness profiles were used in PILE 1 in an attempt to find the best match. Higher stiffness gave higher resonant frequencies, as expected. The profile used for Figs. 5.20-5.22 was $G = 300 \sqrt{d}$, since this gave results as good as any other and it is in line with the pattern found by earlier researchers.

The differences between the theoretical and experimental curves are attributed to three main causes. First the soil behaves nonlinearly, and linear response analysis can only approximate the true response. Second, the top 10" or so of the soil was observed to
lose contact with the bag and to behave like a fluid during excitation. Thus the elastic continuum model used in PILE I cannot be expected to match the true behavior exactly. Third, the pile was modeled as prismatic. The steel section was indeed prismatic, so the pile's structural properties were constant along its length, but the coatings over the strain gages added noticeably to the overall pile diameter, thus providing a larger projected area for soil resistance. This was not taken into account in the PILE I model, because there was no obvious way to do it. The additional overall pile width would meet more soil resistance and attract more bending moment. If the pile's stiffness was the same, the extra bending moment would lead to more curvature and deformation. This is in agreement with most of the data points, which show measured deformations larger than the theoretical ones, especially at forcing frequencies below resonance.

If the soil was really acting as a homogeneous continuum the curvatures would be smoothly distributed along the pile. They were in fact found to vary considerably, sometimes inducing reversed curvature and sometimes not. The effect of double integrating the curvatures to give a deformation is to smooth out these variations to some extent. Figs. 5.20-5.22 thus show smoother relationships than if the maximum curvature had been used in place of the deformation. These fluctuations in curvature are partly caused by the complex behavior of the soil but are exacerbated by the fact that the strains were small and were therefore more sensitive to electronic noise in the data acquisition system.

The sensitivity of the pile bending moments to the soil profile can also be seen by inspecting the mode shapes of a shear column under different assumptions of shear modulus distribution. Since the pile displacements follow closely those of the surrounding soil, the soil mode shapes will dictate the bending moments in the pile. If the modulus is constant with depth, the first mode shape is a simple sine curve, for which the sense of the curvature is concave when looked at from the undisplaced position. On the other hand a linear distribution of modulus leads to a mode shape which is convex. (The modulus
cannot drop quite to zero at the soil surface or a singularity results). There must be some intermediate soil modulus profile which gives rise to a first mode shape which is exactly linear, and which would therefore induce no bending moments at all in the pile, but just rigid body displacements. These observations help to explain the sensitivity of the pile bending moments to the precise values of the soil modulus profile.

Approximate calculations were also done to check the resonant frequency inferred from the deformation vs frequency plots. First the soil was treated as having a constant modulus with depth, and the modulus needed to give the measured resonant frequency of 10 Hz was calculated, based a density of 120 pcf. This was 460 psi, which lies within the range used in all the selected profiles, and therefore seems reasonable. Then the soil modulus was considered as varying linearly with depth and, because there is no closed form solution for this problem, an approximate natural frequency was found by using the Rayleigh-Ritz procedure and a guessed mode shape. This gave 12 Hz, which must be an upper bound to the true solution. Using the same technique on the constant modulus case, for which the true solution is known, revealed that the approximate value in that case was about 10% high. Assuming a similar error in the variable modulus case leads to a natural frequency of about 11 Hz, which is close enough to the measured value of 10 Hz to add credence to it.

Thus the PILE 1 program was able to reproduce well the general features of behavior, while some discrepancies remained in the details because the actual properties were more complex than the elastic continuum model assumed in the program.

5.4.2.2 With the Steel Ring

Fig. 5.23 shows the experimental results obtained using the bag and the top ring with its supporting rods. A theoretical curve is plotted using a soil shear modulus profile of 300 $\sqrt{d}$. It is clear from the plot that the resonant frequency has shifted from 10 Hz (Fig 5.20, without the ring) to 12 Hz (Fig 5.23 with the ring).
The fit between predicted and measured values is good at all frequencies except 8 Hz. It is believed that the discrepancy there was caused by spurious amplification introduced by rocking mode vibrations.

5.4.3 Pile head displacement

5.4.3.1 Without the ring

Figs. 5.24-5.26 represent the absolute pile head displacement divided by the amplitude of the base motion. The three figures are drawn for nominal base motions of 0.05", 0.10" and 0.25". The experimental pile head displacements were obtained by integrating twice the accelerations obtained from the pile head accelerometer.

In the first two figures, the measured response shows two peaks, at about 6 Hz and 10 Hz, but for the nominal base motion of 0.25", only the first peak is present. It is believed that the first peak is caused by the rocking effect which was seen during the experimental runs for 5-8 Hz. The base plate of the bag could be seen by the naked eye to lift up along the line A-A shown in Fig. 3.4. The rocking did not happen for frequencies higher than 9 Hz, so the second peak is believed to correspond to resonance caused by shear deformation of the soil (which is the response of interest) while the first peak is a spurious one which existed in the experiments but which would not be present in a real soil deposit.

The experimental and predicted curves agree only moderately well. If the soil stiffness and damping that gave the best fit for the pile deformations are used, then the predicted maximum amplification (in the soil deformation mode) is about 2.5. This happens to correspond to the measured amplification (at a different frequency) in the rocking mode, but it is different from the measured amplification of about 1.75 at the measured deformation-mode resonance at 10 Hz. In order to match the predicted and measured amplifications at the latter frequency, it is necessary to use a much higher damping factor, in the range of 10-20%. 

33
Thus while the measured responses in the pile deformation mode and pile head
displacement modes both show resonance at 10 Hz, the measured results suggest the need
for different levels of damping in the two modes. This in turn implies that a linear model,
as used here, is not capable of reproducing perfectly the behavior of a nonlinear physical
system.

5.4.3.2 With the steel ring

Fig. 5.27 shows the pile head displacement response for 0.05" base motion. The
measured curve is similar in shape to that of Fig. 5.24, while the predicted ones are
necessarily identical, since there is no way of including a steel ring in the PILE 1 program.

Two differences can be seen. First, the rocking mode resonance occurs at 5 Hz
rather than 6 Hz. This seems reasonable because the stiffening ring adds mass and inertia
at the top of the system, thus lowering its natural frequency. Second, the amplification at
the soil deformation resonance (10 Hz) is smaller than in Fig. 5.24 (without the ring). This
could be explained by the existence of more damping when the ring was present, but the
reason for it is not clear.

The shear stiffness of the bag is approximately 1/1000 that of the soil, so any
stiffening of it by the vertical tensioning caused by the supporting rods will be immaterial.
However, the low shear stiffness of the rubber means that the steel ring plus the 3" of
rubber bag projecting above the soil acts as a flexible single-degree-of-freedom system
attached to the top of the soil column. Thus the presence of the ring might be expected to
influence locally the pile-head displacement, but not the pile deformation. This hypothesis
is borne out by the good fit between measured and predicted response for deformations
(Fig. 5.23) and the poorer one for pile-head displacements (Fig. 5.27).

5.4.4 Soil displacements

The top 5"-10" of the soil acted like a fluid, so meaningful measurements of soil
displacement there could not be made. However, accelerometers in the soil and an LVDT
outside the bag measured the displacement of soil and bag respectively at 28" above the base. Their responses corresponded closely.

Values for nominal base motions of 0.05" and 0.10" are shown in Figs. 5.28 and 5.29. The measured amplifications at this level are in the range 1.0 to 1.5 for frequencies between 5 and 11 Hz, which is smaller than the corresponding values at the pile head, shown in Fig. 5.24. This verifies the fact that the majority of the soil deformation is occurring in the upper layers because the shear stiffness is smaller there. Furthermore, the rocking-mode displacements are also probably responsible for the measured displacements' being much larger than the predicted ones for frequencies in the range 5-8 Hz, where the fit is worst.

5.5 Prediction of full-scale response

The PILE 1 program proved relatively successful in predicting the results for the shaking table experiments. It was considered desirable to find out how well it could reproduce the behavior of a full scale pile under field conditions. Field data from O'Neill et al (1983) was used for the PILE 1 program. The pile was a steel pipe 16" in diameter with a 1/2 in thick wall. The soil was sandy and had a density of 124 pcf and an internal angle of friction of 42°. Loading was applied horizontally 1.2 in above the ground surface. Figs. 5.30, 5.31, and 5.32a and b represent the comparison between the results obtained from PILE 1 and the measured ones for maximum moment, distance to maximum moment, and top displacement respectively. Three soil shear modulus profiles were used to compare the predicted and measured results. These were 325 times the square root of depth, 325 times the square root of depth except for the top layer (in which the modulus was changed from 1292 to 600 psi) and linear 38 times the depth, where depth is measured in inches for all three profiles. Fig. 5.33 shows the moment distribution along the pile for the top 20 feet, using the three soil shear modulus profiles mentioned above, and assuming a free-head pile. Only one experimental point is available, at the point of maximum moment, and
it correlates closely with the modified $32 \sqrt{d}$ and the linear modulus profiles. Figs. 5.32-5.35 show comparisons of measured and predicted values for different response quantities. PILE 1 gives predictions which correlate very well with measured results, and in all cases are much closer then the other methods used in O'Neill (1983). The fact that the program is able to predict so well the measured results from a full-scale test is most encouraging.
CHAPTER 6
Theoretical Prediction of Fixed-head Pile Response

The comparisons of Chapter 5 show that the computer program PILE 1 can predict reasonably accurately the pile response measured in the shaking table experiments and in the field test in which the pile head was free to rotate. However, in practice most pile heads are fixed against rotation, so PILE 1 was used to predict the bending moment for a fixed-head pile under static load using three different soil shear modulus profiles; linear, bilinear, and 300 times the square root of depth (Fig. 6.1). No experiments were done to confirm these calculations. They are presented for illustration purposes only.

Fig. 6.2 represents the bending moment distribution for dynamic loading at 10, 50 and 100 Hz using the same soil profiles as in the Phase 1 series. In both static and dynamic fixed-head runs, the horizontal applied load was the same as was used for the free-head.

Figs. 6.1 and 6.2 represent the absolute value of bending moment. The actual moment changes sign at about the 30” level.

The maximum bending moments in the fixed and free head conditions may be compared. They occur respectively at the head and part way down the pile. If the problem is treated as a classical beam on elastic foundation with a constant foundation modulus, the two moments are equal in magnitude. When the non-uniform shear modulus is taken into account (as it is in PILE 1) the maximum moment in the fixed head condition is found to be 10% larger than that in the free headed pile. This appears reasonable, because the pile-head shear must be large, due to the scant support the pile receives from the upper layers of soil.

Response was predicted using PILE 1, Matlock and Reese’s method and the beam on elastic foundation analysis. The results are compared in Table 6.1, which shows that the predictions using PILE 1 and Matlock and Reese’s method are virtually identical for this loading.
Table 6.1 - Fixed-head pile loaded at the surface

<table>
<thead>
<tr>
<th>Response/Method</th>
<th>PILE 1</th>
<th>Reese and Matlock</th>
<th>Beam on elastic foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-head moment</td>
<td>1640</td>
<td>1650</td>
<td>1540</td>
</tr>
<tr>
<td>Pile-head displacement</td>
<td>0.04&quot;</td>
<td>0.04&quot;</td>
<td>0.048&quot;</td>
</tr>
<tr>
<td>Distance from soil surface to point of</td>
<td>10.5&quot;</td>
<td>10.5&quot;</td>
<td>10.7&quot;</td>
</tr>
</tbody>
</table>
CHAPTER 7
REFERENCES


APPENDIX A

Input for PILE I program

First card:  NCASES

NCASES: Number of case

Second card:  NLAY

NLAY: Number of layer (maximum 25 layer)

Third card:  \((H(I),W(I),ES(I),POIS(I),AT(I)), I = 1, NLAY\)

H: Layer thickness (in)

W: Specific weight

ES: Maximum shear modulus of a layer (PSI)

POIS: Poisson ratio of a layer

AT: Maximum damping ratio of a layer

Fourth card:  NPILE

NPILE: Number of pile

Fifth card:  NSEC

NSEC: Number of pile sections = number of layers

Sixth card:  RO, HP, WP, EP, FP, WTP, PBS, TKP

RO: Pile's radius (in)

HP: Height of pile section above soil surface (in)

WP: Specific weight of pile's material

EP: Modulus of elasticity of pile's material divided by 1000 (PSI)
FP: Axial force (lb)

WTP: Weight at top of the pile (LB)

PBS: Tip condition

\[ PBS = 0. \text{ Free tip} \]

\[ PBS = 1. \text{ Fixed tip} \]

Note: The program is set now such that PBS has to be = 0 for a fixed tip see the last card.

TKP: Total length of the pile

Seventh card: \((\text{PAR}(I),\text{PIN}(I)) I = 1, \text{NSES})\)

PAR: Area of the Pile section (in2)

PIN: Moment of inertia of the pile section (in4)

Eighth card: \((\text{ICOD},\text{ISOL},\text{IIFIX})\)

\[ \text{ICOD} = 1 \text{ calculate only spring constants} \]

\[ \text{ICOD} = 2 \text{ calculate everything} \]

\[ \text{ISOL} = 1 \text{ input a unit horizontal load} \]

\[ \text{ISOL} = 2 \text{ input a unit moment} \]

\[ \text{ISOL} = 3 \text{ input a base motion (earthquake)} \]

\[ \text{IIFIX} = 0 \text{ Free head pile} \]

\[ \text{IIFIX} = 1 \text{ fixed head pile} \]

Ninth card: \(\text{NFR}\)

NFR: Number of different sets of frequencies /1
Tenth card: NOM, OM, DOM

NOM: Number of different frequencies of this set /1

OM: Initial frequency /0.1

DOM: Frequency increment /0.01

Eleventh card: Repeat card No. Five

Twelfth card: Repeat card No. Six

Thirteenth card: Repeat card No. Seven

Fourteenth card: Repeat card No. Eight

Fifteenth card: Repeat card No. Nine

Sixteenth card: Repeat card No. Ten

Seventeenth card: NLOADS

NLOADS: Number of loads

Eighteenth card: (PTOP(I), BMTOP(I), I = 1, NLOADS)

PTOP: applied horizontal load at pile head (lb)

BMTOP: applied bending moment at pile head (lb - in)

Note: If BMTOP = 99999 the pile head is considered fixed. This is only needed when loads are applied at the pile head.
APPENDIX B

Example Calculation of the Response of Laterally Loaded Pile

The following example describes how the response of a laterally loaded pile can be computed using two approaches for a given pile loaded by a concentrated horizontal load of 224 lb and a maximum moment of 335 lb-in applied at the soil surface. The quantities calculated for both free- and fixed-head piles, include 1) maximum moment, 2) depth, and 3) pile-head displacement.

1. Free-head pile

   a. Beam on elastic foundation method

      \[ P = \text{Horizontal load at pile head} \]

      \[ EI = \text{Pile stiffness} \]

      \[ K = \text{Soil modulus (average)} \]

      \[ \beta = \left( \frac{K}{4EI} \right)^{0.25} \]

      \[ v(x) = \text{deflection of pile at a distance } x \text{ from surface} \]

      \[ \frac{d^3}{dx^3} (V(x)) = 4 \beta \frac{P}{K}, f_3(\beta x) = 0 \text{ at max. moment} \]

      \[ f_3(\beta x) = e^{\beta x} (\cos \beta x - \sin \beta x) = 0 \]

      Hence \[ x = 1 \]

      \[ x = \frac{\pi}{4\beta} \text{ distance from soil surface to point of } M_{\text{max}}. \]

      To evaluate \( K \), the average of the soil shear modulus of the upper layers can be taken.

      For this example \( K = 340 \text{ psi} \)

      \[ \therefore \beta = 0.0735 \]
1) Maximum moment

\[ M_{\text{max}} = (M_{\text{max}})_p + (M_{\text{max}})_m \]

\( (M_{\text{max}})_p = \text{max. moment due to applied concentrated load} \)

\( (M_{\text{max}})_m = \text{max. moment due to applied concentrated moment} \)

For an applied concentrated load \( P = 224 \text{ lb} \)

\( (M_{\text{max}})_p = -EIv'' = \frac{P}{\beta} f_2(\beta x); \)

where \( v'' = \frac{d^2 x}{dx^2} \)

\[ f_2(x) = e^{-\beta x} \sin \beta x \]

\[ = 0.3224 \]

\[ \therefore (M_{\text{max}})_p = 982 \text{ lb-in} \]

For an applied concentrated moment, \( M_o = 335 \text{ lb-in} \)

\[ M(x) = M_o f_4(x) \]

\[ f_4 = e^{-\beta x} (\cos x + 0) \]

\[ = 0.3223 \]

\[ (M_{\text{max}})_m = 335 (0.3224) = 108 \text{ lb-in} \]

\[ M_{\text{max}} = 982 + 108 = 1090 \text{ lb-in} \]

compared with the measured value of 1530 lb-in

\[ M_{\text{max}}(\text{B.E.F})/M_{\text{max}}(\text{measured}) = 71\% \]

which is a good estimate considering that only a rough estimate of the modulus was taken.
2) Distance from the soil surface to point of maximum moment can be estimated:

\[ x = \frac{\pi}{4\beta} = 10.7'' \] compared with the measured value of 10.5''

3) Pile-head displacement, \( Y_{\text{max}} \)

\[ Y_{\text{max}} = (Y_{\text{max}})_p + (Y_{\text{max}})_m \]

\[ (Y_{\text{max}})_p = \frac{2P_0}{K} \]

\[ = \frac{224(0.0735)^2}{340} \]

\[ = 0.10'' \]

\[ (Y_{\text{max}})_m = \frac{2M_0 g b^2}{K} \]

\[ = \frac{2(335)(0.0054)^2}{340} \]

\[ = .000054'' \]

Hence, \( Y_{\text{max}} = 0.10'' \)

\[ = 0.10'' \] compared with the measured value of 0.1''

b. Reese and Matlock approach

Reese and Matlock have developed a non-dimensional solution for lateral load versus pile-head displacement relationship. This solution assumes that the soil modulus \( K \) to vary linearly with the depth \( (K = nZ) \), where \( Z \) is the depth.

\[ EI = \text{pile stiffness} \]

\[ P_l = \text{lateral applied load} \]

\[ M_l = \text{moment applied at soil surface} \]
\( Y = \) pile-head displacement
\( \theta_s = \) slope of pile-head in rad.

\( A_0, B_0, A_m, B_m, A_y \) and \( B_y \) are non-dimensional solution coefficients given in graphical form in Reese and Matlock

\( T = \frac{E_1}{n}^{0.2} \)

\( T \) is also tabulated in relation with \( Z \)

For \( \frac{Z}{T} = 1.32 \)

\( Z = 10.5'' \)

\( T = 8 \)

1) The maximum moment

\[ M_{\text{max}} = A_m P_l T + B_m M_l \]

\( A_m = 0.77 \quad B_m = 0.69 \)

\( M_{\text{max}} = 1611 \text{ lb-in compared to measure of 1530 lb-in} \)

2) Pile-head displacement

\[ Y(\text{top}) = \frac{A_y P_l T^3}{EI} + \frac{B_y M_l T^2}{EI} \]

\( A_y = 2.43 \quad B_y = 1.62 \)

\( Y(\text{top}) = 0.108'' \) compared to measure of 0.10''
2. Fixed-head pile
   a. Beam on elastic foundation
      1) Fixed-head moment
         \[ M_{\text{max}} = M(0) = \frac{P}{2} f_3(x) = \frac{P}{2\beta} \]
         \[ = \frac{224 \times 0.0727}{2} = 1540 \text{ lb-in} \]
         compared with the actual of 1640 lb-in
      2) Distance from soil surface to point of zero moment
         \[ X = \frac{x}{4\beta} = 10.7" \text{ compared with 10.5"} \]
      3) Pile-head displacement
         \[ V(x) = V(0) = \frac{P\alpha}{K} \]
         \[ = \frac{224(0.0727)}{340} \]
         \[ = 0.048" \text{ compared with 0.04" using PILE 1} \]
   b. Reese and Matlock method
      1) Fixed-head moment
         \[ M_f = \frac{A_\theta P_1 T - \Theta_s E I}{B_\theta} \frac{\Theta_s = 0}{B_\theta T} \]
         where \( A_\theta = -1.62 \) \( B_\theta = -1.75 \)
         \[ M_f = 1650 \text{ lb-in compared with PILE 1 of } M_f = 1640 \text{ lb-in} \]
2) Pile-head displacement

\[ Y_f = \frac{P_t T}{EI} \left( A_y - \frac{A_\phi B_y}{B_\phi} \right) \]

\[ A_\phi = -1.62, B_\phi = -1.75 \]

\[ Y_f = 0.04" \text{ compared with PILE 1 of 0.04"} \]
Figure 3.1  Soil box with pile - section
Figure 3.2  Soil box with pile - plan
Figure 3.3  Rubber bag - elevation
Figure 3.5  Loading system
Figure 3.6  Load cell
Figure 4.1--Finite element idealization
(adapted from Ref. 4)
Figure 5.1  Variations of soil shear modulus with depth used in theoretical predictions.
Figure 5.2  Variation of moment down pile using three different soil shear moduli (P=224 lb., M=335 lb.-in used for theoretical results)
Figure 5.3  Variation of moment down pile for static load of
P=224 lb. & M=335 lb.-in
Figure 5.4 Variation of moment down pile for static load of P=185 lb. & M=231 lb.-in.
Figure 5.5 Variation of moment down pile for static load of $P=100\ lb.$ & $M=125\ lb.\ -in.$
Figure 5.6  Static load versus pile displacement at soil surface
Figure 5.7  Variation of moment down pile for 10 Hz frequency (P=188 lb., M=250 lb.-in).
Figure 5.8  Variation of moment down pile for 20 Hz frequency (P=178 lb., M=240 lb.-in).
Figure 5.9  Variation of moment down pile for 30 Hz frequency (P=167 lb., M=225 lb.-in).
Figure 5.10  Variation of moment down piles for 40 Hz frequency
(P=113 lb., M=180 lb.-in).
Figure 5.11  Variation of moment down pile for 50 Hz frequency (P=88 lb., M=140 lb.-in).
Figure 5.12  Variation of moment down pile for 60 Hz frequency
(P=84 lb., M=135 lb.-in.)
Figure 5.13  Variation of moment down pile for 70 Hz frequency (P=66 lb., M=105 lb.-in).
Figure 5.14  Variation of moment down pile for 80 Hz frequency (P=38 lb., M=100 lb.-in).
Figure 5.15  Variation of moment down pile for 90 Hz frequency
(P=36 lb., M=95 lb.-in).
Figure 5.16  Variation of moment down pile for 100 Hz frequency (P=32.7 lb., M 85 lb.-in.).
Figure 5.17  Variation of moment down pile for 110 Hz frequency
(P=27 lb., M=70 lb.-in).
Figure 5.18  Pile-head displacement versus frequency, comparison between measured and predicted (PILE1)
Figure 5.19  Pile-head lateral stiffness versus frequency
Figure 5.20  Dynamic amplification of pile deformation versus frequency for maximum applied nominal motion of 0.05" (without steel ring)
Figure 5.21  Dynamic amplification of pile deformation versus frequency for maximum applied nominal motion of 0.1" (without steel ring)
Figure 5.22  Dynamic amplification of pile deformation versus
frequency for maximum applied nominal motion of 0.25"
(without steel ring)
Figure 5.23  Dynamic amplification of pile deformation versus frequency for maximum applied nominal motion of 0.05" (with steel ring)
Figure 5.24  Dynamic amplification of pile-head displacement versus frequency for maximum applied nominal motion of 0.05" (without steel ring)
Figure 5.25  Dynamic amplification of pile-head displacement versus frequency for maximum applied nominal motion of 0.1" (without steel ring)
Figure 5.26  Dynamic amplification of pile-head displacement versus frequency for maximum applied nominal motion of 0.25" (without steel ring)
Figure 5.27  Dynamic amplification of pile-head displacement versus frequency for maximum applied nominal motion of 0.05" (with steel ring)
Figure 5.28  Dynamic amplification of soil displacement at 28" versus frequency for maximum applied nominal motion of 0.05" (without steel ring)
Figure 5.29  Dynamic amplification of soil displacement at 28" versus frequency for maximum applied nominal motion of 0.1" (without steel ring)
Figure 5.30  Comparison of measured and predicted pile-head load versus moment. (Tests reported by O'Neill and Murchison, 1983).
Figure 5.31  Comparison of measured and predicted pile-head load versus displacement (Tests reported by O'Neill and Murchison, 1983)
Figure 5.32a Comparison of measured load vs. distance to maximum moment with values predicted by PILE1 (Tests reported by O'Neill and Murchison, 1983)
Figure 5.32b  Comparison of measured load vs distance to maximum moment with values predicted by various methods (Tests reported by O'Neill and Murchison, 1983)
Figure 5.33  Variation of moment down pile (Tests reported by O'Neill and Murchison, 1983)
Figure 5.34  Comparison of measured load vs. deflection with values predicted by various methods (Tests reported by O'Neill and Murchison, 1983)
Figure 5.35 Comparison of measured load versus maximum moment with values predicted by various methods (Tests reported by O'Neill and Murchison, 1983)
Figure 6.1  Variation of moment for a fixed-head pile under static loading using three different soil shear modulus profiles
Figure 6.2  Variation of moment for a fixed-head pile under dynamic loading using profile C