Precamber Design Example

PGSuper Training

Richard Brice, PE
WSDOT Bridge and Structures Office

Revisions

10/18/2018 – Initial version

04/2019 – Updated for precamber deflection equations

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1 Introduction

The purpose of this document is to illustrate how the PGSuper computer program performs its computations. PGSuper is a computer program for the design, analysis, and load rating of precast, prestressed concrete girder bridges.

A design evaluation followed by a load rating analysis illustrates the engineering computations performed by PGSuper. PGSuper uses a state-of-the-art iterative design algorithm and other iterative computational procedures. Only the final iterative steps are of interest. To avoid lengthy iterations in this document, trial variables are "guessed" based on the final iterations produced by the software.

PGSuper uses 16 decimals of precision. There will be minor differences between these "hand" calculations and numbers reported by PGSuper. When noted, these calculations adopt numeric values reported by PGSuper.

1.1 Sign Convention

This document and PGSuper use the following sign convention.

Item	Value
Compression	< 0
Tension	> 0
Upward Deflection	> 0
Downward Deflection	< 0
Top Section Modulus	< 0
Bottom Section Modulus	> 0
Strand Eccentricity above Centroid	< 0
Strand Eccentricity below Centroid	> 0

2 Bridge Description

2.1 Site Conditions

Normal Exposure

Average Ambient Relative Humidity: 75%

2.2 Roadway

Alignment

PI Station	Back Tangent	Delta	Radius
	N 90 E		

Profile

PVI Station	PVI Elevation	Grade in (g_1)	Grade out (g_2)	Length
102+64	31.15	9%	-9%	201 ft

Superelevations

Left	Right
$-0.02\frac{ft}{ft}$	$-0.02\frac{ft}{ft}$

2.3 Bridge Layout

This bridge has a very steep crest vertical curve. The girders are precambered to eliminate much of the slab haunch build-up dead load.

Back of Pavement Seat, Abutment 1, 102+00

Back of Pavement Seat, Abutment 2, 103+20

Abutments are Normal to the alignment

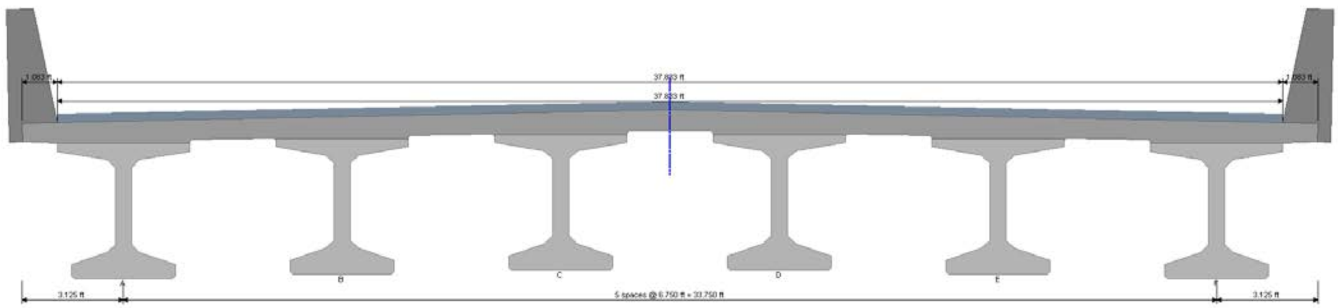

Figure 2-1: Bridge Section at Station 102+60.0

Girders

6 WF50G @ 6'-9"

$A = 776.531\ in^2$
$I_x = 282559.4\ in^4$
$I_y = 71558.9\ in^4$
$Y_t = 25.849\ in$
$Y_b = 24.151\ in$
$S_t = 10931.2\ in^3$
$S_b = 11699.6\ in^3$
Perimeter = 241.284 in

$W_{tf} = 49.0\ in$
$W_{bf} = 38.375\ in$
$t_{web} = 6.125\ in$

$f'_{ci} = 6.1\ ksi$

$f'_c = 7.2\ ksi$

$\gamma_c = 155\ lb/ft^3$
$\gamma_c = 165\ lb/ft^3$ (including rebar)

Precamber = 15"

Pick Points 3.75ft

Bunk Points 4.167ft

Haul Configuration: HT40-72

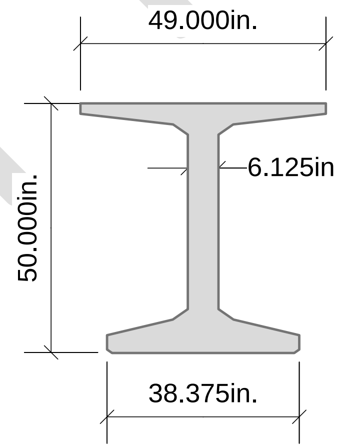

Figure 2-2: Girder Dimensions

Harping points at 0.4L from the end of the girder.

Interior Diaphragms

Rectangular – Between girders only. H = 31.5 in, T = 8.00 in. Located at $0.33L_s$ and $0.67L_s$.

Slab

Gross Depth = 7.5 in
Overhang = 3'-1.5"
Slab Offset ("A" Dimension) = 8.75"
Fillet = ¾"
Sacrificial Depth = ½"
f'c = 4 ksi
γ_c = 150 lb/ft^3
γ_c = 155 lb/ft^3 (including rebar)
Future Wearing Surface, 0.035 k/ft^2

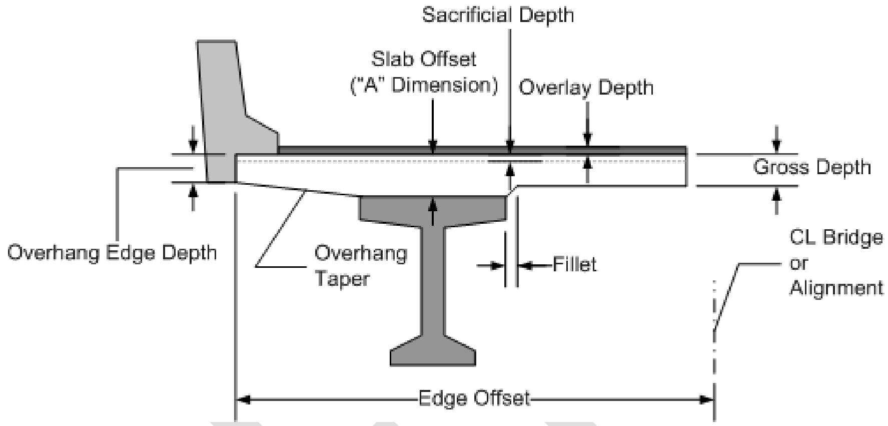

Figure 2-3: Slab Detail

Strands

0.6" Diameter	f_{pu} = 270.0 ksi
Grade 270	f_{py} = 243.0 ksi
Low Relaxation	E_{ps} = 28500 ksi
	a_{ps} = 0.217 in^2/per strand
	Straight Strands = 30
	Harped Strands = 13

Traffic Barrier

42" Single Slope

Design weight = 0.690 kip/ft/barrier

Load is distributed to 3 exterior girders

Load Modifiers

Ductility	Redundancy	Importance
η_D = 1.0	η_R = 1.0	η_I = 1.0

Criteria

Design in accordance with the AASHTO LRFD Bridge Design Specification, Eighth Edition, 2017 and the WSDOT Bridge Design Manual

Load Rate in accordance with AASHTO, The Manual for Bridge Evaluation, Second Edition, 2011 with 2015 interim revisions and the WSDOT Bridge Design Manual

WSDOT policy is to design using gross section properties (BDM 5.6.2.I) using refined estimate of prestress losses (BDM 5.4.1.C). PGSuper supports stress analysis with transformed section properties, the LRFD approximate method for estimating prestress losses, and a non-linear time-step analysis.

3 Design Preliminaries

Evaluate the first interior girder (Girder B).

3.1 Construction Sequence

Figure 3-1 shows the assumed construction sequence. PGSuper models the various construction stages with Construction Events.

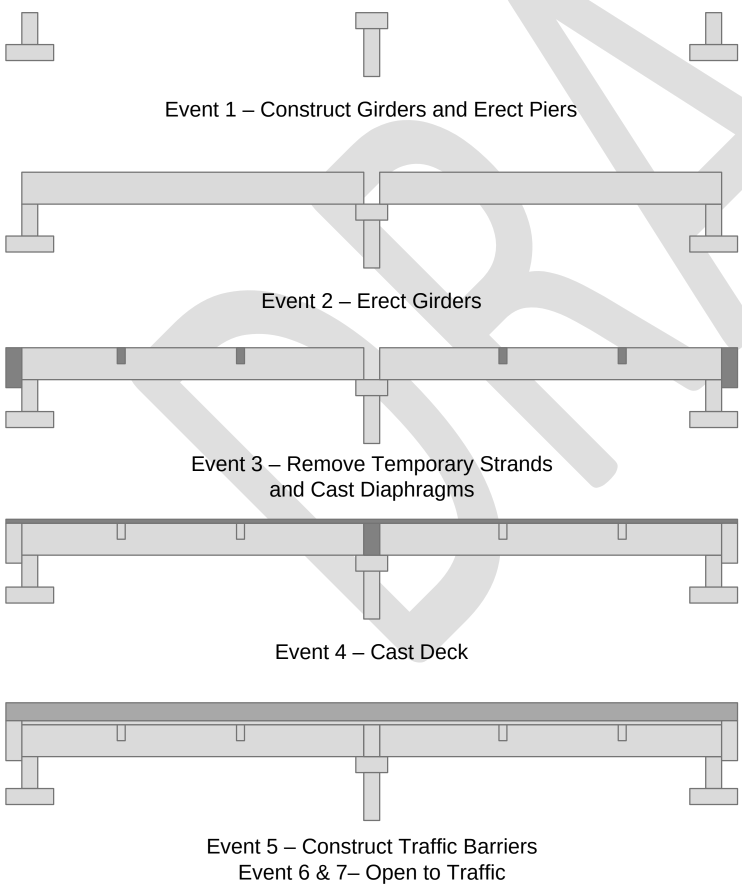

Figure 3-1 Assumed Construction Sequence

3.2 Girder Length

For a typical stub abutment with a Type A connection, the centerline of bearing is located 2'-8.5" from, and measured normal to, the back of pavement seat. The distance from the centerline bearing to the end of the girder is 1'-8.5" measured normal to the CL Bearing, which is parallel to the back of pavement seat.

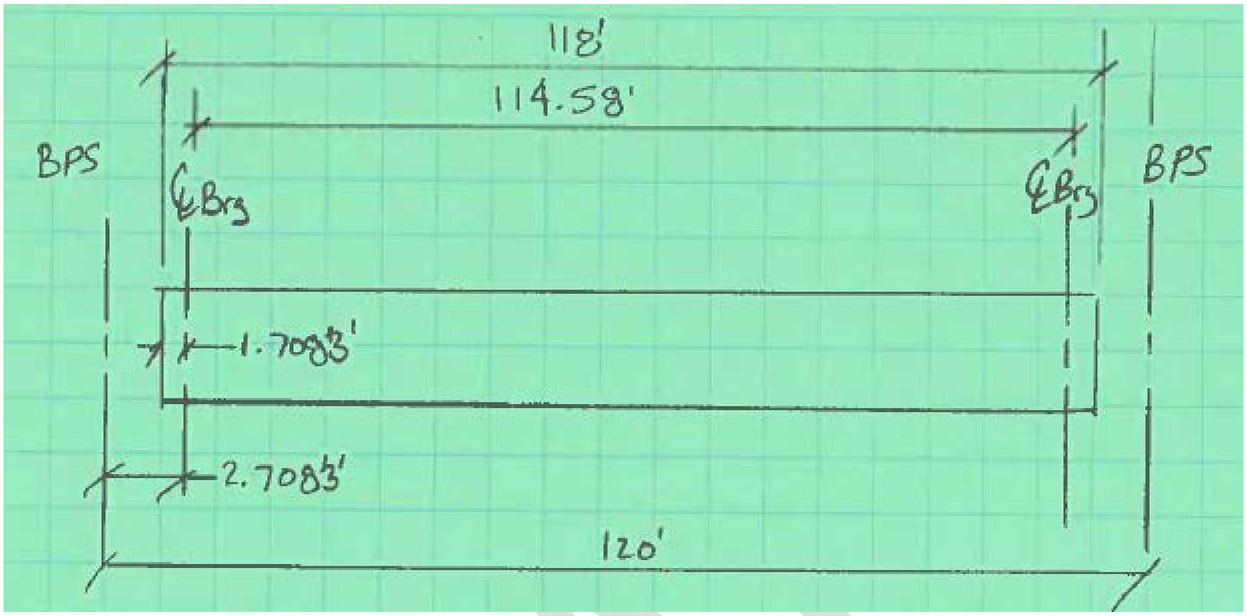

Figure 3-2 Girder Length Geometry

The bearing-to-bearing span length is $L_s = 120ft - 2(2.7083ft) = 114.58ft$.

The overall girder length is $L_g = 114.58ft + 2(1.7083ft) = 118.00ft$.

3.3 Section Properties

Compute the composite section properties. The basic girder section properties are in the bridge description.

3.3.1 Effective Flange Width

The effective flange width of a composite concrete deck slab is the tributary width of the member (LRFD 4.6.2.6.1).

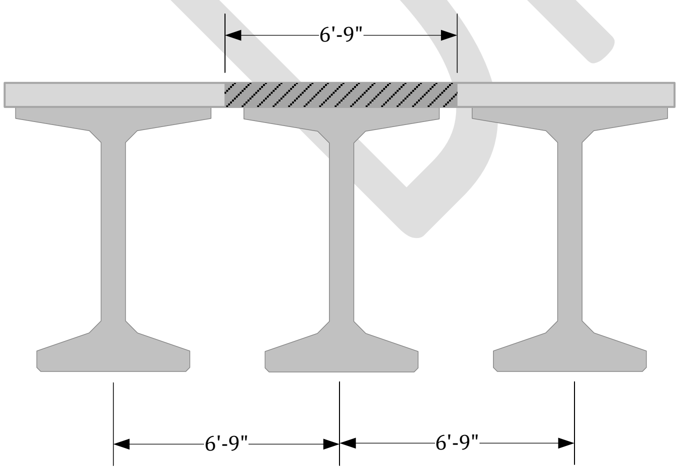

Figure 3-3 Effective Flange Width

$$w_{eff} = 6.75ft = 81in$$

3.3.2 Composite Girder Properties

Transform the slab to equivalent girder material and use the parallel axis theorem to compute the composite girder properties. At mid-span the bottom of the slab is above the top of the girder by the fillet amount (¾ “). If the actual camber exceeds the predicted camber, the ¾” fillet can be easily lost. Assume the bottom of the slab is directly on top of the girder. This provides the least stiff section where the maximum demand occurs. For simplicity, use this section model at all locations (BDM 5.6.2.B.1).

PGSuper has options to include the haunch depth in the section properties calculations. Each section can use the minimum haunch depth (fillet dimension) or the actual haunch depth. Using the actual haunch depth means there is a different set of section properties at every cross section. Using more precise section properties may be desirable for load rating.

Modulus of elasticity of slab concrete

$$E_c = 120{,}000 K_1 w_c^2 {f_c'}^{0.33} = (120{,}000)(1.0)(0.150)^2(4.0)^{0.33} = 4266.223\ ksi$$

Modulus of elasticity of girder concrete assuming a concrete strength of $f_c' = 7.1ksi$

$$E_c = 120{,}000 K_1 w_c^2 {f_c'}^{0.33} = (120{,}000)(1.0)(0.155)^2(7.2)^{0.33} = 5530.500\ ksi$$

$$n = \frac{E_{c\ slab}}{E_{c\ girder}} = \frac{4266.223ksi}{5530.500ksi} = 0.771$$

The sacrificial wearing surface is not part of the structural section. Use the structural slab depth for computing section properties.

$$t_{slab} = t_{gross\ slab\ depth} - t_{sacrifical\ depth} = 7.5in - 0.5in = 7.0in$$

	Area	*Y_b*	*(Area)(Y_b)*
Slab	$(0.771)(81in)(7.0in) = 437.157in^2$	$50.0in + \frac{7.0in}{2} = 53.5in$	$23387.900in^3$
Girder	$776.531in^2$	$24.151in$	$18754.000in^3$
Total	$A_c = 1213.688in^2$		$42141.9in^3$

$$Y_{bc} = \frac{\sum(Area)(Y_b)}{\sum(Area)} = \frac{42141.9in^3}{1213.668in^2} = 34.723in$$

$$Y_{tc\ girder} = H_g - Y_{bc} = 50.0in - 34.723in = 15.277in$$

	Area	*d*	*(Area)(d²)*	*I_o*	*I_o + (Area)(d²)*
Slab	$437.157in^2$	$50.0in + \frac{7.0in}{2} - 34.723in = 18.777in$	$154130.948in^4$	$\frac{1}{12}(0.771)(81in)(7.0in)^3 = 1785.058in^4$	$155916.006in^4$
Girder	$776.531in^2$	$24.151in - 34.723in = -10.572in$	$86790.683in^4$	$282559.4in^4$	$369350.083in^4$
					$I_x = 525266.089in^4$

$$S_{bc} = \frac{I_x}{Y_{bc}} = \frac{525266.089 in^4}{34.723 in} = 15127.325 in^3$$

$$S_{tc\ girder} = \frac{I_x}{Y_{tc\ girder}} = \frac{525266.089 in^4}{15.277 in} = 34382.804 in^3$$

3.3.3 First Moment of Area of deck slab,

$$Q_{slab} = A_{slab}\left(Y_{tc\ girder} + \frac{t_{slab}}{2}\right) = 437.157 in^2\left(15.277 in + \frac{7 in}{2}\right) = 8208.497 in^3$$

3.3.4 Section Property Summary

Below are the section properties from PGSuper. They are slightly different then the properties computed above. Use the section properties reported by PGSuper for better agreement between these calculations and the software.

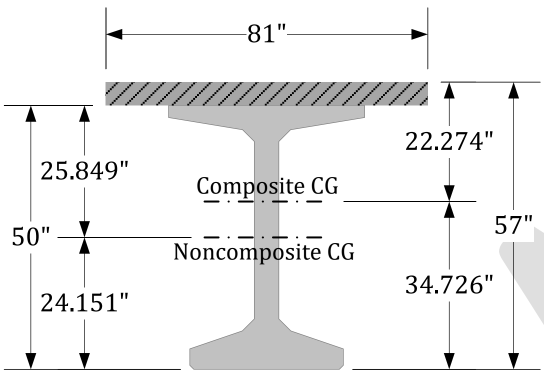

Figure 3-4 Centroid of Non-composte and Composite Section

Table 3-1: Section Properties from PGSuper

	Girder	*Composite Girder*
*Area, **A***	$776.531\ in^2$	$1213.915\ in^2$
$\boldsymbol{I_x}$	$282559.4\ in^4$	$525343.2 in^4$
$\boldsymbol{I_y}$	$71558.9\ in^4$	-
$\boldsymbol{Y_{top\ girder}}$	$25.849\ in$	$15.274\ in$
$\boldsymbol{Y_{top\ slab}}$	-	$22.274\ in$
$\boldsymbol{Y_b}$	$24.151\ in$	$34.726\ in$
$\boldsymbol{S_{top\ girder}}$	$10931.2\ in^3$	$34394.2\ in^3$
$\boldsymbol{S_{top\ slab}}$	-	$30574.7\ in^3$
$\boldsymbol{S_b}$	$11699.6\ in^3$	$15128.3\ in^3$
$\boldsymbol{Q_{slab}}$	-	$8211.5\ in^3$
Effective Flange Width, $\boldsymbol{W_{eff}}$	-	$81.0\ in$
Perimeter	$241.284\ in$	-

3.4 Structural Analysis

There are several significant stages during the life of a prestressed girder. PGSuper automatically models these stages as Construction Events. The events are:

1) Construct girders (aka Casting Yard Stage)
    a) Tension strands, form girders, cast concrete, concrete curing. Initial relaxation of the prestressing strand occurs.
    b) Strip forms and impart the precompression force into the girder (aka Release)
    c) Move girders into storage area (Initial lifting)
    d) Elapsed time during storage (creep, shrinkage, and relaxation losses occur)
2) Erect girders
    a) Prior to erection, the girders must be transported from the fabrication facility to the bridge site
    b) Erect and brace girders
    c) De-tension temporary strands (if applicable)
3) Cast diaphragms and deck (dead load applied to non-composite girder section)
4) Install railing system (traffic barriers, sidewalks, etc). (dead load applied to composite section)
5) Final without Live Load (includes future overlay if applicable)
6) Final with Live Load

PGSuper models the individual steps within a Construction Event with Analysis Intervals. For example, Event 1 – Construct Girders, models five analysis intervals: Tension Strands and Cast Concrete, Elapsed Time during Curing, Prestress Release, Lifting, Placement into Storage, and Elapsed Time during Storage.

The analysis intervals are a general modelling approach associated with time-step analysis. Precast girder design normally uses a pseudo time-step analysis. However, the PGSuper can perform a refined non-linear time-step analysis. PGSplice uses the non-linear time-step analysis as well.

3.4.1 Girder Construction (Casting Yard)

Girder construction at the casting yard consists of tensioning strands, placing mild reinforcement, installing girder forms, and placing concrete. Stripping of girder forms occurs after the concrete reaches adequate strength to accommodate the stresses and stability of the girder. The strands are the detensioned but because of bond with the girder concrete, the precompression force imparts into the girder. If the prestress force is eccentric to the centroid of the girder and it is sufficient to overcome the self-weight of the girder, the girder cambers upwards. In this condition, the girder bears on its ends and bending stresses develop.

$$w_{girder} = \gamma_c A_g = (0.165kcf)(776.531in^2)\left(\frac{1ft^2}{144in^2}\right) = 0.890\ klf$$

where:

A_g = Gross cross sectional area of the girder

γ_c = Unit weight of concrete

$$M_g = \frac{wx}{2}(l - x)$$

Moment at point of prestress transfer (PSXFR)

Prestress transfer occurs over 60 strand diameters (LRFD 5.9.4.3.1)

$$l_t = 60d_b = (60)(0.6in) = 36in = 3ft$$

$$M_g = \frac{(0.890klf)(3ft)}{2}(118ft - 3ft) = 153.525k \cdot ft$$

Moment at harp point (HP)

Harp point is 0.4L from the end of the girder $(0.4)(118ft) = 47.2ft$

$$M_g = \frac{(0.890klf)(47.2ft)}{2}(118ft - 47.2ft) = 1487.08k \cdot ft$$

Moment at mid-span (0.5L)

$$M_g = \frac{(0.89klf)\left(\frac{118ft}{2}\right)}{2}\left(118ft - \frac{118ft}{2}\right) = 1549.05k \cdot ft$$

3.4.2 Erected Girder

Substructure elements support the girder at permanent bearing locations once erected. Bracing stabilizes the girder. Temporary top strands are detensioned, followed by diaphragm and roadway slab casting. Installation of the railing system occurs after the roadway slab gains adequate strength.

3.4.2.1 *Diaphragm and Deck Placement*

In this stage, the girder supports its self-weight along with the weight of the diaphragms and slab.

3.4.2.1.1 Diaphragm Loads

The diaphragm load for an interior girder is $P = HW\gamma_c(S - t_{web})$, where:

H	=	Height of the interior diaphragm
W	=	Width of the interior diaphragm
t_{web}	=	Width of the girder web
S	=	Spacing of the girders

$$P = HW\gamma_c(S - t_{web}) = (38.875in)(8in)(0.155kcf)(81in - 6.125in)\left(\frac{1ft^3}{1728in^3}\right) = 2.09kip$$

Diaphragms are located at 38.194 ft $(0.33L)$ and 76.389 ft $(0.67L)$ from the left bearing.

3.4.2.1.2 Slab Loads

The slab load consists of the main slab and the slab haunch.

3.4.2.1.2.1 Main Slab Load

The main slab load is

$$w_{slab} = W_{trib}t_{slab}\gamma_c = (81in)(7.5in)(0.155kcf)\left(\frac{1ft^2}{144in^2}\right) = 0.654klf$$

3.4.2.1.2.2 Slab Haunch Load

The slab haunch load accounts for the buildup of concrete between the top of the girder and the bottom of the main slab. This concrete element has a width equal to the top flange width (W_{tf}) and varies in depth along the length of the girder because of camber and variations in the roadway surface.

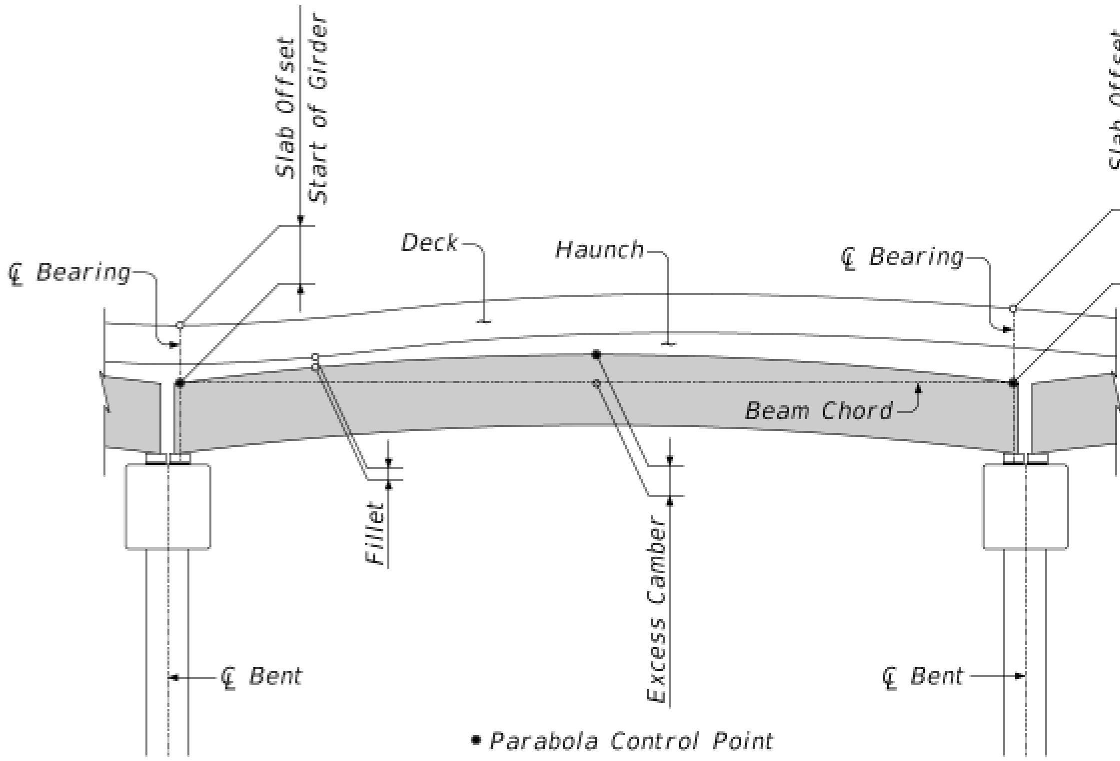

Figure 3-5: Slab Haunch

WSDOT's design policy is to assume zero natural camber for purposes of determining the slab haunch load (BDM 5.6.2.D.3.iv).

PGSuper provides the option to consider excess camber when determining loading. This option may be desirable for load rating as it reduces the haunch dead load.

The basic haunch dead load at any given section is

$$w_{haunch} = W_{tf} t_{haunch} \gamma_c$$

The slab offset ("A" dimension) is 8.75 in. The slab haunch load at the start of the span is

$$t_{haunch} = \mathrm{A} - t_{slab} = 8.75in - 7.5in = 1.25in$$

$$w_{haunch} = (49in)(1.25in)(0.155kcf)\left(\frac{1ft^2}{144in^2}\right) = 0.066\ klf$$

In general, the haunch thickness is computed as $t_{haunch} = EL_{bottom\ slab} - EL_{top\ girder}$ using the elevations of the bottom of the slab and the top of the girder (neglecting natural camber). For this bridge, the roadway profile is a vertical curve and the girder is precambered.

The elevation of the top of the slab over Girder B is

$$EL_{top\ slab} = \frac{50(g_2 - g_1)}{L_{vc}} x^2 + g_1 x + EL_{BVC} - 0.02\frac{ft}{ft}(10.125ft)$$

$$x\ in\ number\ of\ stations$$

The elevation of the bottom of the slab is the top of slab elevation reduced by the slab thickness.

$$EL_{bottom\ slab} = EL_{top\ slab} - t_{slab}$$

The elevation of the top of the girder is computed from the top of girder elevation at the CL Bearing plus the precamber along the length of the girder, measured relative to the bearings.

$$\delta_{pc}(x) = \frac{4\Delta_{pc}}{L_g}\left(x - \frac{x^2}{L_g}\right) - \frac{4\Delta_{pc}}{L_g}\left(x_{clbrg} - \frac{{x_{clbrg}}^2}{L_g}\right)$$

$$x\ is\ distance\ from\ end\ of\ girder$$

$$x_{clbrg} is\ distance\ from\ end\ of\ girder\ to\ CL\ Bearing$$

$$EL_{top\ girder} = EL_{top\ slab} - A + \delta_{pc}(x)$$

The parabolic curves cause the haunch depth to vary along the length of the girder. The table below lists the haunch depth and loading for half the span. Linear load segments model the slab haunch load.

Location (ft)	$t_{haunch}(in)$	$w_{haunch}(klf)$
0.0	1.250	0.066
11.458	2.507	0.132
22.917	3.485	0.184
34.375	4.184	0.221
45.833	4.603	0.243
57.292	4.742	0.250

3.4.2.2 *Superimposed Dead Loads*

Application of superimposed dead loads occurs after the deck has reached adequate strength. The superimposed dead loads consist of the traffic barrier and the overlay, if present. The composite section is resisting these loads.

3.4.2.2.1 Traffic Barrier

The traffic barrier weight is distributed over n exterior girders, if there are $2n$ or more girders, otherwise the weight of the traffic barrier per girders is $w_{tb} = \frac{W_{tb\ left} + W_{tb\ right}}{N}$, where N is the number of girders in the span. From BDM 5.6.3.2.B.2.d, $n = 3$.

$$2n = 6, N = 6, 2n \le N$$

$$w_{tb} = \frac{W_{tb}}{n} = \frac{0.690klf}{3\ girders} = 0.230\frac{klf}{girder}$$

AASHTO permits equal distribution for barrier loads to all girders.

3.4.2.3 *Open to Traffic*

3.4.2.3.1 Future Overlay

Evenly distribute the weight of the future wearing surface to all girders. The curb to curb width of the deck is $38.833ft$.

$$w_o = \frac{(37.833ft)(0.035ksf)}{6\ girder} = 0.221\frac{klf}{girder}$$

Take care when applying the future overlay loading. Certain stress conditions are worse before the overlay is applied and others are worse after it is applied.

3.4.2.3.2 Live Load

The design live load is the HL93 notional model defined in the AASHTO LRFD BDS.

The vehicular live loading is the combination of the:

- design truck or design tandem, and (LRFD 3.6.1.1)
- design lane load (LRFD 3.6.1.2.1)

The design truck consists of three axles. Axle weights and spacing are, 8.0 kip, 14.0 ft, 32.0 kip, 14.0 to 30.0 ft, 32.0 kip. See Figure 3-6 below.

The design tandem consists of a pair of 25.0 kip axles spaced 4.0 ft apart.

The design lane load is 0.640 klf, uniformly distributed along the length of the span.

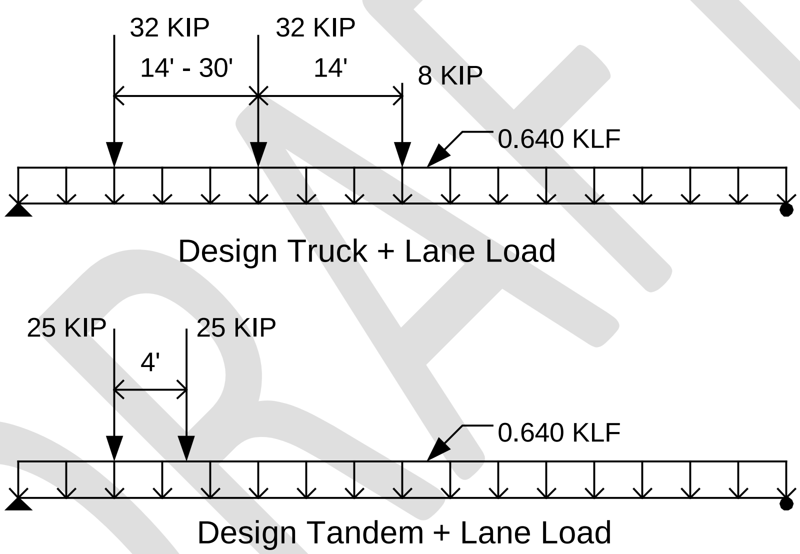

Figure 3-6: HL93 Live Load Model

Apply a dynamic load allowance (impact) of 33% to the design truck and design tandem portions of the live load response.

The fatigue live load is the design truck with the rear axle spacing fixed at 30 ft. The dynamic load allowance for fatigue is 15%.

3.4.3 Analysis Results Summary

3.4.3.1 *At Release*

Loading	Transfer Point	Harp Point	Mid-Span
Girder	$153.49\ k \cdot ft$	$1486.71\ k \cdot ft$	$1548.65\ k \cdot ft$

3.4.3.2 *At Bridge Site*

Loading	$0.5L_s$
Girder after erection	$1460.27\ k\cdot ft$
Diaphragm	$79.78\ k\cdot ft$
Slab	$1073.17\ k\cdot ft$
Haunch	$358.11\ k\cdot ft$
Traffic Barrier	$377.47\ k\cdot ft$
Future Overlay	$362.20\ k\cdot ft$
Design LLIM (HL-93)	$3421.07\ k\cdot ft$
Fatigue LLIM	$1755.47\ k\cdot ft$

Live loads are per lane

3.4.4 Limit State Responses

Group the structural responses into load cases and compute limit state responses. The total factored load, or limit state response, is $Q = \sum \eta_i \gamma_i q_i$. (LRFD Eqn. 3.4.1-1)

LRFD Table 3.4.1-1 gives the load factors. The limit states of importance are:

- Service I, Q = 1.0DC + 1.0DW + 1.0(LL+IM)
- Service III, Q = 1.0DC + 1.0DW + 0.8(LL+IM)
- Strength I, Q = 1.25DC + 1.50DW + 1.75(LL+IM)
- Fatigue I, Q = 0.5DC + 0.5DW + 1.5(LL+IM)

The live load factor for Service III is 0.8 for design and 1.0 for load rating. See BDM 3.5.2

3.4.5 Live Load Distribution Factors

Compute the live load distribution factors. Select the appropriate cross section type from LRFD Table 4.6.2.2.1-1. A precast I-beam with cast-in-place concrete deck corresponds to cross section k.

WSDOT deviates from the LRFD BDS for exterior girders in type k sections as described in BDM 3.9.3.A.

Compute the longitudinal stiffness parameter K_g.

$$K_g = n\left(I + Ae_g^2\right)$$

where:

n	=	modular ratio between beam and deck material $n = \frac{E_{beam}}{E_{slab}}$
I	=	moment of inertia of the beam (in^4)
A	=	area of beam (in^2)
e_g	=	distance between the centers of gravity of the basic beam and deck (in)

$$n = \frac{5530.5ksi}{4266.223ksi} = 1.296$$

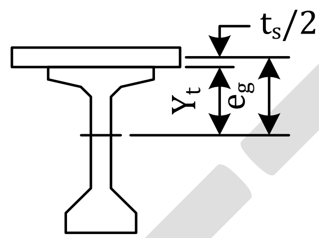

Figure 3-7: e$_g$ Detail

$$e_g = Y_t + \frac{t_s}{2} = 25.849in + \frac{7.0in}{2} = 29.349in$$

$$K_g = 1.296[282559.4in^4 + (776.531in^2)(29.349in)^2] = 1233060in^4$$

3.4.5.1 *Number of Design Lanes*

The number of design lanes is equal to the integer portion of the roadway width divided by 12 ft (LRFD 3.6.1.1.1).

$$N_L = \left\lfloor \frac{37.833ft}{12ft} \right\rfloor = 3\ Design\ Lanes$$

3.4.5.2 *Distribution of Live Loads per Lane for Moments in Interior Beams*

LRFD Table 4.6.2.2.2b-1 gives the live load distribution factors for moments in interior beams.

3.4.5.2.1 Compute Distribution Factor for Moment

Check the range of applicability for live load distribution factors.

$3.5\ ft \le S \le 16\ ft$	$S = 6.75\ ft$	**OK**
$4.5\ in \le t_s \le 12\ in$	$t_s = 7.5\ in$	**OK**
$20\ ft \le L \le 240\ ft$	$L = 114.58\ ft$	**OK**
$N_b \ge 4$	$N_b = 6$	**OK**
$10{,}000 in^4 \le K_g \le 7{,}000{,}000\ in^4$	$K_g = 1233060 in^4$	**OK**

3.4.5.2.1.1 One Design Lane Loaded

The live load distribution factor for one loaded lane is

$$gM_1^i = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$

$$gM_1^i = 0.06 + \left(\frac{6.75}{14}\right)^{0.4}\left(\frac{6.75}{114.58}\right)^{0.3}\left(\frac{1233060}{12.0 \cdot 114.58 \cdot 7^3}\right)^{0.1} = 0.412$$

3.4.5.2.1.2 Two or More Design Lanes Loaded

The live load distribution factor for two or more design lanes loaded is

$$gM_{2+}^{i} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2} \left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$

$$gM_{2+}^{i} = 0.075 + \left(\frac{6.75}{9.5}\right)^{0.6} \left(\frac{6.75}{114.58}\right)^{0.2} \left(\frac{1233060}{12.0 \cdot 114.58 \cdot 7^3}\right)^{0.1} = 0.584$$

3.4.5.3 *Distribution of Live Loads per Lane for Shear in Interior Beams*

LRFD Table 4.6.2.2.3a-1 gives the live load distribution factors for shear in interior beams.

3.4.5.3.1 Compute Distribution Factor for Shear

Check the range of applicability for live load distribution factors.

3.5 ft $\leq S \leq$ *16 ft*	*S = 6.75 ft*	**OK**
4.5 in $\leq t_s \leq$ *12 in*	t_s *= 7.5 in*	**OK**
20 ft $\leq L \leq$ *240 ft*	*L =114.58 ft*	**OK**
$N_b \geq 4$	$N_b = 6$	**OK**

3.4.5.3.1.1 One Design Lane Loaded

The live load distribution factor for one design lane loaded is

$$gV_1^i = 0.36 + \frac{S}{25.0}$$

$$gV_1^i = 0.36 + \frac{6.75}{25.0} = 0.630$$

3.4.5.3.1.2 Two or More Design Lanes Loaded

The live load distribution factor for two or more loaded lanes is

$$gV_{2+}^i = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0}$$

$$gV_{2+}^i = 0.2 + \frac{6.75}{12} - \left(\frac{6.75}{35}\right)^{2.0} = 0.725$$

3.4.5.4 *Live Load Distribution Factor Summary*

Distribution Factor Summary for Strength and Service Limit States

Distribution Factor	1 Loaded Load	2+ Loaded Lanes	Controlling Factor
Moment (gM)	0.412	0.584	0.584
Shear (gV)	0.630	0.725	0.725

3.4.5.5 *Live Load Distribution Factor for Fatigue Limit State*

The fatigue live load distribution uses the factor for one loaded lane (LRFD 3.6.1.4.3b). The single lane distribution factors include a multiple presence factor of 1.2. The multiple presence factor for fatigue loading is 1.0 (LRFD 3.6.1.1.2). Divide the one loaded lane distribution factors by 1.2 to get the fatigue distribution factors.

Distribution Factor Summary for Fatigue Limit States

Distribution Factor	1 Loaded Load
Moment (gM)	$0.412/1.2 = 0.343$
Shear (gV)	$0.630/1.2 = 0.525$

4 Losses and Effective Prestress

Effective prestress is the stress or force remaining in prestressing steel after time dependent losses and elastic effects have occurred. Time dependent losses consist of concrete shrinkage, concrete creep, and prestressing steel relaxation. Elastic effects are changes in the prestress due to externally applied or internal restraining forces. Elastic effects are often called elastic gains.

4.1 Losses before Prestress Transfer

Losses before prestress transfer are due to relaxation of the strand. Prior to the 2005 interim revisions to the LRFD 3rd Edition, relaxation before prestress transfer was included in prestress loss calculations. Since the 2005 interim revisions, this is no longer required based on the idea that fabricators can overstress strands to achieve an effective prestress of $0.75f_{pu}$ at release. However, WSDOT retains the practice of including relaxation prior to prestress transfer because it reflects the production practices used by local fabricators.

$$\Delta f_{pR0} = \frac{\log(24.0t)}{40.0}\left[\frac{f_{pj}}{f_{py}} - 0.55\right] f_{pj}$$

$$f_{pj} = 0.75f_{pu} = 0.75(270) = 202.5ksi$$

$$f_{py} = 0.9f_{pu} = 243ksi$$

$$t = 1\ day$$

$$\Delta f_{pR0} = \frac{\log(24.0 \cdot 1day)}{40}\left[\frac{202.5ksi}{243.0ksi} - 0.55\right](202.5ksi) = 1.980\ ksi$$

This calculation is for intrinsic relaxation of the strand. Intrinsic relaxation is associated with strand tensioned between two stationary points such as in a testing machine or between tensioning bulkheads.

4.2 Losses immediate after transfer

As the force in the pretensioned strands is released from the stressing equipment, it is transferred to the girder as a compression force. This force is typically eccentric and causes axial shortening and bending in the girder. The shortening causes a reduction in the elongation of the strand and a reduction in the precompression force. This is known as the elastic shortening losses.

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

$$f_{cgp} = \frac{P}{A} + \frac{Pe^2}{I} - \frac{M_g e}{I}$$

$$P = N(a_{ps})(f_{pj} - \Delta f_{pR0} - \Delta f_{pES})$$

Solve this equation iteratively for P and Δf_{pES}.

$$E_{ci} = 120000(1.0)(0.155)^2(6.1)^{0.33} = 5236.046\ ksi$$

Assume $P = 1696\ kip$

$$f_{cgp} = \frac{1696kip}{776.531in^2} + \frac{(1696kip)(21.007in)^2}{282559.4in^4} - \frac{(1548.65k \cdot ft)\left(\frac{12in}{1ft}\right)(21.007in)}{282559.4in^4} = 3.447ksi$$

$$\Delta f_{pES} = \frac{28500ksi}{5236.046ksi}(3.447ksi) = 18.763ksi$$

$$P = (43)(0.217in^2)(202.5ksi - 1.98ksi - 18.763ksi) = 1695.9\ kip$$

PGSuper performs this calculation with a very small convergence tolerance and at many points along the girder. The effective prestress force at release and initial lifting for various points (as determined by PGSuper) are given below.

Location	Effective Prestress after release
PSXFR	$1725.93\ kip$
HP	$1694.86\ kip$
0.5Lg	$1695.80\ kip$

4.3 Losses at Hauling

Assume hauling to occur as soon as possible (10 days).

4.3.1.1 *Shrinkage of Girder Concrete*

$$\Delta f_{SRH} = \varepsilon_{bih} E_p K_{ih}$$

$$\varepsilon_{bih} = k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3}$$

$$K_{ih} = \frac{1}{1 + \frac{E_p}{E_{ci}}\frac{A_{ps}}{A_g}\left(1 + \frac{A_g e^2}{I_g}\right)\left[1 + 0.7\psi_b(t_f, t_i)\right]}$$

$$\psi_b(t_f, t_i) = 1.9 k_s k_{hc} k_f k_{td} t_i^{-0.118}$$

$$k_s = 1.45 - 0.13\left(\frac{V}{S}\right) \geq 1.0$$

$$\frac{V}{S} = \frac{AL_g}{PL_g + 2A} = \frac{(776.531in^2)(118ft)(\frac{12in}{1ft})}{(241.284in)(118ft)\left(\frac{12in}{1ft}\right) + 2(776.531in^2)} = 3.204in$$

$$k_s = 1.45 - 0.13(3.204) = 1.03$$

$$k_{hs} = 2.00 - 0.014H = 2.00 - 0.014(75) = 0.95$$

$$k_{hc} = 1.56 - 0.008H = 1.56 - 0.005(75) = 0.96$$

$$k_f = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 6.1} = 0.704$$

$$k_{td} = \frac{t}{12\left(\frac{100 - f'_{ci}}{f'_{ci} + 20}\right) + t}$$

$$k_{td}(t = 9days) = \frac{9}{12\left(\frac{100 - 4(6.1)}{6.1 + 20}\right) + 9} = 0.206$$

$$k_{td}(t = 1999days) = \frac{1999}{12\left(\frac{100 - 4(6.1)}{6.1 + 20}\right) + 1999} = 0.983$$

$$\psi_b(t_f, t_i) = 1.9(1.03)(0.96)(0.704)(0.983)(1)^{-0.118} = 1.30$$

$$\varepsilon_{bih} = (1.03)(0.95)(0.704)(0.206)(0.48 \times 10^{-3}) = 0.0000683$$

$$A_{ps} = N(a_{ps}) = 43(0.217in^2) = 9.331in^2$$

$$K_{ih} = \frac{1}{1 + \frac{28500ksi}{5236.046ksi}\frac{9.331in^2}{776.531in^3}\left(1 + \frac{776.531in^2(21.007in)^2}{282559.4in^4}\right)[1 + 0.7(1.30)]} = 0.783$$

$$\Delta f_{pSRH} = (0.0000683)(28500ksi)(0.783) = 1.524ksi$$

4.3.1.2 *Creep of Girder Concrete*

$$\Delta f_{pCRH} = \frac{E_P}{E_{ci}} f_{cgp}\psi_b(t_h, t_i)K_{ih}$$

$$\psi_b(t_h, t_i) = 1.9(1.03)(0.96)(0.704)(0.206)(1)^{-0.118} = 0.273$$

$$\Delta f_{CRH} = \frac{28500ksi}{5236.046ksi}(3.469ksi)(0.273)(0.783) = 4.016ksi$$

4.3.1.3 *Relaxation of Prestressing Strands*

The girder concrete holds the prestressing strand in tension. The concrete undergoes creep and shrinkage deformations. The strands are between two points that move toward one another. Relaxation occurs at a reduced rate compared to intrinsic relaxation. The relaxation equations given by the AASHTO LRFD BDS are for reduced relaxation.

$$\Delta f_{pR1H} = \left[\frac{f_{pt}}{K_L'}\frac{\log(24t_h)}{\log(24t_i)}\left(\frac{f_{pt}}{f_{py}} - 0.55\right)\right]\left[1 - \frac{3(\Delta f_{pSRH} + \Delta f_{pCRH})}{f_{pt}}\right]K_{ih}$$

$$K_L' = 45$$

$$f_{pt} = \frac{1695.80kip}{9.331in^2} = 181.738ksi$$

$$\Delta f_{pR1H} = \left[\frac{181.738ksi}{45}\frac{\log(24 \cdot 10)}{\log(24 \cdot 1)}\left(\frac{181.738ksi}{243ksi} - 0.55\right)\right]\left[1 - \frac{3(1.524ksi + 4.016ksi)}{181.738ksi}\right](0.783) = 0.981ksi$$

PGSuper supports all three methods of computing relaxation described in the AASHTO LRFD BDS (LRFD 5.9.3.4.2c, C5.9.3.4.2c)

4.3.1.4 *Losses at Hauling*

$$\Delta f_{pH} = \Delta f_{pR0} + \Delta f_{pES} + \Delta f_{pLTH}$$

$$\Delta f_{pLTH} = \Delta f_{pSRH} + \Delta f_{pCRH} + \Delta f_{pR1H}$$

$$\Delta f_{pLTH} = 1.524ksi + 4.016ksi + 0.981ksi = 6.520ksi$$

$$\Delta_{pH} = 1.98ksi + 18.782ksi + 6.520ksi = 27.282ksi$$

4.4 Losses between prestress transfer and deck placement

4.4.1.1 *Shrinkage of Girder Concrete*

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id}$$

$$\varepsilon_{bid} = k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3}$$

$$K_{id} = \frac{1}{1 + \frac{E_p}{E_{ci}} \frac{A_{ps}}{A_g} \left(1 + \frac{A_g e^2}{I_g}\right) \left[1 + 0.7\psi_b\left(t_f, t_i\right)\right]}$$

$$\psi_b\left(t_f, t_i\right) = 1.9 k_s k_{hc} k_f k_{td} t_i^{-0.118}$$

$$k_s = 1.45 - 0.13\left(\frac{V}{S}\right) \geq 1.0 = 1.03$$

$$k_{hs} = 2.00 - 0.014H = 0.95$$

$$k_{hc} = 1.56 - 0.008H = 0.96$$

$$k_f = \frac{1}{1 + f'_{ci}} = 0.704$$

$$k_{td} = \frac{t}{12\left(\frac{100 - 4f'_{ci}}{f'_{ci} + 20}\right) + t} = \begin{matrix} 0774 \; with \; t = (t_d - t_i) = 199 \; day \\ 0.983 \; with \; t = \left(t_f - t_i\right) = 1999 \; day \end{matrix}$$

$$t_i = 1 \; day$$

$$t_d = 120 \; day$$

$$t_f = 2000 \; day$$

$$\varepsilon_{bid} = (1.03)(0.95)(0.82)(0.704)(0.48 \times 10^{-3}) = 0.000257$$

$$\psi_b\left(t_f, t_i\right) = 1.9(1.03)(0.96)(0.704)(0.983)(1)^{-0.118} = 1.30$$

$$K_{id} = \frac{1}{1 + \left(\frac{28500ksi}{5236.046ksi}\right)\left(\frac{9.331in^2}{776.531in^2}\right)\left(1 + \frac{(776.531in^2)(21.007in)^2}{282559.4in^4}\right)\left(1 + 0.7(1.30)\right)} = 0.783$$

$$\Delta f_{pSR} = (0.000257)(28500ksi)(0.783) = 5.733 \; ksi$$

4.4.1.2 *Creep of Girder Concrete*

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \psi_b(t_d, t_i) K_{id}$$

$$\psi_b(t_d, t_i) = 1.9(1.03)(0.96)(0.704)(0.774)(1)^{-0.118} = 1.030$$

$$\Delta f_{pCR} = \frac{28500ksi}{5236.046ksi}(3.451ksi)(1.030)(0.783) = 15.113 \; ksi$$

4.4.1.3 *Relaxation of Prestressing Strands*

$$\Delta f_{pR1} = \left[\frac{f_{pt}}{K'_L} \frac{\log(24t_d)}{\log(24t_i)} \left(\frac{f_{pt}}{f_{py}} - 0.55\right)\right]\left[1 - \frac{3\left(\Delta f_{pSRH} + \Delta f_{pCRH}\right)}{f_{pt}}\right] K_{id}$$

$$f_{pt} = f_{pj} - \Delta f_{pR0} - \Delta f_{pES} = 202.5ksi - 1.98ksi - 18.782ksi = 181.738ksi$$

$$\Delta f_{pR1} = \left[\frac{181.738\ ksi}{45}\frac{\log(24\cdot 120)}{\log(24\cdot 1)}\left(\frac{181.738ksi}{243ksi} - 0.55\right)\right]\left[1 - \frac{3(5.733ksi + 15.113ksi)}{181.738\ ksi}\right](0.783) = 1.029\ ksi$$

4.4.1.4 *Time dependent losses*

$$\Delta f_{pLT_{id}} = \Delta f_{pCR} + \Delta f_{pSH} + \Delta f_{pR1}$$

$$\Delta f_{pLT_{id}} = 5.733ksi + 15.113ksi + 1.029ksi = 21.874\ ksi$$

4.5 Losses between deck placement and final

4.5.1.1 *Shrinkage of Girder Concrete*

$$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df}$$

$$\varepsilon_{bdf} = \varepsilon_{bif} - \varepsilon_{bid}$$

$$\varepsilon = k_s k_{hs} k_f k_{td} 0.48\times 10^{-3}$$

$$K_{df} = \frac{1}{1 + \frac{E_p}{E_{ci}}\frac{A_{ps}}{A_c}\left(1 + \frac{A_c e_c^2}{I_c}\right)\left[1.0 + 0.7\psi_b\left(t_f, t_i\right)\right]}$$

From before

$$k_s = 1.03$$

$$k_{hs} = 0.95$$

$$k_{hc} = 0.96$$

$$k_f = 0.704$$

$$\psi_b\left(t_f, t_i\right) = 1.30$$

$$\varepsilon_{bid} = 0.000257$$

$$k_{td}\left(t = t_f - t_i\right) = 0.983$$

$$\varepsilon_{bif} = (1.03)(0.95)(0.704)(0.983)(0.48\times 10^{-3}) = 0.000326$$

$$\varepsilon_{bdf} = 0.000326 - 0.000257 = 0.0000694$$

$$e_c = e + y_{bc} - y_b = 21.007in + 34.726in - 24.151in = 31.582in$$

$$K_{df} = \frac{1}{1 + \left(\frac{28500ksi}{5236.046ksi}\right)\left(\frac{9.331in^2}{1213.915in^2}\right)\left(1 + \frac{(1213.915in^2)(31.582in)^2}{525343.2in^4}\right)\left(1 + 0.7(1.30)\right)} = 0.791$$

$$\Delta f_{pSD} = (0.0000694)(28500ksi)(0.791) = 1.563\ ksi$$

4.5.1.2 *Creep of Girder Concrete*

$$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp}\left[\psi_b\left(t_f, t_i\right) - \psi_b(t_d, t_i)\right]K_{df} + \frac{E_p}{E_c}(\Delta f_{cd})\psi_b\left(t_f, t_d\right)K_{df}$$

$$\Delta f_{cd} = -\left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)\left(\frac{A_{ps}}{A_g}\right)\left(1 + \frac{A_g e^2}{I_g}\right) - (\Delta f'_{cd} + \Delta f''_{cd})$$

$$M_{adl} = M_{diaphragm} + M_{slab} + M_{haunch}$$

$$M_{adl} = 79.78k\cdot ft + 1073.17k\cdot ft + 358.11k\cdot ft = 1511.06k\cdot ft$$

$$\Delta f'_{cd} = \frac{M_{adl} e}{I_g}$$

$$\Delta f'_{cd} = (1511.06k \cdot ft)\left(\frac{12in}{1ft}\right)\left(\frac{21.007in}{282559.4in^4}\right) = 1.348\ ksi$$

$$\Delta f''_{cd} = \frac{M_{sidl}\left(Y_{bc} - Y_{bg} + e\right)}{I_c}$$

$$M_{sidl} = M_{barrier} + M_{overlay}$$

$$M_{sidl} = 377.47k \cdot ft + 362.20k \cdot ft = 739.67k \cdot ft$$

$$\Delta f''_{cd} = \frac{(739.67k \cdot ft)(34.726in - 24.151in + 21.007in)}{525343.2in^4}\left(\frac{12in}{1ft}\right) = 0.534ksi$$

$$\Delta f_{cd} = -(21.875\ ksi)\left(\frac{9.331in^2}{776.531in^3}\right)\left(1 + \frac{(776.531in^2)(21.007in)^2}{282559.4in^4}\right) - (1.348ksi + 0.534ksi) = -2.463\ ksi$$

$$k_{td} = \frac{t}{12\left(\frac{100 - 4f'_{ci}}{f'_{ci} + 20}\right) + t} = 0.982\ with\ t = \left(t_f - t_d\right) = 1880\ day$$

$$\psi_b\left(t_f, t_d\right) = 1.9(1.03)(0.96)(0.704)(0.982)(120)^{-0.118} = 0.741$$

$$\Delta f_{pCD} = \left(\frac{28500ksi}{5236.046ksi}\right)(3.451ksi)(1.30 - 1.03)(0.791) + \left(\frac{28500ksi}{5530.50ksi}\right)(-2.463ksi)(0.741)(0.791) = -3.317\ ksi$$

4.5.1.3 *Relaxation of Prestressing Strands*

$$\Delta f_{pR2} = \Delta f_{pR1} = 1.029\ ksi$$

4.5.1.4 *Shrinkage of Deck Concrete*

$$\Delta f_{pSS} = \frac{E_p}{E_c} \Delta f_{cdf} K_{df}\left[1 + 0.7\psi_d\left(t_f, t_d\right)\right]$$

$$\Delta f_{cdf} = \frac{\varepsilon_{ddf} A_d E_{c\ deck}}{\left[1 + 0.7\psi_d\left(t_f, t_d\right)\right]}\left(\frac{1}{A_c} - \frac{e_c\ e_d}{I_c}\right)$$

$$\varepsilon_{ddf} = K_{sh} k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3}$$

$$k_s = 1.45 - 0.13\left(\frac{V}{S}\right) \geq 1.0$$

$$A_d = (81in)(7.5in) = 607.5in^2$$

$$\frac{V}{S} = \frac{A}{P} = \frac{W_{trib} t_{gross\ slab\ depth}}{2W_{trib} - W_{tf}} = \frac{(81in)(7.5in)}{2(81in) - 49in} = 5.376in$$

Use the gross slab depth when computing slab shrinkage effects. Shrinkage is an early age effect; therefore, the sacrificial depth is part of the deck slab that is shrinking.

$$k_s = 1.45 - 0.13(5.376) = 0.751 < 1.0\ \therefore 1.0$$

$$k_{hs} = 2.00 - 0.014H = 2.00 - 0.014(75) = 0.95$$

Slab concrete age at time of initial loading is $f'_{ci} = 0.8f'_c$. (LRFD 5.4.2.3.1)

$$f'_{ci} = 0.8f'_c = 0.8(4ksi) = 3.2\ ksi$$

$$k_f = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 3.2} = 1.19$$

$$t = t_f - t_d = 2000 - 120 = 1880\ days$$

$$k_{td} = \frac{t}{12\left(\frac{100 - 4f'_{ci}}{f'_{ci} + 20}\right) + t} = \frac{1880}{12\left(\frac{100 - 4(3.2)}{3.2 + 20}\right) + 1880} = 0.977$$

$K_{sh} = 0.5$ (BDM 5.1.4.3.D – use 50% slab shrinkage strain)

$$\varepsilon_{ddf} = (0.5)(1.0)(0.95)(1.19)(0.978)(0.48 \times 10^{-3}) = 0.265 \times 10^{-3}$$

$$\Delta f_{cdf} = \frac{(0.000265)(607.5in^2)(4266.223ksi)}{1 + 0.7(2.12)}\left(\frac{1}{1213.915in^2} - \frac{(31.582in)(19.024in)}{525343.2in^4}\right) = 0.088\ ksi$$

$$\Delta f_{pSS} = \left(\frac{28500ksi}{5530.5ksi}\right)(0.088ksi)(0.791)\big(1 + 0.7(2.12)\big) = 0.547\ ksi$$

4.5.1.5 *Time Dependent Losses*

$$\Delta f_{pLT_{df}} = \Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR1} - \Delta f_{pSS} = 1.563ksi - 3.317ksi + 1.029ksi - 0.547ksi = -1.272\ ksi$$

4.6 Elastic Gains

4.6.1.1 *Dead load on noncomposite section*

$$\Delta f_{pED} = \frac{E_p}{E_c}\Delta f'_{cd}$$

$$\Delta f'_{cd} = \frac{M_{adl}e}{I_g} = 1.348\ ksi$$

$$\Delta f_{pED} = \left(\frac{28500ksi}{5530.5ksi}\right)(1.348ksi) = 6.947\ ksi$$

4.6.1.1.1 Superimposed dead loads

$$\Delta f_{pSIDL} = \frac{E_p}{E_c}\Delta f''_{cd} = \left(\frac{28500ksi}{5530.5ksi}\right)(0.534ksi) = 2.750\ ksi$$

4.6.1.1.2 Live Loads

$$\Delta f_{pLL} = \frac{E_P}{E_c}\Delta f'''_{cd}$$

$$\Delta f'''_{cd} = \frac{M_{llim}\big(Y_{bc} - Y_{bg} + e\big)}{I_c}$$

$$\Delta f'''_{cd} = \begin{cases} \frac{(1997.7k \cdot ft)(34.726in - 24.151in + 21.007in)}{525343.2in^4}\left(\frac{12in}{1ft}\right) = 1.441\ ksi\ (Design\ Live\ Load) \\ \frac{(602.09k \cdot ft)(34.726in - 24.151in + 21.007in)}{525343.2in^4}\left(\frac{12in}{1ft}\right) = 0.434\ ksi\ (Fatigue\ Live\ Load) \end{cases}$$

$$\Delta f_{pLL} = \begin{cases} \left(\dfrac{28500ksi}{5530.5ksi}\right)(1.441ksi) = 7.426ksi = \Delta f_{pLL-Design}(Design\ Live\ Load) \\ \left(\dfrac{28500ksi}{5530.5ksi}\right)(0.434ksi) = 2.238ksi = \Delta f_{pLL-Fatigue}(Fatigue\ Live\ Load) \end{cases}$$

4.7 Effective Prestress Summary

$$\Delta f_{pLT} = \Delta f_{pLT_{id}} + \Delta f_{pLT_{df}} = 21.874ksi - 1.272ksi = 20.602ksi$$

$$\Delta f_{pT} = \Delta f_{pR0} + \Delta f_{pES} + \Delta f_{pLT} - \Delta f_{pED} - \Delta f_{pSIDL} = 1.98ksi + 18.782ksi + 20.602ksi - 6.947ksi - 2.750ksi = 31.667ksi$$

$$f_{pe} = f_{pj} - \Delta f_{pT} + \begin{cases} 1.0\Delta f_{pLL-Design}\ (Service\ I) \\ 0.8\Delta f_{pLL-Design}\ (Service\ III) \\ 1.5\Delta f_{pLL-Fatigue}\ (Fatigue\ I) \end{cases}$$

$$Service\ I\ f_{pe} = 202.5ksi - 31.667ksi + 1.0(7.426ksi) = 178.259\ ksi$$
$$Service\ III\ f_{pe} = 202.5ksi - 31.667ksi + 0.8(7.426ksi) = 176.774\ ksi$$
$$Fatigue\ I\ f_{pe} = 202.5ksi - 31.667ksi + 1.5(2.238ksi) = 174.190\ ksi$$

5 Stresses

5.1 Final Stresses

Check the final stress conditions first. If the final stresses exceed the limiting stresses, there is not point evaluating the remainder of the design.

$$f = \frac{P}{A} + \frac{Pe}{S} + \frac{M_g + M_{adl}}{S} + \frac{M_{sidl} + \gamma_{llim} M_{llim}}{S_c} + f_{ss}$$

5.1.1 Stress due to slab shrinkage

$$f_{ss} = \frac{-\varepsilon_{ddf} A_d E_{cdeck}}{[1 + 0.7\psi_d(t_f, t_d)]}\left(\frac{1}{A_c} - \frac{e_d}{S}\right)$$

$$\psi_d(t_f, t_d) = 1.9 k_s k_{hc} k_f k_{td} t_i^{-0.118}$$

$$t_i = 1\ days$$

$$\psi_d(t_f, t_d) = 1.9(1.0)(0.96)(1.19)(0.978)(1)^{-0.118} = 2.12$$

$$A_c = 1213.915in^2$$

$$e_d = Y_{tc} + \frac{t_{gross\ slab\ depth}}{2} = 15.274in + \frac{7.5in}{2} = 19.024in$$

$$S_{tgc} = -34394.2in^3$$

$$S_{bc} = 15128.3in^3$$

$$f_{top} = \frac{(-0.265 \times 10^{-3})(607.5in^2)(4266.223ksi)}{[1 + 0.7(2.12)]}\left(\frac{1}{1213.915in^2} - \frac{19.024in}{-34394.2in^3}\right) = -0.381\ ksi$$

$$f_{bot} = \frac{(-0.265 \times 10^{-3})(607.5in^2)(4266.223ksi)}{[1 + 0.7(2.12)]}\left(\frac{1}{1213.915in^2} - \frac{19.024in}{15128.3in^3}\right) = 0.120\ ksi$$

5.1.2 Service III

$$P = -(43)(0.217in^2)(176.774ksi) = -1649.48kip$$

$$f_b = \frac{-1649.48kip}{776.531in^2} + \frac{(-1649.48kip)(21.007in)}{11699.6in^3} + \frac{(1460.27 + 79.78 + 1073.17 + 358.11k \cdot ft)\left(\frac{12in}{1ft}\right)}{11699.6in^3} + \frac{(377.47 + 362.20 + 0.8 \cdot 1997.7k \cdot ft)\left(\frac{12in}{1ft}\right)}{15128.3in^3} + 0.120ksi = -5.086ksi + 4.902ksi + 0.120ksi = -0.064ksi < 0ksi\ OK$$

5.1.3 Service I

$$P = -(43)(0.217in^2)(178.259ksi) = -1663.34kip$$

Stress limit $-0.6f'_c = -0.6(7.2ksi) = -4.320ksi$

$$f_t = \frac{-1663.34kip}{776.531in^2} + \frac{(-1663.34kip)(201.007in)}{-10931.2in^3} + \frac{(1460.27 + 79.78 + 1073.14 + 358.11k \cdot ft)\left(\frac{12in}{1ft}\right)}{-10931.2in^3} + \frac{(377.47 + 362.20 + 1.0 \cdot 1997.7k \cdot ft)\left(\frac{12in}{1ft}\right)}{-34394.2in^3} - 0.381ksi = 1.054ksi - 4.217ksi - 0.381ksi = -3.543ksi < -4.320ksi\ OK$$

5.1.4 Fatigue I

$$P = -(43)(0.217in^2)(174.190ksi) = -1625.37kip$$

Stress limit $-0.4f'_c = -0.4(7.2ksi) = -2.880ksi$

$$f_t = 0.5\left[\frac{-1625.37kip}{776.531in^2} + \frac{(-1625.37kip)(21.007in)}{-10931.2in^3}\right] + \frac{0.5(1460.27 + 79.78 + 1073.14 + 358.11k \cdot ft)\left(\frac{12in}{1ft}\right)}{-10931.2in^3} + \frac{(0.5 \cdot (377.47 + 362.20k \cdot ft) + 1.5 \cdot 602.09k \cdot ft)\left(\frac{12in}{1ft}\right)}{-34394.2in^3} + 0.5(-0.381ksi) = 0.515ksi - 0.942ksi - 0.191ksi = -1.750ksi < -2.880ksi\ OK$$

5.2 Initial Stresses

Evaluate stresses immediately after release.

$$f = \frac{P}{A} + \frac{Pe}{S} + \frac{M_g}{S}$$

The governing stress immediately after release occurs at the point of prestress transfer. From PGSuper, the effective prestress is $P = -1725.93kip$.

Stress limit $-0.65f'_{ci} = -0.65(6.1ksi) = -3.965ksi$

$$f_b = \frac{-1725.93kip}{776.531in^2} + \frac{(-1725.93kip)(10.741in)}{11699.6in^3} + \frac{(153.49k \cdot ft)\left(\frac{12in}{1ft}\right)}{11699.6in^3} = -3.807ksi + 0.157ksi = -3.650ksi < -3.965ksi\ OK$$

Stress limit $0.0948\lambda\sqrt{f'_{ci}} \leq 0.200ksi = 0.0948(1.0)\sqrt{6.1} = 0.234ksi \rightarrow 0.200ksi$

$$f_t = \frac{-1725.93kip}{776.531in^2} + \frac{(-1725.93kip)(10.741in)}{-10931.2in^3} + \frac{(153.49k \cdot ft)\left(\frac{12in}{1ft}\right)}{-10931.2in^3} = -0.527ksi - 0.168ksi = -0.695ksi < 0.200ksi\ OK$$

5.3 After Deck Casting

Evaluate stresses after the deck has been cast.

This is not an AASHTO LRFD requirement. BDM 5.2.1C provides stress limits at erection. The governing erection stress case is for the noncomposite girder carrying the weight of the deck concrete.

$$f = \frac{P}{A} + \frac{Pe}{S} + \frac{M_g + M_{adl}}{S}$$

The governing stress immediately after deck placement occurs at the point of prestress transfer. From PGSuper, the effective prestress is $P = -1528.92kip$.

Stress limit $-0.45f_c' = -0.45(7.2ksi) = -3.240ksi$

$$f_b = \frac{-1528.92kip}{776.531in^2} + \frac{(-1528.92kip)(10.741in)}{11699.6in^3} + \frac{(65.10 + 64.36k \cdot ft)\left(\frac{12in}{1ft}\right)}{11699.6in^3} = -3.373ksi + 0.133ksi$$
$$= -3.240ksi \leq -3.240ksi\ OK$$

Stress limit $0.19\lambda\sqrt{f_c'} = 0.19(1.0)\sqrt{7.2} = 0.510ksi$

$$f_t = \frac{-1528.92kip}{776.531in^2} + \frac{(-1528.92kip)(10.741in)}{-10931.2in^3} + \frac{(65.10 + 64.36k \cdot ft)\left(\frac{12in}{1ft}\right)}{-10931.2in^3} = -0.467ksi - 0.142ksi = -0.609ksi$$
$$< 0.510ksi\ OK$$

5.4 After Superimposed Dead Loads (Permanent Loads Only)

$$f = \frac{P}{A} + \frac{Pe}{S} + \frac{M_g + M_{adl}}{S} + \frac{M_{sidl}}{S_c} + f_{ss}$$

$$P = -1594.04kip$$

Stress limit $-0.45f_c' = -0.45(7.2ksi) = -3.240ksi$

$$f_t = \frac{-1594.04kip}{776.531in^2} + \frac{(-1594.04kip)(21.007in)}{-10932.2in^3} + \frac{(65.10 + 64.36k \cdot ft)\left(\frac{12in}{1ft}\right)}{-10932.2in^3} + \frac{(739.67k \cdot ft)\left(\frac{12in}{1ft}\right)}{-33629.0in^3} - 0.381ksi$$
$$= 0.987ksi - 3.262ksi - 0.381ksi = -2.275ksi < -3.240ksi\ OK$$

5.5 Lifting

5.5.1 Check girder stability

Designing precast, prestressed concrete bridge girders for lateral stability ensures safety and constructability. PCI's Aspire Magazine[3] presents WSDOT's perspective on stability design.

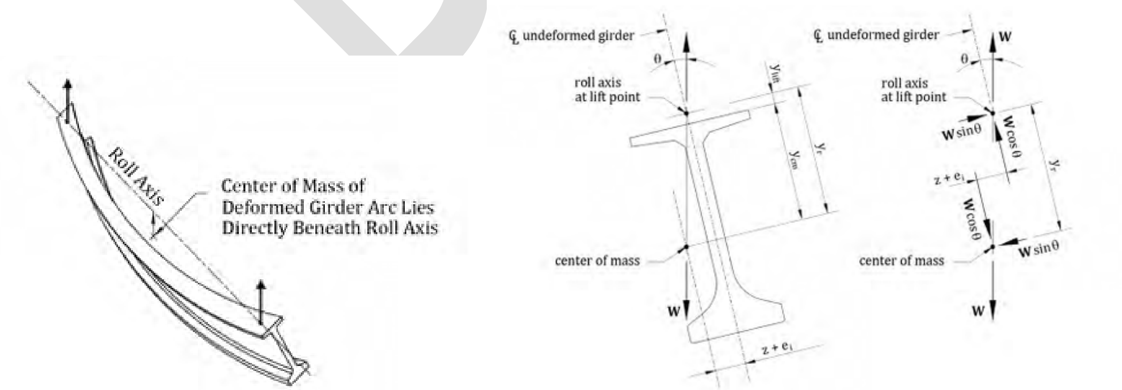

Figure 5-1: Equilibrium of Hanging Girder

5.5.1.1 *Vertical Location of Center of Gravity*

5.5.1.1.1 Estimate Camber

Compute camber for the girder in the hanging configuration. However, the stability analysis procedure needs the camber measured from a datum at the ends of the girder, not the lift points.

5.5.1.1.1.1 Girder

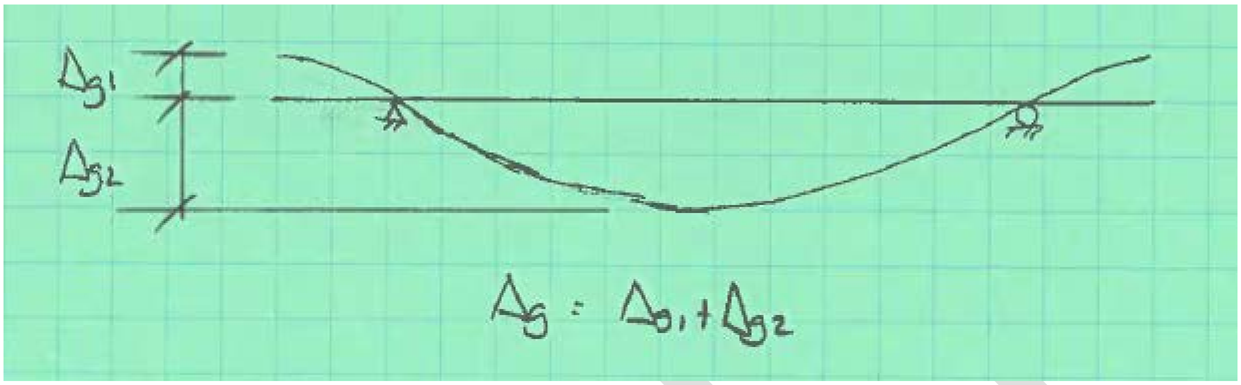

Figure 5-2: Girder Self-Weight Deflection during Lifting

$$L_s = L_g - 2a = 118ft - 2(3.75ft) = 110.5ft$$

At girder ends

$$\Delta_{g1} = \frac{w_g a}{24E_{ci}I_x}\left[3a^2(a+2L_s) - L_s^3\right]$$
$$= \frac{(-0.890klf)(3.75ft)}{24(5236.046ksi)(282559.4in^4)}\left[3(3.75ft)^2\big(3.75ft + 2(110.5ft)\big) - (110.5ft)^3\right]\left(\frac{1728in^3}{1ft^3}\right)$$
$$= 0.218\ in$$

Mid-span

$$\Delta_{g2} = \frac{5w_g L_s^4}{384E_{ci}I_x} - \frac{w_g a^2 L_s^2}{16E_{ci}I_x} = \left[\frac{5(-0.890klf)(110.5ft)^4}{384(5236.046ksi)(282559.4in^4)} - \frac{(-0.890klf)(3.75ft)^2(110.5ft)^2}{16(5236.046ksi)(282559.4in^4)}\right]\left(\frac{1728in^3}{1ft^3}\right)$$
$$= -2.018in + 0.011in = -2.007in$$

Total

$$\Delta_g = -0.218in - 2.007in = -2.225in$$

5.5.1.1.1.2 Prestressing

The customary equations for prestress induced deflections must be modified for precambered girders. See Appendix A for a derivation of the equations.

5.5.1.1.1.2.1 Straight Strands

$$P = \left(\frac{30}{43}\right)(1695.8kip) = 1183.12kip$$

$$\Delta_{ss} = \frac{P(e)L^2}{8E_{ci}I_x} = \left[\frac{(1183.12kip)(21.218in)(118ft)^2}{8(5236.046ksi)(282559.4in^4)}\right]\left(\frac{144in^2}{1ft^2}\right) = 4.253in$$

5.5.1.1.1.2.2 Harped Strands

$$P = \left(\frac{13}{43}\right)(1695.8kip) = 512.68\ kip$$

$$\delta_{pc}(x) = 4\Delta_{pc}\left(\frac{x}{L_g} - \frac{x^2}{L_g^2}\right)$$

$$\delta_{hp} = \delta_{pc}(0.4L = 47.2ft) = 4(15in)\left(\frac{47.2ft}{118ft} - \frac{(47.2ft)^2}{(118ft)^2}\right) = 14.4in$$

$$e' = e_{hp} - e_e - \delta_{hp} = 19.920in - (-16.310in) - 14.4in = 21.831in$$

$$b = 0.4$$

$$N = \frac{Pe'}{bL} = \frac{(512.68kip)(21.831in)}{(0.4)(118ft)}\left(\frac{1ft}{12in}\right) = 19.76kip$$

$$\Delta_{hs} = \frac{b(3-4b^2)NL^3}{24E_{ci}I_x} + \frac{Pe_eL^2}{8E_{ci}I_x} + \frac{5P\Delta_{pc}L^2}{48E_{ci}I_x}$$
$$= \frac{0.4(3-4(0.4)^2)(19.76kip)(118ft)^3}{24(5236.046ksi)(282559.4in^4)}\left(\frac{1728in^3}{1ft^3}\right) + \frac{(512.68kip)(-16.310in)(118ft)^2}{8(5236.046ksi)(282559.4in^4)}\left(\frac{144in^2}{1ft^2}\right)$$
$$+ \frac{5(512.68kip)(15in)(118ft)^2}{48(5236.046ksi)(282559.4in^4)}\left(\frac{144in^2}{1ft^2}\right) = 1.492in - 1.417in + 1.086in = 1.161in$$

5.5.1.1.1.3 Initial Camber

$$\Delta_{ps} = \Delta_{ss} + \Delta_{hs} = 4.253in + 1.161in = 5.414in$$

$$\Delta_{camber} = \Delta_g + \Delta_{ps} = -2.225in + 5.414in = 3.189in$$

5.5.1.1.2 Offset factor

The offset factor locates the center of mass of the girder with respect to the roll axis.

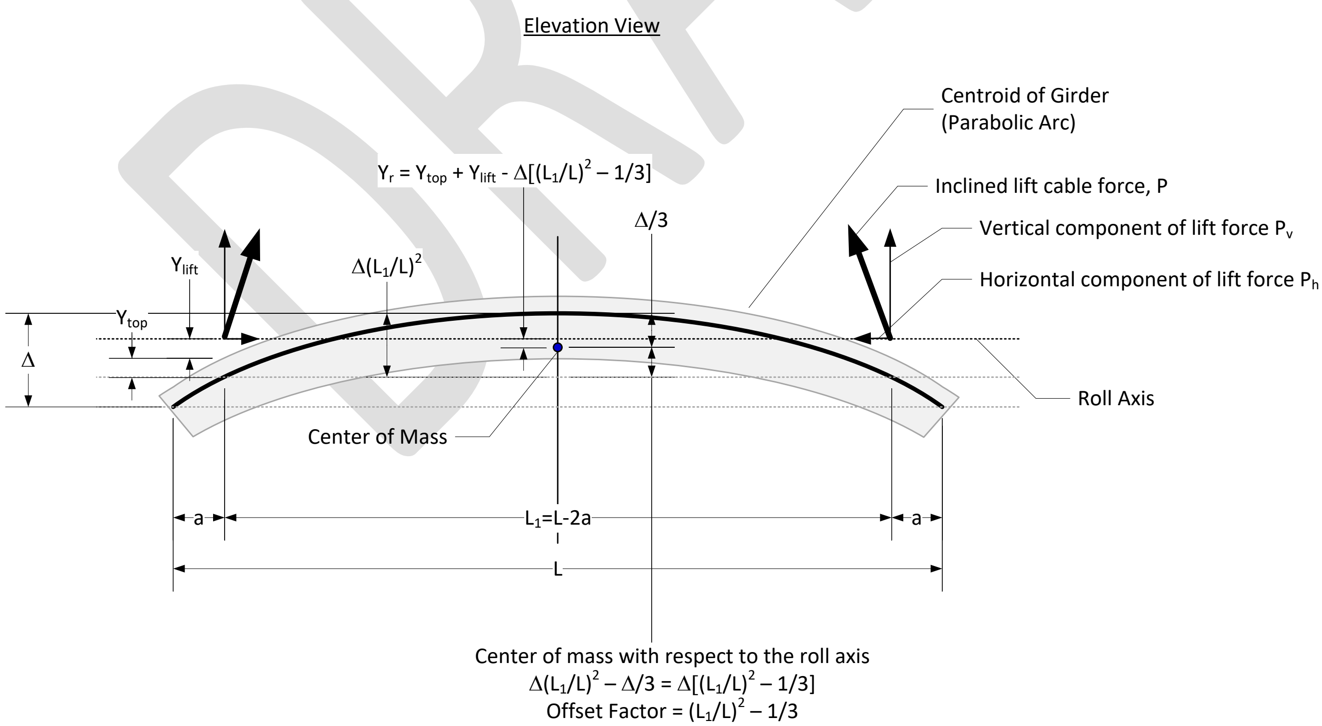

Figure 5-3: Offset Factor

$$F_o = \left(\frac{L_s}{L_g}\right)^2 - \frac{1}{3} = \left(\frac{110.5ft}{118ft}\right)^2 - \frac{1}{3} = 0.544$$

5.5.1.1.3 Location the roll axis above the top of girder

$$y_{rc} = 0in$$

5.5.1.1.4 Location of CG below roll axis

$$y_r = Y_{top} - F_o(\Delta_{camber} + \Delta_{pc}) + y_{rc} = 25.849in - (0.544)(3.189in + 15in) - 0in = 15.961in$$

5.5.1.2 *Lateral Deflection Parameters*

5.5.1.2.1 Lateral Sweep

Sweep tolerance is 1/8” per 10 ft

$$e_{sweep} = \frac{118ft}{10ft}\left(\frac{1}{8}in\right) = 1.475in$$

5.5.1.2.2 Initial Lateral Eccentricity

Initial lateral eccentricity of center of gravity of girder due to lateral sweep and eccentricity of lifting devices from CL girder

$$e_{lift} = 0.25in$$

$$e_i = F_o e_{sweep} + e_{lift} = (0.544)(1.475in) + 0.25in = 1.052in$$

5.5.1.2.3 Lateral Deflection of CG

Lateral deflection of center of gravity due to total girder weight applied to weak axis

$$W_g = w_g L_g = (0.89klf)(118ft) = 104.99kip$$

$$a = 3.75ft$$

$$L_s = L_g - 2a = 118ft - 2(3.75ft) = 110.5ft$$

$$z_o = \left(\frac{W_g}{12E_{ci}I_{yy}L_g^2}\right)\left(\frac{L_s^5}{10} - a^2L_s^3 + 3a^4L_s + \frac{6a^5}{5}\right)$$

$$= \left(\frac{104.99kip}{12(5236.046ksi)(71558.9in^4)(118ft)^2}\right)\left(\frac{(110.5ft)^5}{10} - (3.75ft)^2(110.5ft)^3 + 3(3.75ft)^4(110.5ft) + \frac{6}{5}(3.75ft)^5\right)\left(\frac{1728in^3}{1ft^3}\right) = 4.719\ in$$

5.5.1.3 *Equilibrium tilt angle*

$$\theta_{eq} = \frac{e_i}{y_r - z_o} = \frac{1.052in}{15.961in - 4.719in} = 0.09356\ rad$$

5.5.1.4 *Girder Stresses in Hanging Girder*

5.5.1.4.1 Direct stress at Prestress Transfer Point and Harp Point

5.5.1.4.1.1 Prestressing

$$f_{ps} = \frac{P}{A} + \frac{Pe}{S}$$

From PGSuper, the effective prestress force at the prestress transfer is $P = 1204.14\ kip$ straight strands and $P = 521.79kip$ harped strands. The strand eccentricities are $21.218in$ and $-13.436in$.

$$f_t = \frac{-(1204.14kip + 521.79kip)}{776.531in^2} + \frac{(-1204.14kip)(21.218in) + (-521.79kip)(-13.436in)}{-10931.2in^3} = -0.527ksi$$

$$f_b = \frac{-(1204.14kip + 521.79kip)}{776.531in^2} + \frac{(-1204.14kip)(21.218in) + (-521.79kip)(-13.436in)}{11699.6in^3} = -3.807ksi$$

From PGSuper, the effective prestress force at the harp point is $P = 1182.46\ kip$ straight strands and $P = 512.40kip$ harped strands. The strand eccentricities are $21.218in$ and $19.920in$.

$$f_t = \frac{-(1182.46kip + 512.40kip)}{776.531in^2} + \frac{(-1182.46kip)(21.218in) + (-512.40kip)(19.920in)}{-10931.2in^3} = 1.046ksi$$

$$f_b = \frac{-(1182.46kip + 512.40kip)}{776.531in^2} + \frac{(-1182.46kip)(21.218in) + (-512.40kip)(19.920in)}{11699.6in^3} = -5.200ksi$$

5.5.1.4.1.2 Girder self-weight

At Transfer point

$$M_g = \frac{(0.89klf)}{2}(3ft)^2 = -4.000k \cdot ft$$

$$f_t = \frac{-4.000k \cdot ft}{-10931.2in^3}\left(\frac{12in}{1ft}\right) = 0.004ksi$$

$$f_b = \frac{-4.000 \cdot ft}{11699.6in^3}\left(\frac{12in}{1ft}\right) = -0.004ksi$$

At Harp Point

$$M_g = \frac{w_g}{2}(L_s x - x^2 - a^2)$$

$$x = 0.4L_g - a = 0.4(118ft) - 3.75ft = 43.45ft$$

$$M_g = \frac{(0.89klf)}{2}\left((110.5ft)(43.45ft) - (44.95ft)^2 - (3.75ft)^2\right) = 1289.85k \cdot ft$$

$$f_t = \frac{1289.85k \cdot ft}{-10931.2in^3}\left(\frac{12in}{1ft}\right) = -1.416ksi$$

$$f_b = \frac{1289.85k \cdot ft}{11699.6in^3}\left(\frac{12in}{1ft}\right) = 1.323ksi$$

5.5.1.4.2 Tilt induced stresses

Top left flange tip at Transfer Point

$$f_{tl} = \frac{M_g W_{tf}}{2I_{yy}}\theta_{eq}$$

$$f_{tl} = \frac{(-4.00k \cdot ft)(49in)}{2(71558.9in^4)}(0.09356rad)\left(\frac{12in}{1ft}\right) = -0.002ksi$$

Bottom right flange tip at Transfer Point

$$f_{br} = -\frac{M_g W_{bf}}{2I_{yy}}\theta_{eq}$$

$$f_{br} = -\frac{(-4.000k \cdot ft)(38.375in)}{2(71558.9in^4)}(0.09356rad)\left(\frac{12in}{1ft}\right) = -0.001ksi$$

Top left flange tip at Harp Point

$$f_{tl} = \frac{M_g W_{tf}}{2I_{yy}}\theta_{eq}$$

$$f_{tl} = \frac{(1289.85k \cdot ft)(49in)}{2(71558.9in^4)}(0.09356rad)\left(\frac{12in}{1ft}\right) = 0.496ksi$$

Bottom right flange tip at Harp Point

$$f_{br} = -\frac{M_g W_{bf}}{2I_{yy}}\theta_{eq}$$

$$f_{br} = -\frac{(1289.85k \cdot ft)(38.375in)}{2(71558.9in^4)}(0.09356rad)\left(\frac{12in}{1ft}\right) = -0.388ksi$$

5.5.1.4.3 Total stress without tilt

Top at Transfer Point

$$f_t = -0.527ksi + 0.004ksi = -0.522ksi$$

Bottom Transfer Point

$$f_b = -3.807ksi - 0.004ksi = -3.811ksi$$

Top at Harp Point

$$f_t = 1.046ksi - 1.416ksi = -0.370ksi$$

Bottom at Harp Point

$$f_b = -5.200ksi + 1.323ksi = -3.877ksi$$

5.5.1.4.4 Total stress including tilt

$$f_t = f_{ps} + f_g + f_{tilt}$$

Top left flange tip at Transfer Point

$$f_{tl} = -0.527ksi + 0.004ksi - 0.002ksi = -0.521ksi$$

Bottom left flange tip at Transfer Point

$$f_{br} = -3.807ksi - 0.004ksi - 0.001ksi = -3.810ksi$$

Top left flange tip at Harp Point

$$f_{tl} = 1.046ksi - 1.416ksi + 0.496ksi = 0.126ksi$$

Bottom left flange tip at Harp Point

$$f_{br} = -5.200ksi + 1.323ksi - 0.388ksi = -4.265ksi$$

5.5.1.5 *Factor of Safety Against Cracking*

Lateral cracking moment

$$M_{cr} = \frac{(f_r - f_{direct})2I_{yy}}{W_{top}}$$

Tilt angle at first crack

$$\theta_{cr} = \frac{M_{cr}}{M_g} \leq 0.4\, rad$$

Cracking moment at Transfer Point

$$f_r = 0.24\lambda\sqrt{f'_{ci}} = (0.24)(1.0)\sqrt{6.1ksi} = 0.593ksi$$

$$f_{direct} = f_{ps} + f_g = -0.527ksi + 0.004ksi = -0.522ksi$$

$$M_{cr} = \frac{\big(0.593ksi - (-0.522ksi)\big)2(71558.9in^4)}{49in}\left(\frac{1ft}{12in}\right) = -271.41\ k\cdot ft$$

Tilt angle at first crack at Transfer Point

$$\theta_{cr} = \frac{-271.41k\cdot ft}{-4.000k\cdot ft} = 67.85\ rad \therefore 0.4\ rad$$

Factor of Safety against Cracking at Transfer Point

$$FS_{cr} = \frac{y_r\theta_{cr}}{e_i + z_o\theta_{cr}} = \frac{(15.961in)(0.4)}{1.052in + (4.719in)(0.4)} = 2.172$$

$$FS_{cr} > 1.0\ \textbf{OK}$$

Cracking moment at Harp Point

$$M_{cr} = \frac{\big(0.593ksi - (-0.370ksi)\big)2(71558.9in^4)}{49in}\left(\frac{1ft}{12in}\right) = 234.24\ k\cdot ft$$

Tilt angle at first crack at Harp Point

$$\theta_{cr} = \frac{234.24k\cdot ft}{1297.47k\cdot ft} = 0.18160rad$$

Factor of Safety against Cracking at Harp Point

$$FS_{cr} = \frac{y_r\theta_{cr}}{e_i + z_o\theta_{cr}} = \frac{(15.961in)(0.18160)}{1.052in + (4.719in)(0.18160)} = 1.518$$

$$FS_{cr} > 1.0\ \textbf{OK}$$

5.5.1.6 *Factor of Safety against Failure*

$$\theta_{max} = \sqrt{\frac{e_i}{2.5z_o}} \leq 0.4\ rad = \sqrt{\frac{1.052in}{2.5(4.719in)}} = 0.29857\ rad$$

$$FS_f = \frac{y_r\theta_{max}}{e_i + (1 + 2.5\theta_{max})(z_o\theta_{max})} = \frac{(15.961in)(0.29857)}{1.052in + \big(1 + 2.5(0.29857)\big)(4.719in)(0.29857)} = 1.357$$

If $FS_f < FS_{cr}, FS_f = FS_{cr}$

$$FS_f = 1.518$$

$$FS_f > 1.5\ \textbf{OK}$$

5.5.2 Check Girder Stresses

5.5.2.1 *Compression stress without tilt*

$$-0.65f'_{ci} = -0.65(6.1ksi) = -3.965\ ksi$$

Bottom at prestress transfer point

$$-3.844ksi < -3.965\ ksi\ \textbf{OK}$$

Bottom at harp point

$$-3.877ksi < -3.965\ ksi\ \textbf{OK}$$

The stress at the prestress transfer point and the harp point are approximately the same. They required concrete strength at these locations is also the same. The girder is optimized for fabrication. See Reference 2 for more information about designing for optimized fabrication.

5.5.2.2 *Compression stress with tilt*

Stress limit

$$-0.70f'_{ci} = -0.70(6.1ksi) = -4.270\ ksi$$

Bottom right at prestress transfer point

$$-3.812ksi < -4.270\ ksi\ \textbf{OK}$$

Bottom right at harp point

$$-4.265ksi < -4.270\ ksi\ \textbf{OK}$$

5.5.2.3 *Tension stress*

$$0.0948\lambda\sqrt{f'_{ci}} \leq 0.200ksi = 0.0948(1.0)\sqrt{6.1ksi} = 0.234ksi\ \therefore 0.200ksi$$

Top right at prestress transfer point

$$-0.484ksi < 0.200\ ksi\ \textbf{OK}$$

Top right at harp point

$$0.126ksi < 0.200\ ksi\ \textbf{OK}$$

5.6 Hauling

5.6.1 Check girder stability

Bunk points are H away from the ends of the girder ($4.167ft$) and hauling is assumed to occur with the HT40-72 haul truck configuration.

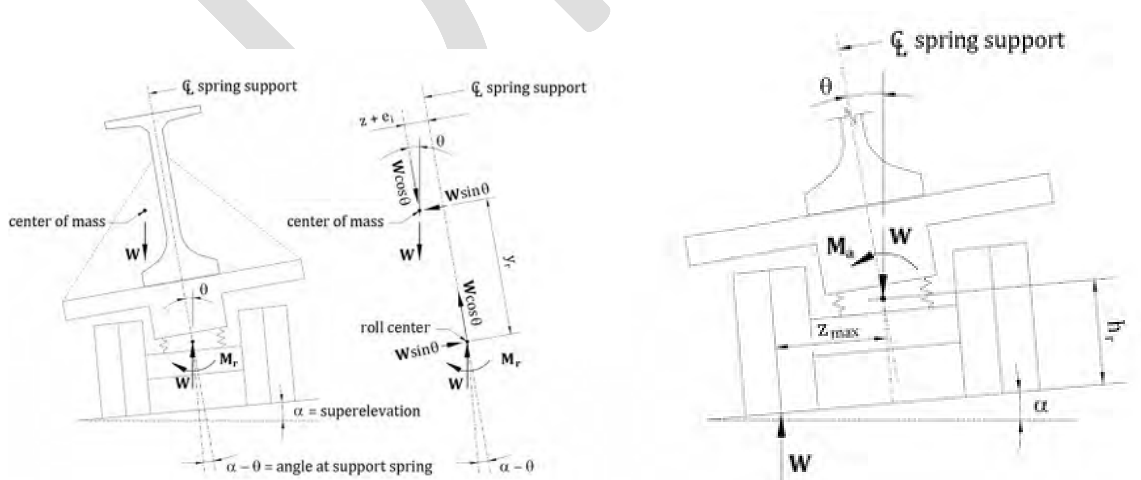

Figure 5-4: Equilibrium during Hauling

5.6.1.1 *Stability Analysis Parameters*

Parameter	Value
Rotational Stiffness	$K_\theta = 40000\dfrac{k \cdot in}{rad}$
Center-to-center wheel spacing	$W_{cc} = 72\ in$
Height of the roll center above the roadway surface	$H_{rc} = 24in$
Height of the bottom of the girder above roadway	$H_{bg} = 72\ in$
Bunk placement tolerance	$e_{bunk} = 1.0\ in$
Normal Crown Slope	$\alpha = 0.02\dfrac{ft}{ft}$
Maximum Superelevation	$\alpha = 0.06\dfrac{ft}{ft}$
Impact for Normal Crown Slope Case	$IM = \pm 20\%$
Impact for Superelevation Case	$IM = 0\%$
Modulus of Rupture	$f_r = 0.24\lambda\sqrt{f'_c} = (0.24)(1.0)\sqrt{7.1ksi} = 0.644ksi$

5.6.1.2 *Vertical Location of Center of Gravity*

5.6.1.2.1 Camber at Hauling

Assume girder transportation occurs as late as possible to maximize camber grown while in storage. Assume transportation occurs at 90 days.

The camber at hauling is equal to the camber at the end of storage plus the change in dead load deflection due to the different support conditions between storage and hauling.

From before, the prestress deflection measured from the ends of the girder is

$$\Delta_{ps} = 5.414in$$

Changing the datum to the storage support location

$$\Delta_{ps1} = 5.135in \text{ at mid-span}$$

$$\Delta_{ps2} = -0.278in \text{ at girder end}$$

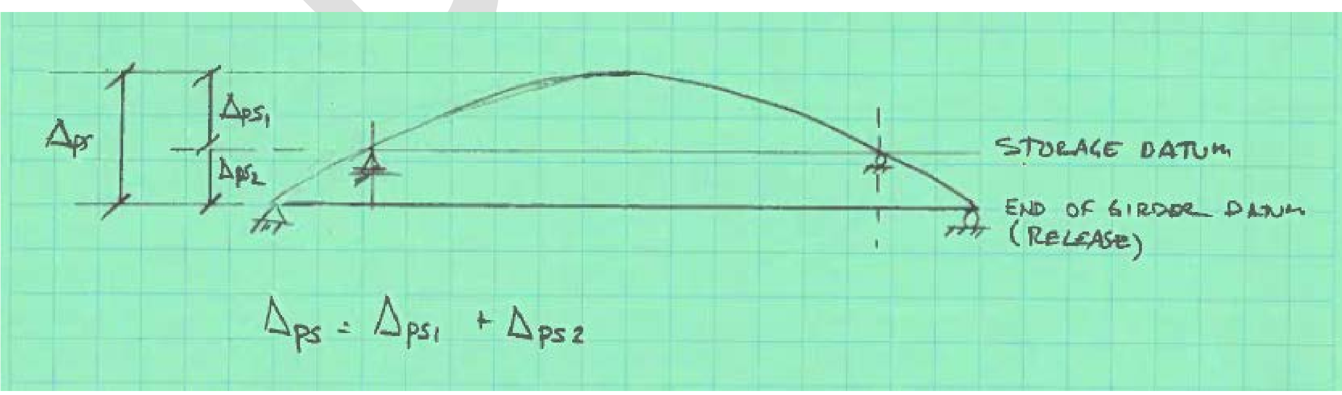

Figure 5-5: Prestress induced Deflection based on Storage Datum

The dead load deflection at mid-span during storage is

$$L_s = L_g - 2a = 118ft - 2(1.708ft) = 114.583ft$$

The dead load deflection at the girder ends during storage is

$$\Delta_{g1} = \frac{w_g a}{24E_{ci}I_x}\left[3a^2(a+2L_s) - L_s^3\right]$$
$$= \frac{(-0.890klf)(1.708ft)}{24(5236.046ksi)(282559.4in^4)}\left[3(1.708ft)^2\big(1.708ft + 2(114.583ft)\big) - (114.583ft)^3\right]\left(\frac{1728in^3}{1ft^3}\right) = 0.111\ in$$

The mid-span deflection during storage is

$$\Delta_{g2} = \frac{5w_gL_s^4}{384E_{ci}I_x} - \frac{w_ga^2L_s^2}{16E_{ci}I_x} = \left[\frac{5(-0.890klf)(114.583ft)^4}{384(5236.046ksi)(282559.4in^4)} - \frac{(-0.890klf)(1.708ft)^2(114.583ft)^2}{16(5236.046ksi)(282559.4in^4)}\right]\left(\frac{1728in^3}{1ft^3}\right)$$
$$= -2.333in + 0.003in = -2.330in$$

Creep deflection during storage is

$$\Delta_{creep} = \psi_b(t_h,t_i)(\Delta_{ps} + \Delta_g)$$

$$k_{td}(t = 89days) = \frac{89}{12\left(\frac{100-4(6.1)}{6.1+20}\right)+89} = 0.719$$

$$\psi_b(t_h,t_i) = 1.9(1.03)(0.96)(0.704)(0.719)(1)^{-0.118} = 0.955$$

At mid-span

$$\Delta_{creep} = (0.955)(5.135in - 2.330in) = 2.678in$$

At end of girder

$$\Delta_{creep} = (0.955)(-0.278in + 0.111in) = -0.159in$$

Girder deflection in the hauling configuration

$$L_s = 118ft - 2(4.167ft) = 109.667ft$$

Mid-span deflection

$$\Delta_g = \frac{5w_gL_s^4}{384E_cI_x} - \frac{w_ga^2L_s^2}{16E_cI_x} = \left[\frac{5(-0.890klf)(109.667ft)^4}{384(5530.5ksi)(282559.4in^4)} - \frac{(-0.890klf)(4.167ft)^2(109.667ft)^2}{16(5530.5ksi)(282559.4in^4)}\right]\left(\frac{1728in^3}{1ft^3}\right)$$
$$= -1.854in + 0.013in = -1.841in$$

Deflection at girder ends

$$\Delta_g = \frac{w_g a}{24E_cI_x}\left[3a^2(a+2L_s) - L_s^3\right]$$
$$= \frac{(-0.890klf)(4.167ft)}{24(5530.5ksi)(282559.4in^4)}\left[3(4.167ft)^2\big(4.167ft + 2(109.667ft)\big) - (109.667ft)^3\right]\left(\frac{1728in^3}{1ft^3}\right)$$
$$= 0.223\ in$$

We want the total camber measured between the girder ends and mid-span

$$\Delta_{camber} = \left(\Delta_g + \Delta_{ps} + \Delta_{creep}\right)_{mid-span} - \left(\Delta_g + \Delta_{ps} + \Delta_{creep}\right)_{end}$$
$$= (-1.841in + 5.135in + 2.678in) - (0.223in - 0.278in - 0.159in) = 6.186in$$

5.6.1.2.2 Offset Factor

$$F_o = \left(\frac{L_s}{L_g}\right)^2 - \frac{1}{3} = \left(\frac{109.667ft}{118ft}\right)^2 - \frac{1}{3} = 0.530$$

5.6.1.2.3 Location of roll axis below top of girder

$$y_{rc} = H_{bg} + H_g - H_{rc} = 72.0in + 50.0in - 24.0in = 98.0in$$

5.6.1.2.4 Location of center of gravity above roll axis

$$y_r = y_{rc} - Y_{top} + F_o\left(\Delta_{camber} + \Delta_{pc}\right) = 98.0in - 25.849in + 0.530(6.186in + 15in) = 83.389in$$

5.6.1.3 *Lateral Deflection Parameters*

5.6.1.3.1 Lateral Sweep

Sweep tolerance = 1/8” per 10 ft

$$e_{sweep} = \left(\frac{118ft}{10ft}\right)\left(\frac{1}{8}in\right) = 1.475in$$

5.6.1.3.2 Initial Lateral Eccentricity

Initial lateral eccentricity of center of gravity of girder due to lateral sweep and eccentricity of bunking devices from CL girder

$$e_i = F_o e_{sweep} + e_{bunk} = (0.560)(1.475in) + 1.000in = 1.782in$$

5.6.1.3.3 Lateral Deflection of CG

Lateral deflection of center of gravity due to total weight of girder applied to the weak axis

$$z_o = \frac{W_g}{12E_cI_yL_g^2}\left(\frac{L_s^5}{10} - a^2L_s^3 + 3a^4L_s + \frac{6}{5}a^5\right)$$

$$z_o = \frac{104.99kip}{12(5530.5ksi)(71558.9in^4)(118ft)^2}\left(\frac{(109.667ft)^5}{10} - (4.167ft)^2(109.667ft)^3 + 3(4.167ft)^4(109.667ft) + \frac{6}{5}(4.167ft)^5\right)\left(\frac{1728in^3}{1ft^3}\right) = 4.290in$$

5.6.1.3.4 Girder Stresses at Harping Point

5.6.1.3.4.1 Stress due to prestressing

$$f_t = \frac{-(1139.81kip + 493.92kip)}{776.531in^2} + \frac{(-1139.81kip)(221.218in) + (-493.92kip)(19.920in)}{-10931.2in^3} = 1.009ksi$$

$$f_b = \frac{-(1139.81kip + 493.92kip)}{776.531in^2} + \frac{(-1139.81kip)(221.218in) + (-493.92kip)(19.920in)}{11699.6in^3} = -5.012ksi$$

5.6.1.3.4.2 Stress due to girder self-weight (without impact)

$$M_g = \frac{w_g}{2}(L_s x - x^2 - a^2)$$

$$x = 0.4L_g - a = 0.4(118ft) - 4.167ft = 43.033ft$$

$$M_g = \frac{0.890klf}{2}\left((109.667ft)(43.033ft) - (43.033ft)^2 - (4.167ft)^2\right) = 1267.97k\cdot ft$$

$$f_t = \frac{1267.97k\cdot ft}{-10931.2in^3}\left(\frac{12in}{1ft}\right) = -1.392ksi$$

$$f_b = \frac{1267.97k \cdot ft}{11699.6in^3}\left(\frac{12in}{1ft}\right) = 1.301ksi$$

5.6.1.4 *Analyze normal crown slope, no impact case*

5.6.1.4.1 Equilibrium tilt angle

$$\theta_{eq} = \frac{\left(K_\theta \alpha + (IM)W_g e_i\right)}{K_\theta - (IM)W_g(y_r + z_o)} = \frac{\left(\left(40000\frac{k\cdot in}{rad}\right)\left(0.02\frac{ft}{ft}\right) + (1.0)(104.99kip)(1.782in)\right)}{\left(40000\frac{k\cdot in}{rad}\right) - (1.0)(104.99kip)(82.64in + 4.160in)} = 0.03196\,rad$$

5.6.1.4.2 Stress due to lateral loading from tilt

Top left flange tip

$$f_{tl} = \frac{(IM)\left(M_g\right)\theta_{eq}W_{top}}{2I_y} = \frac{(1.0)(1299.92k \cdot ft)(0.03196)(49in)}{(2)(71558.9in^4)}\left(\frac{12in}{1ft}\right) = 0.166\,ksi$$

Bottom right flange tip

$$f_{br} = -\frac{(IM)\left(M_g\right)\theta_{eq}W_{bot}}{2I_y} = -\frac{(1.0)(1299.92k \cdot ft)(0.03196)(38.375in)}{(2)(71558.9in^4)}\left(\frac{12in}{1ft}\right) = -0.130\,ksi$$

5.6.1.4.3 Factor of Safety against Cracking

Lateral cracking moment

$$f_{direct} = f_{ps} + (IM)f_g = 1.009ksi + (1.0)(-1.392ksi) = -0.389ksi$$

$$M_{cr} = \frac{(f_r - f_{direct})2I_y}{W_{top}} = \frac{\left(0.644ksi - (-0.389ksi)\right)(2)(71558.9in^4)}{49in}\left(\frac{1ft}{12in}\right) = 250.05k \cdot ft$$

Tilt angle at first crack

$$\theta_{cr} = \frac{M_{cr}}{(IM)M_g} \leq 0.4$$

$$\theta_{cr} = \frac{250.05\;k \cdot ft}{(1.0)(1267.41\;k \cdot ft)} = 0.19720\,rad$$

Factor of Safety against Cracking

$$FS_{cr} = \frac{K_\theta(\theta_{cr} - \alpha)}{(IM)W_g[(z_o + y_r)\theta_{cr} + e_i]}$$

$$FS_{cr} = \frac{\left(\left(40000\,\frac{k\cdot in}{rad}\right)\left(0.19720\,rad - 0.02\frac{ft}{ft}\right)\right)}{(1.0)(104.99kip)[(4.290in + 82.640in)(0.19720rad) + 1.782in]} = 3.540$$

$$FS_{cr} > 1.0\ \textbf{OK}$$

5.6.1.4.4 Factor of Safety against Failure

$$\theta'_{max} = \sqrt{\alpha^2 + \frac{e_i + ((IM)z_o + y_r)\alpha}{2.5(IM)z_o}} + \alpha \leq 0.4\,rad$$

$$\theta'_{max} = \sqrt{0.02^2 + \frac{1.782in + ((1.0)(4.290n) + 82.640in)0.02}{2.5(1.0)(4.290in)}} + 0.02 = 0.589\,rad\;\;\therefore 0.4\,rad$$

$$FS_f = \frac{K_\theta(\theta'_{max} - \alpha)}{(IM)W_g[((IM)z_o\theta'_{max})(1 + 2.5\theta'_{max}) + y_r\theta'_{max} + e_i]}$$

$$FS_f = \frac{40000\,\frac{k\cdot in}{rad}(0.4 - 0.02)}{(1.0)(104.99kip)[((1.0)(4.290in)(0.4)(1 + 2.5(0.40) + (82.640in)(0.4) + 1.782in]} = 3.753$$

$$FS_f > 1.5 \textbf{ OK}$$

5.6.1.4.5 Factor of Safety against Rollover

$$\theta_{ro} = \frac{(IM)W_g\left(\frac{W_{cc}}{2} - H_{rc}\alpha\right)}{K_\theta} + \alpha$$

$$\theta_{ro} = \frac{(1.0)(104.99kip)\left(\frac{72in}{2} - (24in)(0.02)\right)}{\left(40000\,\frac{k\cdot in}{rad}\right)} + 0.02 = 0.1132\,rad$$

$$FS_r = \frac{K_\theta(\theta_{ro} - \alpha)}{(IM)W_g[(z_o(1 + 2.5\theta_{ro}) + y_r)\theta_{ro} + e_i]}$$

$$FS_r = \frac{\left(40000\,\frac{k\cdot in}{rad}\right)(0.1132 - 0.02)}{(1.0)(104.99kip)\left[\left((4.290in)\left(1 + 2.5(0.1132)\right) + 82.640in\right)(0.1132) + 1.782in\right]} = 2.998$$

$$FS_r > 1.5 \textbf{ OK}$$

5.6.1.5 *Analyze normal crown slope, impact up*

5.6.1.5.1 Equilibrium tilt angle

$$\theta_{eq} = \frac{\left(K_\theta\alpha + (IM)W_g e_i\right)}{K_\theta - (IM)W_g(y_r + (IM)z_o)} = \frac{\left(\left(40000\frac{k\cdot in}{rad}\right)\left(0.02\frac{ft}{ft}\right) + (0.8)(104.99kip)(1.782in)\right)}{\left(40000\frac{k\cdot in}{rad}\right) - (0.8)(104.99kip)(82.64in + (0.8)(4.290in))} = 0.02904\,rad$$

5.6.1.5.2 Stress due to lateral loading from tilt

Top left flange tip

$$f_{tl} = \frac{(IM)\left(M_g\right)\theta_{eq}W_{top}}{2I_y} = \frac{(0.8)(1267.97k\cdot ft)(0.02904)(49in)}{(2)(71558.9in^4)}\left(\frac{12in}{1ft}\right) = 0.121\,ksi$$

Bottom right flange tip

$$f_{br} = -\frac{(IM)\left(M_g\right)\theta_{eq}W_{bot}}{2I_y} = -\frac{(0.8)(1267.97k\cdot ft)(0.02904)(38.375in)}{(2)(71558.9in^4)}\left(\frac{12in}{1ft}\right) = -0.095\,ksi$$

5.6.1.5.3 Factor of Safety against Cracking

Lateral cracking moment

$$f_{direct} = f_{ps} + (IM)f_g = 1.009ksi - (0.8)(-1.392ksi) = -0.105ksi$$

$$M_{cr} = \frac{(f_r - f_{direct})2I_y}{W_{top}} = \frac{\left(0.644ksi - (-0.105ksi)\right)(2)(71558.9in^4)}{49in}\left(\frac{1ft}{12in}\right) = 182.29k\cdot ft$$

Tilt angle at first crack

$$\theta_{cr} = \frac{M_{cr}}{(IM)M_g} \leq 0.4$$

$$\theta_{cr} = \frac{182.29\ k \cdot ft}{(0.8)(1267.41\ k \cdot ft)} = 0.17970\ rad$$

Factor of Safety against Cracking

$$FS_{cr} = \frac{K_\theta(\theta_{cr} - \alpha)}{(IM)W_g[((IM)z_o + y_r)\theta_{cr} + e_i]}$$

$$FS_{cr} = \frac{\left(\left(40000\ \frac{k \cdot in}{rad}\right)\left(0.17970\ rad - 0.02\frac{ft}{ft}\right)\right)}{(0.8)(104.99kip)[((0.8)(4.290in) + 82.640in)(0.17970rad) + 1.782in]} = 4.375$$

$$FS_{cr} > 1.0 \textbf{ OK}$$

5.6.1.5.4 Factor of Safety against Failure

$$\theta'_{max} = \sqrt{\alpha^2 + \frac{e_i + ((IM)z_o + y_r)\alpha}{2.5(IM)z_o}} + \alpha \leq 0.4\ rad$$

$$\theta'_{max} = \sqrt{0.02^2 + \frac{1.782in + ((0.8)(4.160in) + 82.640in)0.02}{2.5(0.8)(4.160in)}} + 0.02 = 0.669\ rad\ \therefore 0.4\ rad$$

$$FS_f = \frac{K_\theta(\theta'_{max} - \alpha)}{(IM)W_g[((IM)z_o\theta'_{max})(1 + 2.5\theta'_{max}) + y_r\theta'_{max} + e_i]}$$

$$FS_f = \frac{40000\ \frac{k \cdot in}{rad}(0.4 - 0.02)}{(0.8)(104.99kip)[((0.8)(4.290in)(0.4)(1 + 2.5(0.40) + (82.640in)(0.4) + 1.782in]} = 4.777$$

$$FS_f > 1.5 \textbf{ OK}$$

5.6.1.5.5 Factor of Safety against Rollover

$$\theta_{ro} = \frac{(IM)W_g\left(\frac{W_{cc}}{2} - H_{rc}\alpha\right)}{K_\theta} + \alpha$$

$$\theta_{ro} = \frac{(0.8)(104.99kip)\left(\frac{72in}{2} - (24in)(0.02)\right)}{\left(40000\ \frac{k \cdot in}{rad}\right)} + 0.02 = 0.09459\ rad$$

$$FS_r = \frac{K_\theta(\theta_{ro} - \alpha)}{(IM)W_g[((IM)z_o(1 + 2.5\theta_{ro}) + y_r)\theta_{ro} + e_i]}$$

$$FS_r = \frac{\left(40000\ \frac{k \cdot in}{rad}\right)(0.094596 - 0.02)}{(0.8)(104.99kip)\left[\left((0.8)(4.290in)\left(1 + 2.5(0.09459)\right) + 82.640in\right)(0.09459) + 1.782in\right]} = 3.527$$

$$FS_r > 1.5 \textbf{ OK}$$

5.6.1.6 *Analyze normal crown slope, impact down*

5.6.1.6.1 Equilibrium tilt angle

$$\theta_{eq} = \frac{\left(K_\theta \alpha + (IM)W_g e_i\right)}{K_\theta - (IM)W_g(y_r + (IM)z_o)} = \frac{\left(\left(40000\frac{k\cdot in}{rad}\right)\left(0.02\frac{ft}{ft}\right) + (1.2)(104.99kip)(1.782in)\right)}{\left(40000\frac{k\cdot in}{rad}\right) - (1.2)(104.99kip)(82.64in + (1.2)(4.290in))} = 0.03552\ rad$$

5.6.1.6.2 Stress due to lateral loading from tilt

Top left flange tip

$$f_{tl} = \frac{(IM)\left(M_g\right)\theta_{eq}W_{top}}{2I_y} = \frac{(1.2)(1267.97k\cdot ft)(0.03538)(49in)}{(2)(71558.9in^4)}\left(\frac{12in}{1ft}\right) = 0.222\ ksi$$

Bottom right flange tip

$$f_{br} = -\frac{(IM)\left(M_g\right)\theta_{eq}W_{bot}}{2I_y} = -\frac{(1.2)(1267.97k\cdot ft)(0.03538)(38.375in)}{(2)(71558.9in^4)}\left(\frac{12in}{1ft}\right) = -0.174\ ksi$$

5.6.1.6.3 Factor of Safety against Cracking

Lateral cracking moment

$$f_{direct} = f_{ps} + (IM)f_g = 1.009ksi + (1.2)(-1.392ksi) = -0.662ksi$$

$$M_{cr} = \frac{(f_r - f_{direct})2I_y}{W_{top}} = \frac{\left(0.644ksi - (-0.662ksi)\right)(2)(71558.9in^4)}{49in}\left(\frac{1ft}{12in}\right) = 317.81k\cdot ft$$

Tilt angle at first crack

$$\theta_{cr} = \frac{M_{cr}}{(IM)M_g} \le 0.4$$

$$\theta_{cr} = \frac{317.81\ k\cdot ft}{(1.2)(1267.41\ k\cdot ft)} = 0.20887\ rad$$

Factor of Safety against Cracking

$$FS_{cr} = \frac{K_\theta(\theta_{cr} - \alpha)}{(IM)W_g[((IM)z_o + y_r)\theta_{cr} + e_i]}$$

$$FS_{cr} = \frac{\left(\left(40000\ \frac{k\cdot in}{rad}\right)\left(0.20887\ rad - 0.02\frac{ft}{ft}\right)\right)}{(1.2)(104.99kip)[((1.2)(4.290in) + 82.640in)(0.20887rad) + 1.782in]} = 2.957$$

$FS_{cr} > 1.0$ **OK**

5.6.1.6.4 Factor of Safety against Failure

$$\theta'_{max} = \sqrt{\alpha^2 + \frac{e_i + ((IM)z_o + y_r)\alpha}{2.5(IM)z_o}} + \alpha \le 0.4\ rad$$

$$\theta'_{max} = \sqrt{0.02^2 + \frac{1.782in + ((1.2)(4.290in) + 82.640in)0.02}{2.5(1.2)(4.290in)}} + 0.02 = 0.553\ rad\ \therefore 0.4\ rad$$

$$FS_f = \frac{K_\theta(\theta'_{max} - \alpha)}{(IM)W_g[((IM)z_o\theta'_{max})(1 + 2.5\theta'_{max}) + y_r\theta'_{max} + e_i]}$$

$$FS_f = \frac{40000\ \frac{k\cdot in}{rad}(0.4 - 0.02)}{(1.2)(104.99kip)[((1.2)(4.290in)(0.4)(1 + 2.5(0.40) + (82.640in)(0.4) + 1.782in]} = 3.073$$

$FS_f > 1.5$ **OK**

5.6.1.6.5 Factor of Safety against Rollover

$$\theta_{ro} = \frac{(IM)W_g\left(\frac{W_{cc}}{2} - H_{rc}\alpha\right)}{K_\theta} + \alpha$$

$$\theta_{ro} = \frac{(1.2)(104.99kip)\left(\frac{72in}{2} - (24in)(0.02)\right)}{\left(40000\,\frac{k\cdot in}{rad}\right)} + 0.02 = 0.13188\,rad$$

$$FS_r = \frac{K_\theta(\theta_{ro} - \alpha)}{(IM)W_g[((IM)z_o(1 + 2.5\theta_{ro}) + y_r)\theta_{ro} + e_i]}$$

$$FS_r = \frac{\left(40000\,\frac{k\cdot in}{rad}\right)(0.13188 - 0.02)}{(1.2)(104.99kip)\left[\left((1.2)(4.290in)\left(1 + 2.5(0.13188)\right) + 82.640in\right)(0.13188) + 1.782in\right]} = 2.596$$

$FS_r > 1.5$ **OK**

5.6.1.7 *Analyze at maximum superelevation, no impact*

5.6.1.7.1 Equilibrium tilt angle

$$\theta_{eq} = \frac{\left(K_\theta\alpha + (IM)W_g e_i\right)}{K_\theta - (IM)W_g(y_r + (IM)z_o)} = \frac{\left(\left(40000\frac{k\cdot in}{rad}\right)\left(0.06\frac{ft}{ft}\right) + (1.0)(104.99kip)(1.782in)\right)}{\left(40000\frac{k\cdot in}{rad}\right) - (1.0)(104.99kip)(82.64in + (1.0)(4.290in))} = 0.08401\,rad$$

5.6.1.7.2 Stress due to lateral loading from tilt

Top left flange tip

$$f_{tl} = \frac{(IM)\left(M_g\right)\theta_{eq}W_{top}}{2I_y} = \frac{(1.0)(1299.92k\cdot ft)(0.08401)(49in)}{(2)(71558.9in^4)}\left(\frac{12in}{1ft}\right) = 0.441\,ksi$$

Bottom right flange tip

$$f_{br} = -\frac{(IM)\left(M_g\right)\theta_{eq}W_{bot}}{2I_y} = -\frac{(1.0)(1299.92k\cdot ft)(0.08401)(38.375in)}{(2)(71558.9in^4)}\left(\frac{12in}{1ft}\right) = -0.345\,ksi$$

5.6.1.7.3 Factor of Safety against Cracking

Lateral cracking moment

$$M_{cr} = \frac{(f_r - f_{direct})2I_y}{W_{top}} = \frac{\left(0.644ksi - (-0.383ksi)\right)(2)(71558.9in^4)}{49in}\left(\frac{1ft}{12in}\right) = 250.05k\cdot ft$$

Tilt angle at first crack

$$\theta_{cr} = \frac{M_{cr}}{(IM)M_g} \le 0.4$$

$$\theta_{cr} = \frac{250.05\,k\cdot ft}{(1.0)(1267.41\,k\cdot ft)} = 0.19720\,rad$$

Factor of Safety against Cracking

$$FS_{cr} = \frac{K_\theta(\theta_{cr} - \alpha)}{(IM)W_g[((IM)z_o + y_r)\theta_{cr} + e_i]}$$

$$FS_{cr} = \frac{\left(\left(40000\,\frac{k\cdot in}{rad}\right)\left(0.19720\,rad - 0.06\,\frac{ft}{ft}\right)\right)}{(1.0)(104.99kip)[((1.0)(4.290in) + 82.640in)(0.19720rad) + 1.782in]} = 2.741$$

$$FS_{cr} > 1.0 \textbf{ OK}$$

5.6.1.7.4 Factor of Safety against Failure

$$\theta'_{max} = \sqrt{\alpha^2 + \frac{e_i + ((IM)z_o + y_r)\alpha}{2.5(IM)z_o}} + \alpha \le 0.4\,rad$$

$$\theta'_{max} = \sqrt{0.06^2 + \frac{1.782in + ((1.0)(4.290in) + 82.640in)0.06}{2.5(1.0)(4.290in)}} + 0.06 = 0.878\,rad \;\therefore\; 0.4\,rad$$

$$FS_f = \frac{K_\theta(\theta'_{max} - \alpha)}{(IM)W_g[((IM)z_o\theta'_{max})(1 + 2.5\theta'_{max}) + y_r\theta'_{max} + e_i]}$$

$$FS_f = \frac{40000\,\frac{k\cdot in}{rad}(0.4 - 0.06)}{(1.0)(104.99kip)[((1.0)(4.290in)(0.4)(1 + 2.5(0.40) + (82.640in)(0.4) + 1.782in]} = 3.358$$

$$FS_f > 1.5 \textbf{ OK}$$

5.6.1.7.5 Factor of Safety against Rollover

$$\theta_{ro} = \frac{(IM)W_g\left(\frac{W_{cc}}{2} - H_{rc}\alpha\right)}{K_\theta} + \alpha$$

$$\theta_{ro} = \frac{(1.0)(104.99kip)\left(\frac{72in}{2} - (24in)(0.06)\right)}{\left(40000\,\frac{k\cdot in}{rad}\right)} + 0.06 = 0.15071\,rad$$

$$FS_r = \frac{K_\theta(\theta_{ro} - \alpha)}{(IM)W_g[((IM)z_o(1 + 2.5\theta_{ro}) + y_r)\theta_{ro} + e_i]}$$

$$FS_r = \frac{\left(40000\,\frac{k\cdot in}{rad}\right)(0.15071 - 0.06)}{(1.0)(104.99kip)\left[\left((1.0)(4.290in)\left(1 + 2.5(0.15071)\right) + 82.640in\right)(0.15071) + 1.782in\right]} = 2.268$$

$$FS_r > 1.5 \textbf{ OK}$$

5.6.2 Check Girder Stresses

5.6.2.1 *Compression stress*

Maximum compression occurs at the harp point with impact up.

Check compression without lateral bending

$$f_b = f_{ps} + (IM)f_g$$

$$f_b = -5.012ksi + (0.8)(1.301ksi) = -3.971ksi$$

$$-0.65f'_c = -0.65(7.2ksi) = -4.680ksi$$

$$-3.971ksi < -4.680\,ksi \textbf{ OK}$$

Check compression stress at bottom right corner of girder

$$f_b = f_{ps} + (IM)\left(f_g + f_{tilt}\right)$$
$$f_b = -5.012ksi + (0.8)(1.301ksi - 0.131ksi) = -4.066\ ksi$$
$$-0.70f_c' = -0.70(7.2ksi) = -5.040ksi$$
$$-4.066ksi < -5.040ksi \ \textbf{OK}$$

5.6.2.2 *Tension stress*

Stress limit

$$0.0948\lambda\sqrt{f_c'} = 0.0948(1.0)\sqrt{7.2ksi} = 0.254ksi$$

Maximum tension stress occurs at top left corner of girder on normal crown slope with impact up at the harp point

$$f_t = 1.009ksi + (1.2)(-1.392ksi) = 0.016\ ksi$$
$$0.016\ ksi < 0.254\ ksi \ \textbf{OK}$$

6 Flexural Capacity

6.1.1.1 *Compute Nominal Moment Capacity at 0.5L_g.*

Strength I limit state

$$Strength\ I = 1.25DC + 1.5DW + 1.75(LL + IM)$$
$$M_u = 1.25(1460.27 + 79.78 + 1073.17 + 358.11 + 377.47) + 1.50(362.20) + 1.75(0.584)(3421.07) = 8225.27k \cdot ft$$
$$c = \frac{A_{ps}f_{pu} - \alpha_1 f_c'(b - b_w)h_f}{\alpha_1 f_c' \beta_1 b_w + kA_{ps}\dfrac{f_{pu}}{d_p}}$$
$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 2\left(1.04 - \frac{243}{270}\right) = 0.28$$
$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$
$$\alpha_1 = 0.85$$
$$d_p = Y_t + e + t_s = 25.849in + 21.205in + 7in = 54.054in$$
$$c = \frac{(9.331in^2)(270ksi) - 0.85(4ksi)(81in - 6.125in)(7in)}{0.85(4ksi)(0.85)(6.125in) + (0.28)(9.331in^2)\left(\dfrac{270ksi}{54.054in}\right)} = \frac{737.345kip}{17.7\dfrac{k}{in} + 13.050\dfrac{k}{in}} = 23.978in$$
$$f_{ps} = 270ksi\left(1 - 0.28\frac{23.978in}{54.054in}\right) = 236.464ksi$$
$$a = \beta_1 c = 0.85(23.978in) = 20.381in$$
$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + \alpha_1 f_c'(b - b_w)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)$$
$$M_n = (9.331in^2)(236.464ksi)\left(54.054in - \frac{20.381in}{2}\right) + 0.85(4ksi)(81in - 6.125in)(7in)\left(\frac{20.381in}{2} - \frac{7in}{2}\right)$$
$$= 108705k \cdot in = 9058.8k \cdot ft$$
$$d_t = 57in - 2in = 55in$$
$$\varepsilon_t = 0.003\left(\frac{d_t}{c} - 1\right) = 0.003\left(\frac{55in}{20.381in} - 1\right) = 0.005$$

$$0.75 \le \phi = 0.75 + \frac{0.25(\varepsilon_t - \varepsilon_{cl})}{\varepsilon_{tl} - \varepsilon_{cl}} \le 1.0 = 0.75 + \frac{0.25(0.005 - 0.005)}{0.005 - 0.002} = 0.75 \therefore \phi = 0.75$$

$$M_r = \phi M_n = 0.75(9058.8k \cdot ft) = 6794.1k \cdot ft$$

$M_r < M_u$ **NO GOOD**

The AASHTO method for computing moment capacity does not account for the large compression flange in the girder or the higher strength of the girder concrete. See Reference 7 for more information. PGSuper uses strain compatibility analysis to compute the moment capacity.

Stress-strain relationship for prestressing strands:

$$f_{ps} = \varepsilon_{ps}\left[877 + \frac{27{,}613}{\left(1 + \left(112.4\varepsilon_{ps}\right)^{7.36}\right)^{\frac{1}{7.36}}}\right] \le 270ksi$$

Stress-strain relationship for concrete:

$$f_c = f_c' \frac{n\left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right)}{n - 1 + \left(\frac{\varepsilon_{cf}}{\varepsilon_c'}\right)^{nk}}$$

where

$$n = 0.8 + \frac{f_c'}{2500}$$

$$k = 0.67 + \frac{f_c'}{9000}$$

$$if\ \frac{\varepsilon_{cf}}{\varepsilon_c'} < 1.0, k = 1.0$$

$$E_c = \frac{40{,}000\sqrt{f_c'} + 1{,}000{,}000}{1000}$$

$$\varepsilon_c' x1000 = \frac{f_c'}{E_c}\frac{n}{n-1}$$

Effective prestress, $f_{pe} = f_{pj} - \Delta f_{pT} = 202.5ksi - 31.667ksi = 170.833ksi$

Initial strain in prestressing strand, $\varepsilon_{psi} = \frac{f_{pe}}{E_p} = \frac{170.833ksi}{28500ksi} = 4.994x10^{-3}$

Discretize the composite girder section into “slices”. Compute the strain at the centroid of each slice. The stress in the slice is determined from the stress-strain relationship for the slice material. Finally, compute the axial force and moment contribution for each slice. Sum the contribution of each slice to determine the capacity of the section.

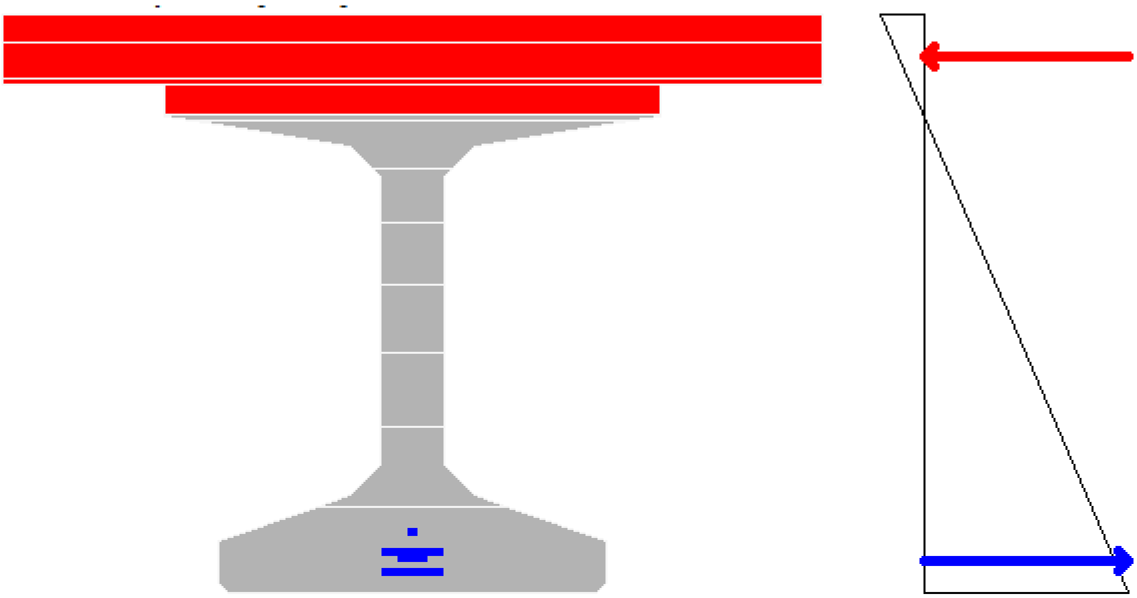

Figure 6-1: Discretized Girder Section for Strain Compatibility Analysis

Slice	Area (in^2)	Y_{cg} (in)	Strain	Stress (KSI)	δF = (Area)(Stress) (kip)	δM = (δF)(Y_{cg}) (kip-ft)
1	230.850	31.424	-0.00258311	-3.603	0.000	-3.603
2	282.150	28.257	-0.0016567	-3.931	0.000	-3.931
3	54.000	26.182	-0.00104965	-3.186	0.000	-3.186
4	159.077	24.225	-0.000477198	-2.094	0.000	-2.094
5	8.729	22.497	2.83273e-05	0.000	0.000	0.000
6	92.717	20.767	0.000534609	0.000	0.000	0.000
7	33.613	15.003	0.00222073	0.000	0.000	0.000
8	36.852	9.257	0.00390179	0.000	0.000	0.000
9	40.731	2.924	0.00575461	0.000	0.000	0.000
10	44.610	-4.043	0.00779273	0.000	0.000	0.000
11	67.310	-12.403	0.0102384	0.000	0.000	0.000
12	0.217	-17.751	0.0177973	261.251	0.000	261.251
13	3.038	-20.151	0.0184994	261.924	0.000	261.924
14	292.892	-20.229	0.0125279	0.000	0.000	0.000
15	2.604	-20.751	0.0186749	262.090	0.000	262.090
16	3.472	-22.151	0.0190845	262.474	0.000	262.474

Resultant Force = ∑(δF) = 0.00 kip

Resultant Moment = ∑(δM) = -10120.56 kip-ft

Depth to neutral axis, *c = 10.255 in*
Compression Resultant, *C = -2446.21 kip*
Depth to Compression Resultant, d_c =*4.210 in*

Tension Resultant, $T = 2446.21\ kip$
Depth to Tension Resultant, $d_e = 53.857\ in$
Nominal Capacity, $M_n = 10120.56\ kip\text{-}ft$
Moment Arm, $d_e - d_c = M_n/T = 49.647\ in$

The capacity reduction factor is

$$\varepsilon_t = 0.003\left(\frac{d_t}{c} - 1\right) = 0.003\left(\frac{55in}{10.255in} - 1\right) = 0.013$$

$$0.75 \le \phi = 0.75 + \frac{0.25(\varepsilon_t - \varepsilon_{cl})}{\varepsilon_{tl} - \varepsilon_{cl}} \le 1.0 = 0.75 + \frac{0.25(0.013 - 0.005)}{0.005 - 0.002} = 1.5 \therefore \phi = 1.0$$

$$M_r = 10120.56k \cdot ft \ge M_u = 8225.27k \cdot ft \ \ \textbf{OK}$$

6.1.1.2 *Minimum Reinforcement and the Cracking Moment*

In order to insure there is sufficient reinforcement in the section to achieve ductile behavior, a minimum amount of reinforcement is required. The minimum reinforcement is such that any section in the girder shall have adequate prestressed reinforcement to develop a factored flexural resistance, M_r, which is at least the lesser of the cracking strength or 133% of the ultimate moment. (LRFD 5.6.3.3)

$$M_{r\ min} = lesser\ of\ \begin{cases} M_{cr} \\ 1.33M_u \end{cases}$$

The cracking moment is

$$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S_c - M_{dnc}\left(\frac{S_c}{S_b} - 1\right)\right]$$

where:

f_r	=	Modulus of rupture
f_{cpe}	=	Compressive stress due to prestressing at the bottom of the girder
S_c	=	Bottom section modulus of the composite section
S_b	=	Bottom section modulus of the non-composite section
M_{dnc}	=	Dead load moment resisted by the non-composite section
γ_1	=	Flexural cracking variability factor = 1.6
γ_2	=	Prestress variability factor = 1.1
γ_3	=	Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement = 1.0 for prestressed concrete

6.1.1.2.1 Compute cracking moment at 0.5L_g.

$$f_r = 0.24\sqrt{f_c'} = 0.24\sqrt{7.2ksi} = 0.644ksi$$

$$f_{cpe} = 4.915ksi$$

$$S_c = 15128.3in^3$$

$$S_{nc} = 11699.6in^3$$

$$M_{dnc} = M_{girder} + M_{diaphragms} + M_{slab} + M_{haunch} = 2971.33k \cdot ft$$

$$M_{cr} = 1.0\left[(1.6 \cdot 0.644ksi + 1.1 \cdot 4.915ksi)(15128.3in^3)\left(\frac{1ft}{12in}\right) - (2971.33k \cdot ft)\left(\frac{15128.3in^3}{11699.6in^3} - 1\right)\right] = 7244.k \cdot ft$$

6.1.1.2.2 Evaluate Minimum Reinforcement Requirement

$M_u = 8225.27k \cdot ft$

$$M_{r\,min} = lesser\ of \begin{cases} M_{cr} = 7244k \cdot ft \\ 1.33M_u = 1.33 \cdot 8225.27k \cdot ft = 10939k \cdot ft \end{cases} = 7244k \cdot ft$$

$M_r = 10120k \cdot ft \geq M_{r\,min} = 7244k \cdot ft$ **OK**

6.2 Check Splitting Resistance

Compute the splitting resistance of the pretensioned anchorage zone provided by the vertical reinforcement in the ends of the girder at the service limit states as $P_r = f_s A_s$ (5.10.10.1) where,

f_s = the stress in the steel not exceeding *20 ksi*

A_s = total area of vertical reinforcement located within the distance h/4 from the end of the beam (in^2)

h = overall depth of the girder (*in*)

The resistance shall not be less than 4% of the prestressing force at transfer.

The splitting force at PSXFR is $P = 0.04A_{ps}(f_{pj} - \Delta f_{pR0} - \Delta f_{pES}) = 0.04(9.331in^2)(202.5ksi - 1.98ksi - 18.782ksi) = 69.04kip$

The splitting zone is $\frac{h}{4} = \frac{4.1667ft}{4} = 1.042ft$. The vertical reinforcement in the splitting zone is $2.569in^2$.

The splitting resistance is $P_r = f_s A_s = (20\ ksi)(2.569in^2) = 51.37\ kip$

$P < P_r$ **NO GOOD, but OK per BDM 5.6.2F if total splitting reinforcement is provided at 2.5" spacing**

If the splitting reinforcement does not fit within H/4 from the end of the girder, BDM 5.6.2F permits the total splitting reinforcement to extend beyond H/4 at a spacing not greater than 2.5"

6.3 Check Confinement Zone Reinforcement

For the distance of *1.5d* from the ends of the girder, reinforcement shall be placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than #3 deformed bars with spacing not exceeding *6 in*.

The length of the confinement zone is $1.5d = 1.5(50\ in) = 75\ in = 6.25\ ft$.

Provide #3 bars spaced at 6" for the end 6.25ft of the girder.

7 Shear Capacity

Ensure the girder has sufficient capacity to resist shear in the Strength I limit state. Verify that shear reinforcement is adequately detailed.

These computations and checks demonstrate shear design at the critical section (LRFD 5.7.3.2 and 5.7.3.3). A complete design will also evaluated shear locations where abrupt changes to the shear force diaphragm occur and at changes in reinforcement size and spacing.

7.1 Locate Critical Section for Shear

The critical section for shear is located at d_v from the face of support where d_v is from the critical section. For purposes of design, the ultimate shear between the support and the critical section is equal to the shear at the critical section.

Determining the location of the critical section can be challenging because d_v varies with position along the girder. To find the critical section plot d_v along the length of the girder and draw a 45° line from the face of support towards the center of the girder. The intersection point of the 45° line and the d_v curve is the location of the critical section. Figure 7-1 illustrates this technique.

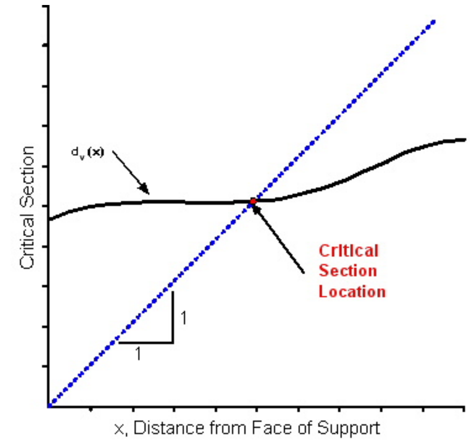

Figure 7-1: Graphical method to Determine Critical Section Location

For this girder, the critical sections are located $4.555\ ft$ and $110.028\ ft$ from the left support. The tables that follow show the details for finding the critical sections.

Table 7-1: Critical Section Calculation Details for Abutment 1

Location from Left Support (ft)	Assumed C.S. Location (in)	d_v (in)	CS Intersects?
(FoS) 0.500	0.000	48.660	No
(Bar Develop.) 1.087	7.041	48.660	No
(PSXFR) 1.292	9.500	48.660	No
2.042	18.500	48.661	No
2.458	23.500	48.661	No
3.125	31.500	48.661	No
4.555	48.661	48.661	*Yes
(H) 4.667	50.000	48.661	No
(1.5H) 6.750	75.000	47.981	No
10.092	115.100	45.733	No

* - Intersection values are linearly interpolated

Table 7-2: Critical Section Calculation Details for Abutment 2

Location from Left Support (ft)	Assumed C.S. Location (in)	d_v (in)	CS Intersects?
104.492	115.100	45.733	No
(1.5H) 107.833	75.000	47.981	No

(H) 109.917	50.000	48.661	No
110.028	48.661	48.661	*Yes
111.458	31.500	48.661	No
112.125	23.500	48.661	No
112.542	18.500	48.661	No
(PSXFR) 113.292	9.500	48.660	No
(Bar Develop.) 113.497	7.041	48.660	No
(FoS) 114.083	0.000	48.660	No

* - Intersection values are linearly interpolated

7.2 Check Ultimate Shear Capacity

7.2.1 Compute Nominal Shear Resistance

The nominal shear resistance, V_n, is the lesser of:

$$V_n = V_c + V_p + V_s$$

$$V_n = 0.25 f_c' b_v d_v + V_p$$

for which

$$V_c = 0.0316\beta\sqrt{f_c'} b_v d_v$$

$$V_s = \frac{A_v f_y d_v \cot\theta}{s}$$

where

b_v = Effective web width taken as the minimum web width within the depth d_v.

d_v = Effective shear depth

s = Stirrup spacing

β = Factor indicating ability of diagonally cracked concrete to transmit tension

θ = Angle of inclination of diagonal compressive stresses

A_v = Area of shear reinforcement within a distance s

V_p = Component in the direction of the applied shear of the effective prestressing force, positive if resisting the applied shear.

7.2.1.1 *Determination of β and θ*

Step 1: Determine b_v

b_v is the effective web width. For this girder $b_v = 6.125 in$.

Step 2: Determine d_v

d_v is the distance measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (internal moment arm), but it need not be taken less than the greater of $0.9d_e$ or $0.72h$.

From a flexural capacity analysis at the critical section the *Moment Arm =41.680 in*, d_e *=54.068 in*, and h *=57 in*.

$$d_v = greatest\ of \begin{cases} Moment\ Arm = 41.680in \\ 0.9d_e = 0.9(54.068in) = 48.661in \\ 0.72h = 0.72(57in) = 41.040in \end{cases}$$

Step 3: Compute stress in prestrssing steel when the stress in the surrounding concrete is 0.0 ksi

$$f_{po} = 0.70f_{pu} = 189ksi$$

Step 4: Compute the longitudinal strain on the flexural tension side of the beam

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v}+0.5N_u+|V_u-V_p|-A_{ps}f_{po}\right)}{E_sA_s + E_pA_{ps} + E_cA_{ct}}\ for\ \varepsilon_s < 0$$

At the critical section

$$f_{pe} = 159.304\ ksi$$

$$P_{eh} = (13)(0.217in^2)(159.304ksi) = 449.396\ kip$$

$$V_p = \frac{P_{eh}}{\sqrt{1^2 + \left(\frac{0.4L}{e'}\right)^2}}$$

$$e' = 24.6in$$

$$0.4L = 47.2ft = 566.4in$$

$$V_p = \frac{449.4kip}{\sqrt{1^2 + \left(\frac{566.4in}{24.6in}\right)^2}} = 17.3\ kip$$

$$M_u = 1266.25\ k\cdot ft$$

$$N_u = 0\ kip$$

$$V_u = 299.68kip$$

$$\left|V_u - V_p\right| = 282.37\ kip$$

$$d_v = 46.881\ in$$

$$A_s = 0\ in^2$$

$$E_s = 29000\ ksi$$

$$A_{ps} = 5.955\ in^2$$

$$E_{ps} = 28500\ ksi$$

$$A_{ct} = 433.906\ in^2$$

$$E_c = 5530.5ksi$$

$$\varepsilon_s = \frac{\left(\frac{|1266.25k\cdot ft|\left(\frac{12in}{1ft}\right)}{46.881in}+0.5(0)+282.37kip-(5.955in^2)(189ksi)\right)}{(29000ksi)(0in^2) + (28500ksi)(5.955in^2) + (5530.5\text{ksi})(433.906in^2)} = -0.207x10^{-3} < 0$$

Step 5: Compute β and θ

$$\beta = \frac{4.8}{(1+750\varepsilon_s)} = \frac{4.8}{\left(1+(750)(-0.207x10^{-3})\right)} = 5.68$$

$$\theta = 29 + 3500\varepsilon_s = 29 + (3500)(-0.207x10^{-3}) = 28.3°$$

7.2.1.2 *Compute Shear Capacity of Concrete*

$$V_c = 0.0316\beta\lambda\sqrt{f_c'}b_v d_v = 0.0316(5.68)(1.0)\sqrt{7.2ksi}(6.125in)(48.661in) = 143.55\ kip$$

7.2.1.3 *Compute Shear Capacity of Transverse Reinforcement*

For #5 stirrups, $A_v = 0.62\ in^2$.

$$V_s = \frac{A_v f_y d_v \cot\theta}{s} = \frac{(0.62\ in^2)(60\ ksi)(48.661in)\cot 28.3}{6\ in} = 560.86\ kip$$

7.2.1.4 *Compute Nominal Shear Capacity of Section*

$$V_n = V_c + V_p + V_s = 143.55\ kip + 17.31\ kip + 560.86\ kip = 721.71\ kip$$

$$V_n = 0.25f_c'b_v d_v + V_p = 0.25(7.2\ ksi)(6.125\ in)(48.661\ in) + 17.31kip = 553.8kip$$

$$V_r = \phi V_n = 0.9(553.8\ kip) = 498.4\ kip$$

7.2.1.5 *Check Ultimate Shear Capacity*

$V_u = 299.68\ kip \le V_r = 498.4\ kip$ **OK**

Repeat these calculations at all locations where stirrup size or spacing changes or where the applied shear abruptly changes.

7.2.2 Check Requirement for Transverse Reinforcement

Transverse reinforcement is required when $V_u > 0.5\phi(V_c + V_p)$. (LRFD 5.8.2.4)

$$0.5\phi(V_c + V_p) = 0.5(0.9)(143.55\ kip + 17.31\ kip) = 72.4\ kip < 299.68\ kip$$

V_u exceeds the limiting value; therefore, transverse reinforcement is required at this section. Transverse reinforcement is provided. **OK**

7.2.3 Check Minimum Transverse Reinforcement

Where transverse reinforcement is required, as specified in LRFD 5.7.2.5, the area of steel shall not be less than $A_{v\ min} = 0.0316\lambda\sqrt{f_c'}\frac{b_v s}{f_y} = 0.0316(1.0)\sqrt{7.2\ ksi}\frac{(6.125\ in)(6\ in)}{60\ ksi} = 0.0519\ in^2 < 0.62\ in^2$

OK

This can also be represented as $\frac{A_v}{s}min = 0.0316\lambda\sqrt{f_c'}\frac{b_v}{f_y} = 0.0316(1.0)\sqrt{7.2\ ksi}\frac{6.125\ in}{60\ ksi} = 0.00866\frac{in^2}{in} = 0.104\frac{in^2}{ft}$.

7.2.4 Check Maximum Spacing of Transverse Reinforcement

The spacing of the transverse reinforcement shall not exceed the following:

- If $v_u < 0.125f_c'$ then $s \le 0.8d_v \le 24\ in$
- If $v_u \ge 0.125f_c'$ then $s \le 0.4d_v \le 12\ in$

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} = \frac{|299.68kip - 0.9(17.31kip)|}{0.9(6.125in)(48.661in)} = 1.059ksi$$

$$0.125f_c' = 0.125(7.2\ ksi) = 0.90ksi < 1.059ksi$$

$$s_{max} = 0.4d_v = 0.4(48.661\ in) = 19.464\ in\ > 12\ in \rightarrow s_{max} = 12\ in$$

The actual spacing is 6.0 in. **OK**

7.3 Check Longitudinal Reinforcement for Shear

At each section, the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy:

$$A_s f_y + A_{ps} f_{ps} \geq \left[\frac{M_u}{d_v \phi_f} + 0.5\frac{N_u}{\phi_a} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta\right]$$

At the inside edge of the bearing area of simple end supports to the section of critical shear, the longitudinal reinforcement on the flexural tension side of the member shall satisfy:

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\phi_v} - 0.5V_s - V_p\right)\cot\theta$$

At the critical section, all of the harped strands are above the mid-height of the girder. The harped strands are not on the flexural tension side (See LRFD Figure 5.7.3.4.2-2)

$$A_{ps} = (30)(0.217in^2) = 6.510in^2$$

From the moment capacity analysis, $f_{ps,avg} = 131.375ksi$. The stress in the strands adjusted for lack of full development in the moment capacity analysis. Do not apply the reduction again in these calculations (See LRFD 5.9.4.3.2).

$M_u = 144.07k \cdot ft$

$d_v = 48.660\ in$

$V_u = 299.68kip$

$V_s = 332..97kip$

$V_p = 17.31kip$

$\theta = 28.3°$

$$\frac{M_u}{d_v \phi_f} + 0.5\frac{N_u}{\phi_a} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

$$= \frac{144.07k \cdot ft\left(\frac{12in}{1ft}\right)}{(48.660in)(1.0)} + 0.5\frac{(0)}{1.0} + \left(\left|\frac{299.68kip}{0.9} - 17.31kip\right| - 0.5(332.97kip)\right)\cot 28.3° = 277.32kip$$

$A_{ps} f_{ps} = (6.510in^2)(131.375ksi) = 855.25kip$

$855.25kip \geq 277.32kip$ **OK**

7.4 Check Horizontal Interface Shear

This entire design is based on the assumption that the slab and girder work together to form a composite section. Verify the slab-girder interface has adequate capacity to develop this composite action.

7.4.1 Check Nominal Capacity

The critical section for shear location is used to demonstrate these calculations. A complete design will verify the slab-girder interface capacity at various sections along the girder.

7.4.1.1 *Compute Nominal Capacity*

The nominal shear resistance at the slab-girder interface is $V_{ni} = cA_{cv} + \mu\left[A_{vf} f_y + P_c\right] \leq minimum \begin{cases} K_1 f_c' A_{cv} \\ K_2 A_{cv} \end{cases}$

where

V_n	=	Nominal shear resistance (kip)
A_{cv}	=	Area of concrete engaged in shear transfer (in^2)

A_{vf} = Area of shear reinforcement crossing the shear plane (in^2)

f_y = Yield strength of reinforcement (ksi)

c = Cohesion factor

μ = Friction factor

P_c = Permanent net compressive force normal to the shear plane, or 0.0 kip if tensile (kip)

f'_c = Specified 28-day strength of the weaker concrete (ksi)

K_1 = 0.3

K_2 = 1.8

The top flange of the girder, which is a roughened surface, supports the deck slab. For this situation $c = 0.280$ *ksi* and $\mu = 1.0$.

The area of concrete engaged in the shear transfer: $A_{cv} = b_{vi}L_{vi} = (49\ in)\left(1\frac{in}{in}\right) = 49\frac{in^2}{in}$.

The area of shear reinforcement consists of the stirrups extending from the web into the slab (#5 @ 6 in): $A_{vf} = \frac{0.62\ in^2}{6\ in} = 0.103\frac{in^2}{in}$.

P_c is the weight of the slab. For this computation, neglect the weight of the sacrificial depth of slab. The sacrificial depth wears away with time and its weight will not contribute to the normal force at the girder/slab interface for the life of the structure.

$$P_c = \gamma_c\left[w_{trib}\left(t_{slab} - t_{wearing}\right) + w_{tf}t_{haunch}\right] = (0.155\ kcf)[81in(7.5in - 0.5in) + 49in(8.75in - 7.5in)]\frac{1\ ft^2}{144\ in^2} = 0.610\ klf$$

$$cA_{cv} + \mu\left[A_{vf}f_y + P_c\right] = (0.280\ ksi)\left(49\frac{in^2}{in}\frac{12in}{1ft}\right) + 1.0\left[\left(0.103\frac{in^2}{in}\frac{12in}{1ft}\right)(60ksi) + 0.610\ klf\right] = 239.650\ \ kip/ft$$

$$K_1f'_cA_{cv} = 0.3(4ksi)\left(49\frac{in^2}{in}\frac{12in}{ft}\right) = 705.6\ \ kip/ft$$

$$K_2A_{cv} = 1.8\left(49\frac{in^2}{in}\frac{12in}{ft}\right) = 1058.4\ \ kip/ft$$

$$V_n = 239.650\ \ kip/ft$$

$$V_r = \phi V_n = 0.9(239.650\ \ kip/ft) = 215.685\ k/ft$$

7.4.1.2 *Compute Demand*

The factored interface shear stress for a concrete girder/slab bridge may be determined as $v_{ui} = \frac{V_u}{b_{vi}d_{vi}}$. The factored interface shear force for a concrete girder/slab bridge may be determined as $V_{ui} = v_{ui}A_{cv}$. Substituing Equation 5.8.4.2-1 into 5.8.4.2-2 the interface shear force is $V_{uh} = \frac{V_u}{d_{vi}}$.

At the critical section, $V_u\ =\ 299.68\ kip$.

$$V_{uh} = \frac{V_uQ}{I} = \frac{(299.68kip)(8211.5in^3)}{(525343.3in^4)} = 56.210\frac{k}{ft}$$

$$V_{uh} \le V_r$$

OK

7.4.2 Check Minimum Reinforcement

The LRFD specification requires a minimum amount of shear reinforcement in the slab-girder interface. Check to make sure this requirement is satisfied.

The cross-sectional area, A_{vf}, of the reinforcement per unit length should not be less than $\frac{0.05 b_v}{f_y}$.

For a cast-in-place concrete slab on clean concrete girder surface free of laitance:

- The minimum interface shear reinforcement need not exceed the lessor of the amount determined using Eqn. 5.8.4.4-1 and the amount needed to resist $\frac{1.33 V_{ui}}{\phi}$ as determined using Eqn 5.8.4.1-3
- The minimum reinforcement provisions shall be waived for girder/slab interfaces with surface roughened to an amplitude of 0.25 in where the factored interface shear stress, v_{ui} of Eqn 5.8.4.2-1 is less than 0.210 ksi, and all vertical (transverse) shear reinforcement required by the provisions of Article 5.8.1.1 is extended across the interface and adequately anchored into the slab.

$v_{ui} = \frac{V_n}{A_{cv}} = \frac{239.65 \frac{kip}{ft}}{49 \frac{in^2}{in} \frac{12 in}{1 ft}} = 0.096 \frac{ksi}{ft} < 0.100 \frac{ksi}{ft}$. This requirement is waived.

OK

The maximum allowable spacing of the transverse reinforcement is $24.0\ in$. The actual spacing at this section is $6.0\ in$. The maximum spacing along the length of the girder is $18.0\ in$. **OK**

8 Check Haunch Dimension

The slab offset is $8.75 in$. Verify the haunch is large enough to accommodate the camber, but not too large that the girder has to carry unnecessary dead load. For such a large girder, an extra inch of concrete over the top flange can add up to a considerable amount of weight.

The haunch depth is to be such that at the mid-span the distance between the bottom of the slab and the top of the girder is equal to the slab fillet dimension, $0.75 in$. Account for geometric effects due to the roadway and camber. The haunch depth at the bearing is $A_{haunch} = A_{slab+fillet} + A_{profile\ effect} + A_{girder\ orientation\ effect} + A_{excess\ camber}$.

8.1 Slab and Fillet

The slab and fillet is the gross slab depth plus the fillet dimension. If the actual camber is exactly equal to the predicted value, and all deflections are as predicted, the top of the girder will be exactly t_{fillet} below the bottom of the deck as its closest point.

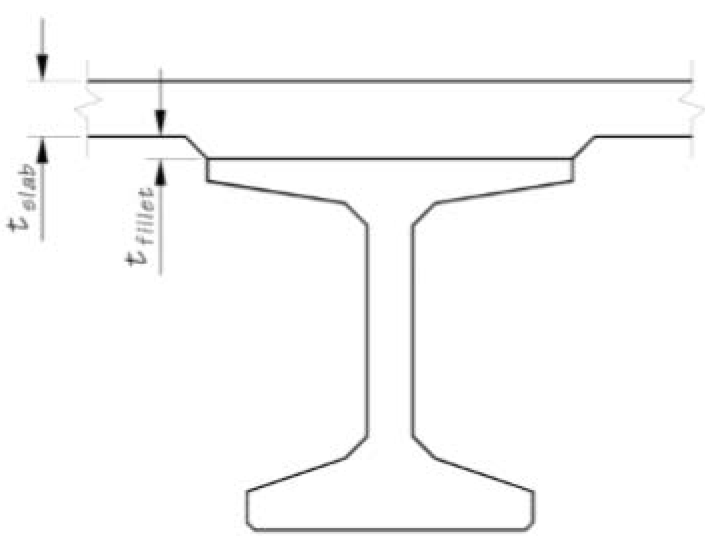

Figure 8-1: Slab + Fillet Effect

$$A_{slab+fillet} = 7.5\ in + 0.75\ in = 8.25 in$$

8.2 Profile Effect

PGSuper uses a general approach to determine the profile effect. Draw a chord line from the point where a vertical line passing through the CL Bearings intersect the deck. Then the profile effect is the maximum difference in elevation between this chord line and the roadway surface.

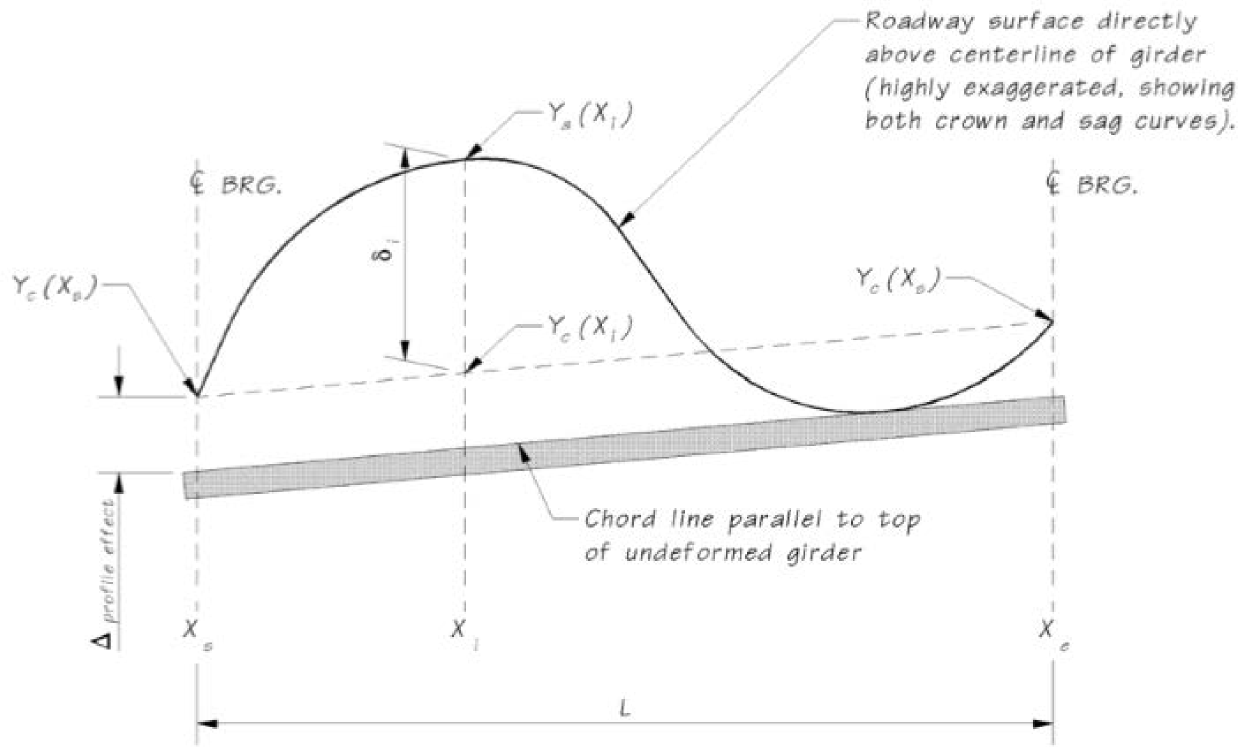

Figure 8-2: General Method for Profile Effect

The entire span of the bridge is within the limits of the horizontal and vertical curves. Use the simplified method of computing the profile effect. See BDM Appendix 5-B1 for additional information.

8.2.1 Vertical Curve

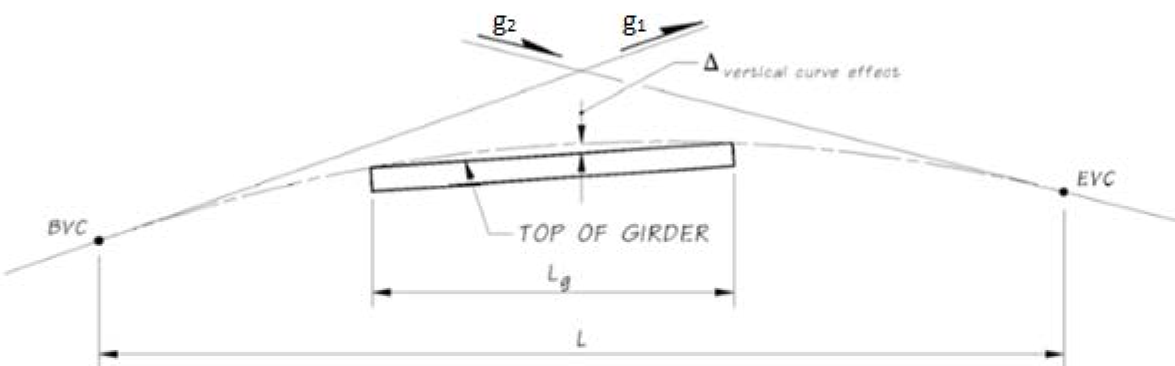

Figure 8-3: Vertical Curve Effect

$$A_{vc} = \frac{1.5(g_2 - g_1)L_g^2}{100L_{vc}}(in) = \frac{1.5(-9\% - 9\%)(114.583ft)^2}{100(201ft)} = -17.636\ in$$

8.2.2 Horizontal Curve

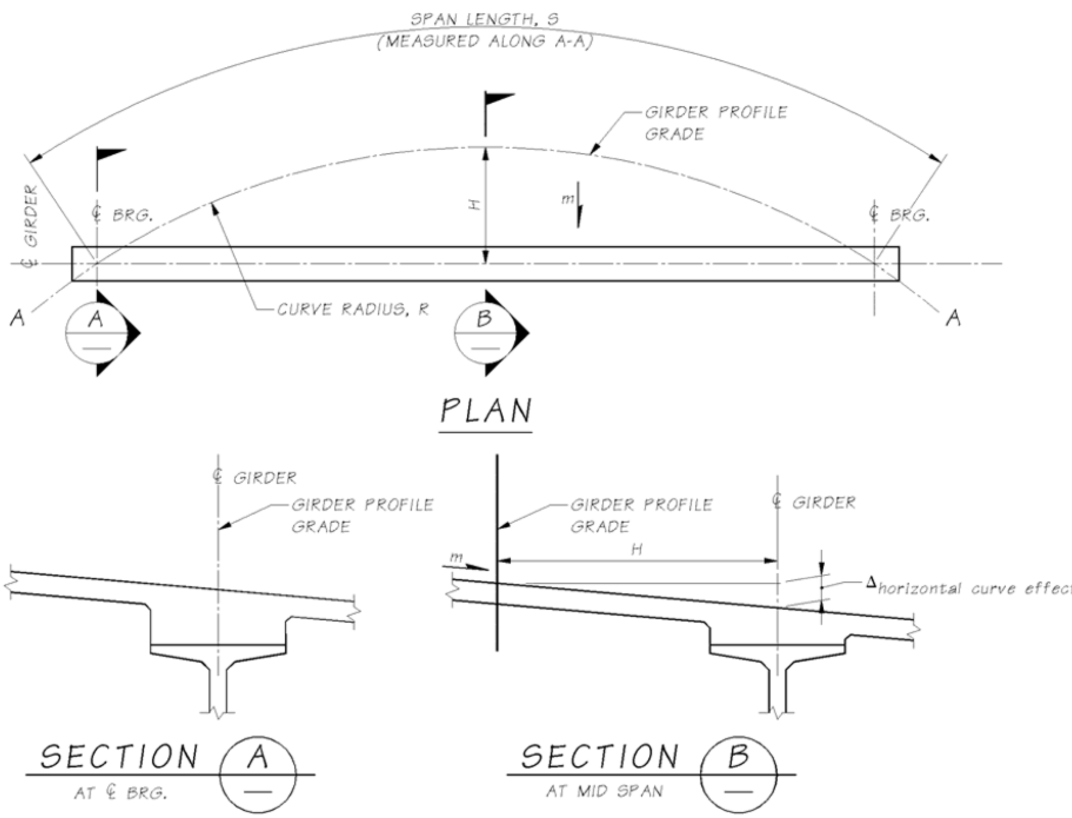

Figure 8-4: Horizontal Curve Effect

$$A_{hc} = \frac{1.5S^2m}{R}(in)$$

There is not a horizontal curve

$$A_{hc} = 0.0\ in$$

8.2.3 Profile Effect

$$A_{profile} = A_{vc} + A_{hc} = -17.636in + 0.0in = -17.636in$$

8.3 Girder Orientation Effect

The girder orientation effect accounts for the crown slope and the orientation of the girder. $A_{girder\ orientation\ effect} = m\frac{w_{tf}}{2}$.

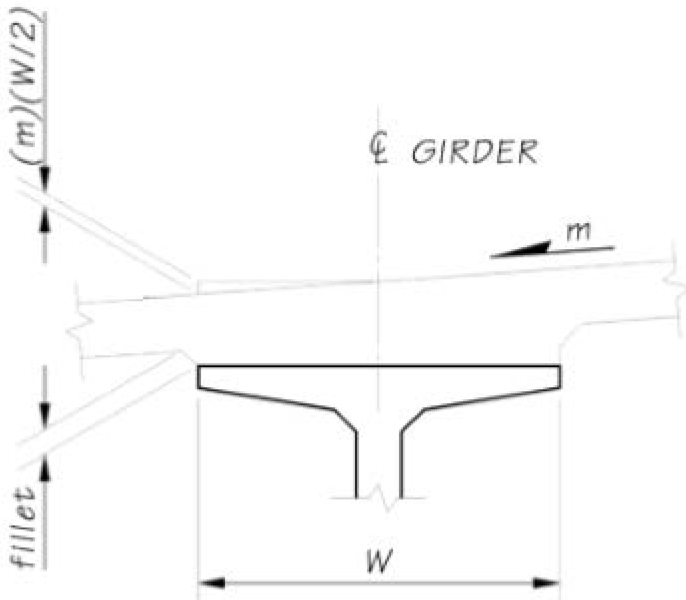

Figure 8-5: Top Flange Effect

$$A_{top\ flange\ effect} = 0.02\frac{49in}{2} = 0.490in$$

8.4 Excess Camber

The excess camber is the camber that remains in the girder after all of the loads are applied.

Figure 8-6: Camber Effect

The graphic below illustrates how the girder deflects over time.

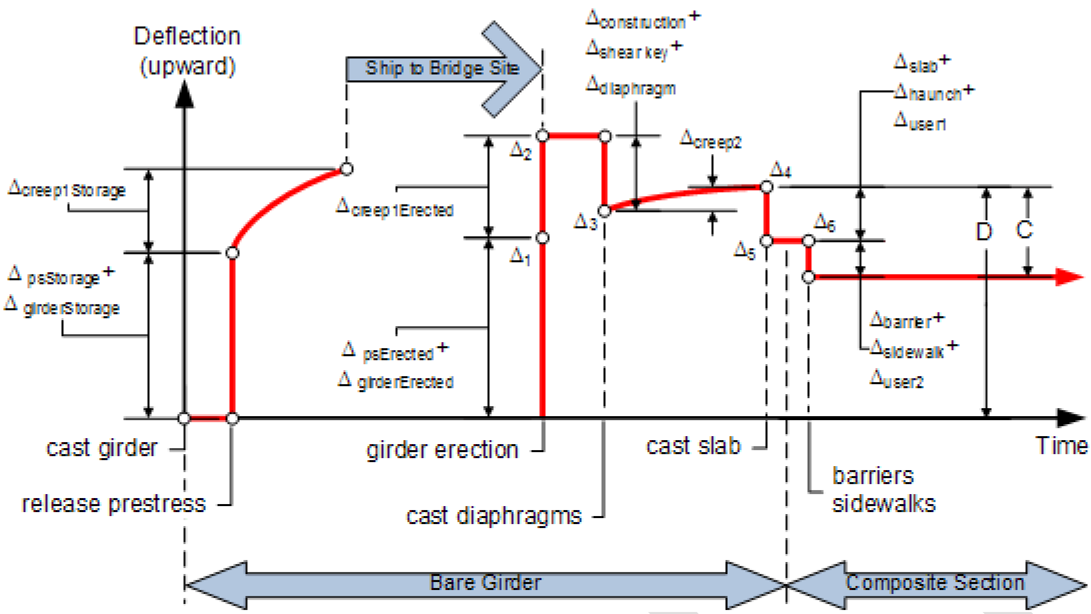

Figure 8-7: Camber Diagram

Assume time-dependent deformations end after deck casting

$$\Delta_{girder} = deflection\ due\ to\ girder\ self$$

$$\Delta_{ps} = deflection\ due\ to\ permanent\ prestressing, based\ on\ inplace\ span\ length$$

$$\Delta_{creep1} = \psi(t_e, t_i)(\Delta_{girder} + \Delta_{ps})$$

$$\Delta_{dia} = deflection\ due\ to\ diaphragm\ self\ weight$$

$$\delta_{girder} = incremental\ girder\ deflection\ due\ to\ change\ in\ support\ location\ between\ storage\ and\ erection$$

$$\Delta_{creep2} = [\psi(t_d, t_i) - \psi(t_e, t_i)](\Delta_{girder} + \Delta_{ps}) + \psi(t_d, t_e)(\Delta_{dia} + \delta_{girder})$$

$$\Delta_{deck} = deflection\ due\ to\ deck\ self\ weight$$

$$\Delta_{haunch} = deflection\ due\ to\ haunch\ self\ weight$$

$$\Delta_{barrier} = deflection\ due\ to\ traffic\ barrier\ self\ weight$$

$$\Delta_{excess} = excess\ camber$$

$$\Delta_1 = (\Delta_{girder} + \Delta_{ps}) + \Delta_{pc}$$

$$\Delta_2 = \Delta_1 + \Delta_{creep1}$$
$$\Delta_3 = \Delta_2 + \Delta_{dia}$$

$$\Delta_4 = \Delta_3 + \Delta_{creep2}$$

$$\Delta_5 = \Delta_4 + \Delta_{deck} + \Delta_{haunch}$$

$$\Delta_6 = \Delta_{excess} = \Delta_5 + \Delta_{barrier}$$

8.4.1 Compute Creep Coefficients

The creep coefficients for release until erection and deck casting are computed above.

Prestress release until erection $\psi(t_h = 90, t_i = 1) = \psi(t_e = 90, t_i = 1) = 0.954$

Prestress release until deck casting $\psi(t_d = 120, t_e = 1) = 1.027$

Compute creep coefficient for erection to deck casting

f'_{ci} is the girder concrete strength at the time of load application to the erected girder and not the initial concrete strength at release.

$$f'_{ci} = 7.2\ ksi$$

$$k_f = \frac{5}{1 + 7.2} = 0.610$$

$$k_{td} = \frac{(120 - 90)}{12\left(\frac{100 - 4(7.2)}{7.2 + 20}\right) + (120 - 90)} = 0.488$$

$$\psi(t_d = 120, t_e = 90) = 1.9(1.03)(0.96)(0.610)(0.488)(90)^{-0.118} = 0.330$$

8.4.2 Compute Deflections

Girder Deflection, for the erected girder

$$\Delta_g = \frac{5wL^4}{384E_{ci}I_x} = \frac{5(-0.890klf)(114.583ft)^4}{384(5236.046ksi)(282559.4in^4)}\left(\frac{1728in^3}{1ft^3}\right) = -2.333in$$

Prestress Deflection, $\Delta_{ps} = 5.413in$. This is the deflection measured relative to the ends of the girder. The deflection at the CL Bearing based on the release datum is $\Delta_{psbrg} = 0.278in$. The prestress deflection measured relative to the bearings is $\Delta_{ps} = 5.413in - 0.278in = 5.135in$

Creep Deflection during Storage, $\Delta_{creep1} = 1.027(5.413in - 2.333in) = 2.678in$

Apply the creep coefficient to the girder and prestress deflections only (do not apply to precamber)

Diaphragm Deflection, $\Delta_{diaphragm} = -0.123in$

Slab Deflection, $\Delta_{slab} = -1.623in$

Haunch Deflection, $\Delta_{haunch} = -0.532in$

Creep Deflection between diaphragm and deck casting, $\Delta_{creep2} = (1.027 - 0.954)(5.413in - 2.333in) + (0.330)(-0.123in) = 0.163in$

Traffic Barrier Deflection, $\Delta_{tb} = -0.307in$

Precamber, $\Delta_{pc} = 15in - \frac{4(15in)}{118ft}\left(1.708ft - \frac{(1.708ft)^2}{118ft}\right) = 14.144in$

$$\Delta_1 = -2.333in + 5.413in + 14.144in = 16.947in$$

$$\Delta_2 = 16.947in + 2.678in = 19.625in$$

$$\Delta_3 = 19.625 - 0.123in = 19.501in$$

$$\Delta_4 = 19.501in + 0.163in = 19.665in = D_{120}$$

$$\Delta_5 = 19.665in - 1.623in - 0.532in = 17.509in$$

$$\Delta_6 = 17.509 - 0.307in = 17.202in = \Delta_{excess}$$

8.5 Check Required Haunch

The required haunch is $A_{haunch} = A_{slab+fillet} + A_{top\ flange\ effect} + A_{profile\ effect} + A_{excess\ camber}$

$$A_{haunch} = 8.25in + 0.49in - 17.636in + 17.202in = 8.306\ in$$

For a crest vertical curve, the minimum slab offset often governs.

$$A_{haunch_{min}} = A_{slab+fillet} + A_{top\ flange\ effect} = 8.25in + 0.49in = 8.74in$$

The provided haunch is 8.75 in. **OK**

8.6 Compute Lower Bound Camber at 40 days

8.6.1 Creep Coefficients

Creep coefficients are computed the same as before, assuming erection at 10 days and deck casting at 40 days.

$$\psi_b(t_d = 10, t_i = 1) = 0.273$$

$$\psi_b(t_d = 40, t_e = 10) = 0.428$$

$$\psi_b(t_f = 40, t_1 = 1) = 0.702$$

8.6.2 Compute Deflections

Creep Deflection, $\Delta_{creep1} = 0.273(5.413in - 2.333in) = 0.766in$

Additional Creep Deflection, $\Delta_{creep2} = (0.702 - 0.273)(5.413in - 2.333in) + (0.428)(-0.123in) = 1.150in$

Traffic Barrier Deflection, $\Delta_{tb} = -0.307in$

$$\Delta_1 = -2.333in + 5.413in + 14.144in = 16.947in$$

$$\Delta_2 = 16.947in + 0.766in = 17.712in$$

$$\Delta_3 = 17.712 - 0.123in = 17.589in$$

$$\Delta_4 = 17.589in + 1.150in = 18.739in = D_{40}$$

This is an upper bound value for D_{40}. There is a $\pm 25\%$ natural variation in camber from the mean value. Therefore, lower bound camber at 40 days $= 0.5(D_{40} - \Delta_{pc}) + \Delta_{pc} = 0.5(18.739in - 14.144in) + 14.144in = 16.442in$.

Natural camber variation does not apply to precamber.

8.7 Check for Possible Girder Sag

When the screed camber, C, exceeds the deflection at slab casting, D, the girder will have a net downward deflection, also known as sag. The sag condition is most likely to occur for rapidly constructed bridges.

Compare the screed camber to the average value of D_{40} to determine the potential for sag. The average value is 75% $(D_{40} - \Delta_{pc}) + \Delta_{pc} = (0.75)(18.739in - 14.144in) + 14.144in = 17.591in$

$$\Delta_{excess} = D - C$$

$$\Delta_5 = 18.739in - 1.623in - 0.532in = 16.584in$$

$$\Delta_6 = 16.584 - 0.307in = 16.227in = \Delta_{excess}$$

$$C = 18.739in - 16.277in = 2.462in$$

$C < Average\ D_{40}$ **OK**

9 Bearing Seat Elevations

From the PGSuper Bridge Geometry Report, the roadway surface elevations at the CL Bearing points for Girder B are:

Abutment 1, Sta. 102+02.71, Offset 10.125ft L, Elev. 24.743ft

Abutment 2, Sta. 103+17.29, Offset 10.125ft L, Elev. 25.153ft

The basic slope of the girder is $\frac{25.153ft - 24.743ft}{114.583ft} = 0.00358\frac{ft}{ft}$

The end of the girder also slopes due to precamber $= 4\Delta_{pc}\left(\frac{1}{L} - \frac{2x}{L^2}\right)$

At the left end of the girder, $x = 1.708ft$ so the girder slope is $4\left(15in\frac{1ft}{12in}\right)\left(\frac{1}{118ft} - \frac{2(1.708ft)}{(118ft)^2}\right) = 0.04115\frac{ft}{ft}$

At the right end of the girder, $x = 116.292ft$ so the girder slope is $4\left(15in\frac{1ft}{12in}\right)\left(\frac{1}{118ft} - \frac{2(116.292ft)}{(118ft)^2}\right) = -0.04115\frac{ft}{ft}$

The left end girder slope is $0.00358\frac{ft}{ft} + 0.04115\frac{ft}{ft} = 0.04473\frac{ft}{ft}$

The right end girder slope is $0.00358\frac{ft}{ft} - 0.04115\frac{ft}{ft} = -0.03757\frac{ft}{ft}$

The left end slope-adjusted height of the girder is $50in\left(\sqrt{(0.04473)^2 + (1)^2}\right) = 50.050in$

The right end slope-adjusted height of the girder is $50in\left(\sqrt{(-0.03757)^2 + (1)^2}\right) = 50.035in$

Deduct the sloped adjusted girder height and the slab offset from the roadway surface elevation to get the bottom of girder elevation.

Bottom of girder elevation at Abutment 1: Elev = $24.743ft - 50.050in\left(\frac{1ft}{12in}\right) - 8.75in\left(\frac{1ft}{12in}\right) = 19.843ft$

Bottom of girder elevation at Abutment 2: Elev = $25.153ft - 50.035in\left(\frac{1ft}{12in}\right) - 8.75in\left(\frac{1ft}{12in}\right) = 20.254ft$

After designing the bearings, add the bearing recess (typically ½") and deduct the bearing depth from the bottom of girder elevation to get the bearing seat elevation.

10 Load Rating

The bridge opens for traffic without the future overlay installed. For this reason, take the DW force effects associated with the overlay as zero. Installing the overlay necessitates updating the load rating analysis.

10.1 Inventory Rating

10.1.1 Moment

$$RF = \frac{\phi_c\phi_s\phi_n K M_n - \gamma_{DC}M_{DC} - \gamma_{DW}M_{DW}}{\gamma_{LL}M_{LLIM}}$$

$$\phi_c\phi_s \geq 0.85$$

$$K = \frac{M_r}{M_{min}} \leq 1.0$$

At 0.5L

$$\phi_c = \phi_s = \phi_n = 1.0$$

$$M_n = 10120.56k \cdot ft$$

$$M_{DC} = 3348.8k \cdot ft$$

$$M_{DW} = 0.0k \cdot ft$$

$$M_{LLIM} = 1997.7\frac{k \cdot ft}{girder}$$

$$M_{cr} = 7244.04k \cdot ft$$

$$M_u = 8225.27k \cdot ft$$

$$M_{min} = min\begin{cases} M_{cr} \\ 1.33M_u \end{cases} = 7244.04k \cdot ft$$

$$K = \frac{10120.56k \cdot ft}{7244.04k \cdot ft} = 1.397 \therefore 1.0$$

$$\gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.75$$

$$RF = \frac{(1)(1)(1)(1)(10120.56k \cdot ft) - (1.25)(3348.8k \cdot ft) - (1.5)(0k \cdot ft)}{(1.75)(1997.7k \cdot ft)} = 1.70$$

10.1.2 Shear

$$RF = \frac{\phi_c\phi_s\phi_n V_n - \gamma_{DC}V_{DC} - \gamma_{DW}V_{DW}}{\gamma_{LL}V_{LLIM}}$$

At 19.67ft (location where stirrup spacing increases)

$$\phi_c = \phi_s = 1.0, \phi_n = 0.9$$

$$V_n = 310.02kip$$

$$V_{DC} = 77.19kip$$

$$V_{DW} = 0.0k$$

$$V_{LLIM} = 70.11\frac{kip}{girder}$$

$$\gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.75$$

$$RF = \frac{(1)(1)(0.9)(310.02kip) - (1.25)(77.19kip) - (1.5)(0kip)}{(1.75)(70.11kip)} = 1.49$$

10.1.3 Bending Stress – Service III limit state

$$RF = \frac{f_R - \gamma_{DC}f_{DC} - \gamma_{DW}f_{DW}}{\gamma_{LL}f_{LLIM}}$$

For load rating we use the AASHTO specified tension limit and live load factor

$$f_R = f_{limit} - f_{ps} = 0.19\lambda\sqrt{f_c'} - f_{ps}$$

$$f_R = 0.19(1.0)\sqrt{7.2}ksi - (-5.129ksi) = 5.638ksi$$

$$\gamma_{LL} = 1.0$$

$$RF = \frac{5.638ksi - (1.0)(3.347ksi) - 1.0(0ksi)}{(1.0)(1.585ksi)} = 1.45$$

10.2 Operating Rating

10.2.1 Moment

$$RF = \frac{\phi_c \phi_s \phi_n K M_n - \gamma_{DC} M_{DC} - \gamma_{DW} M_{DW}}{\gamma_{LL} M_{LLIM}}$$

$$\phi_c \phi_s \geq 0.85$$

$$K = \frac{M_r}{M_{min}} \leq 1.0$$

At 0.5L

$$\phi_c = \phi_s = \phi_n = 1.0$$

$$M_n = 10120.56k \cdot ft$$

$$M_{DC} = 3348.8k \cdot ft$$

$$M_{DW} = 0.0k \cdot ft$$

$$M_{LLIM} = 1997.7 \frac{k \cdot ft}{girder}$$

$$M_{cr} = 7244.04k \cdot ft$$

$$M_u = 8225.27k \cdot ft$$

$$M_{min} = min\begin{cases} M_{cr} \\ 1.33M_u \end{cases} = 7244.04k \cdot ft$$

$$K = \frac{10120.56k \cdot ft}{7244.04k \cdot ft} = 1.397 \therefore 1.0$$

$$\gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.35$$

$$RF = \frac{(1)(1)(1)(1)(10120.56k \cdot ft) - (1.25)(3348.8k \cdot ft) - (1.5)(0k \cdot ft)}{(1.35)(1997.7k \cdot ft)} = 2.20$$

10.2.2 Shear

$$RF = \frac{\phi_c \phi_s \phi_n V_n - \gamma_{DC} V_{DC} - \gamma_{DW} V_{DW}}{\gamma_{LL} V_{LLIM}}$$

At 19.67ft (location where stirrup spacing increases)

$$\phi_c = \phi_s = 1.0, \phi_n = 0.9$$

$$V_n = 310.02kip$$

$$V_{DC} = 77.19kip$$

$$V_{DW} = 0.0k$$

$$V_{LLIM} = 70.11 \frac{kip}{girder}$$

$$\gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.35$$

$$RF = \frac{(1)(1)(0.9)(310.02kip) - (1.25)(77.19kip) - (1.5)(0kip)}{(1.35)(70.11kip)} = 2.00$$

10.3 Legal Loads

Type 3, $M_{LLIM} = 821.09k \cdot ft$

Type 3S2, $M_{LLIM} = 1017.78k \cdot ft$

Type 3-3, $M_{LLIM} = 1048.08k \cdot ft$

Type 3-3 rating will govern so we will show calculations of the rating factors for this loading. The rating factor calculations for the other loadings will be similar. The rating factor calculations for NRL, EV2, and EV3 are similar.

10.3.1 Moment

$$RF = \frac{\phi_c \phi_s \phi_n K M_n - \gamma_{DC} M_{DC} - \gamma_{DW} M_{DW}}{\gamma_{LL} M_{LLIM}}$$

$$\phi_c \phi_s \geq 0.85$$

$$K = \frac{M_r}{M_{min}} \leq 1.0$$

At 0.5L

$$\phi_c = \phi_s = \phi_n = 1.0$$

$$M_n = 10120.56k \cdot ft$$

$$M_{DC} = 3348.8k \cdot ft$$

$$M_{DW} = 0.0k \cdot ft$$

$$M_{LLIM} = 1048.08 \frac{k \cdot ft}{girder}$$

$$M_{cr} = 7244.04k \cdot ft$$

$$M_u = 8225.27k \cdot ft$$

$$M_{min} = min \begin{cases} M_{cr} \\ 1.33 M_u \end{cases} = 7244.04k \cdot ft$$

$$K = \frac{10120.56k \cdot ft}{7244.04k \cdot ft} = 1.397 \therefore 1.0$$

$$\gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.45$$

$$RF = \frac{(1)(1)(1)(1)(10120.56k \cdot ft) - (1.25)(3348.8k \cdot ft) - (1.5)(0k \cdot ft)}{(1.45)(1048.08k \cdot ft)} = 3.91$$

10.3.2 Shear

$$RF = \frac{\phi_c \phi_s \phi_n V_n - \gamma_{DC} V_{DC} - \gamma_{DW} V_{DW}}{\gamma_{LL} V_{LLIM}}$$

At 19.67ft (location where stirrup spacing increases)

$$\phi_c = \phi_s = 1.0, \phi_n = 0.9$$

$$V_n = 329.93kip$$

$$V_{DC} = 77.19kip$$

$$V_{DW} = 0.0kip$$

$$V_{LLIM} = 39.55 \frac{kip}{girder}$$

$$\gamma_{DC} = 1.25, \gamma_{DW} = 1.50, \gamma_{LL} = 1.45$$

$$RF = \frac{(1)(1)(0.9)(329.93kip) - (1.25)(77.19kip) - (1.5)(0k)}{(1.45)(39.55kip)} = 3.50$$

10.3.3 Bending Stress – Service III limit state

This is a WSDOT requirement, not in MBE

$$RF = \frac{f_R - \gamma_{DC} f_{DC} - \gamma_{DW} f_{DW}}{\gamma_{LL} f_{LLIM}}$$

For load rating we use the AASHTO specified tension limit and live load factor

$$f_R = f_{limit} - f_{ps} = 0.19\lambda\sqrt{f_c'} - f_{ps}$$

Before we can compute the stress in the girder due to the prestressing, we must compute the effective prestress accounting for the elastic gain for to the Type 3-3 loading.

$$\Delta f_{pLL} = \frac{E_p}{E_c}\frac{M_{LLIM}(Y_{bc} - Y_{bg} + e)}{I_c} = \frac{28500ksi}{5530.5\ ksi}\frac{(1048.08k \cdot ft)(34.726in - 24.151in + 21.205in)}{525343.2in^4}\left(\frac{12in}{1ft}\right) = 3.921ksi$$

$$P = (9.331in^2)(202.5ksi - 20.602ksi - 9.697ksi + 3.921ksi) = 1643.39kip$$

$$f_{ps} = -\frac{1643.39kip}{776.531in^2} - \frac{(1643.39kip)(21.205in)}{11699.6in^3} = -5.027ksi$$

$$f_R = 0.19(1.0)\sqrt{7.2}ksi - (-5.027ksi) = 5.537ksi$$

$$\gamma_{LL} = 1.0$$

$$RF = \frac{5.537ksi - (1.0)(3.347ksi) - 1.0(0ksi)}{(1.0)(0.831ksi)} = 2.63$$

10.4 Permit Loads

The load ratings for the permit loads are the same as the legal loads (with the obvious exception of the live load effects and load factors being different).

WSDOT also evaluates the optional reinforcement yielding check (MBE 6A.5.4.2.2b). The stress in the prestressing steel nearest the extreme tension fiber should not exceed $0.9f_y$. The analysis method used by PGSuper follows MBE A3.13.4.2b.

$$f_r = 0.9f_y = (0.9)(0.9)f_{pu} = (0.9)(0.9)(270ksi) = 218.7ksi$$

Moment beyond cracking

$$M_{bcr} = \gamma_{DC} M_{DC} + \gamma_{DW} M_{DW} + \gamma_{LL} M_{LLIM} - M_{cr}$$

Unlike the other permit rating cases where the one loaded lane live load distribution factor is used (MBE 6A.4.5.4.2b), use the governing of one loaded lane and two or more loaded lanes for these calculations (MBE C6A.5.4.2.2b).

For OL1, $M_{LLIM} = 1500.49k \cdot ft$ per girder.

For OL2, $M_{LLIM} = 2540.87k \cdot ft$ per girder

$$M_{bcr} = (1.0)(3348.8k \cdot ft) + (1.0)(0) + (1.0)(2540.87k \cdot ft) - 7244.04k \cdot ft = -1354.34k \cdot ft$$

Because $M_{bcr} < 0$, the loads aren't enough to cause cracking, so take $M_{bcr} = 0.0k \cdot ft$

The additional stress transferred to the reinforcement due to cracking is

$$f_{bcr} = \frac{E_s}{E_g}\frac{M_{bcr}(d_s - c)}{I_{cr}} = 0.0ksi$$

$$f_s = f_{pe} + f_{bcr}$$

Compute the effective prestress

For OL1

$$\Delta f_{pLL} = \frac{E_p}{E_c}\frac{M_{LLIM}\left(Y_{bc} - Y_{bg} + e\right)}{I_c} = \frac{28500ksi}{5530.5\ ksi}\frac{(1500.49k \cdot ft)(34.726in - 24.151in + 21.205in)}{525343.2in^4}\left(\frac{12in}{1ft}\right) = 5.613ksi$$

$$f_{pe} = 202.5ksi - 31.672ksi + 5.613ksi = 176.441\text{ ksi}$$

$$f_s = f_{pe} + f_{brc} = 176.411ksi + 0ksi = 176.441ksi$$

For OL2

$$\Delta f_{pLL} = \frac{E_p}{E_c}\frac{M_{LLIM}\left(Y_{bc} - Y_{bg} + e\right)}{I_c} = \frac{28500ksi}{5530.5\ ksi}\frac{(2540.87k \cdot ft)(34.726in - 24.151in + 21.205in)}{525343.2in^4}\left(\frac{12in}{1ft}\right) = 9.505ksi$$

$$f_{pe} = 202.5ksi - 31.672ksi + 9.505ksi = 180.278\text{ ksi}$$

$$f_s = 180.278\ ksi$$

Yield stress ratio

$$SR = \frac{f_r}{f_s}$$

OL1

$$SR = \frac{218.7ksi}{176.441ksi} = 1.24$$

OL2

$$SR = \frac{218.7ksi}{180.278ksi} = 1.21$$

11 Software

PGSuper is precast-prestressed girder design, analysis, and load rating software. PGSuper is part of the BridgeLink Bridge Engineering Application Suite jointly developed by the Washington State and Texas Departments of Transportation.

Download from http://www.wsdot.wa.gov/eesc/bridge/software

12 References

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9. WSDOT, *PGSuper Theoretical Manual*, Washington State Department of Transportation

13 Appendix A

Derivation of prestress deflection equations

Deflection equation is found by solving the following differential equation

$$M(x) = -Pe(x) = EI\frac{d^2y}{dx^2}$$

Some other useful relationships

$$y(x) = \int \theta(x)\,dx$$

$$\theta(x) = \int \phi(x)\,dx$$

$$\phi(x) = \frac{M(x)}{EI}$$

Straight Strands

$$e(x) = e$$

$$\theta(x) = -\frac{Pe}{EI}\int dx$$

$$\theta(x) = -\frac{Pe}{EI}(x + K_1)$$

$$\theta\left(\frac{L}{2}\right) = 0$$

$$K_1 = -\frac{L}{2}$$

$$y(x) = -\frac{Pe}{EI}\left(\frac{x^2}{2} + K_1x + K_2\right)$$

$$y(0) = 0$$

$$K_2 = 0$$

$$\Delta_{ss} = y\left(\frac{L}{2}\right) = -\frac{Pe}{EI}\left(\frac{\left(\frac{L}{2}\right)^2}{2} + \left(-\frac{L}{2}\right)\left(\frac{L}{2}\right)\right)$$

$$\Delta_{ss} = \frac{PeL^2}{8EI}$$

Harped Strands

$$e(x) = Y_{cg}(x) - Y_h(x)$$

$$Y_{cg}(x) = Y_b + \frac{4\Delta_{pc}}{L}\left(x - \frac{x^2}{L}\right)$$

$$Y_h(x) = \begin{cases} Y_b - \frac{e'}{bL}x - e_e & 0 \le x \le bL \\ Y_b + \delta_h - e_h & bL \le x \le L(1-b) \\ Y_b - \frac{e'}{bL}(L-x) - e_e & L(1-b) \le x \le L \end{cases}$$

$$e' = e_h - e_e - \delta_h$$

$$e(x) = \begin{cases} \frac{4\Delta_{pc}}{L}\left(x - \frac{x^2}{L}\right) + \frac{e'}{bL}x + e_e & 0 \le x \le bL \\ \frac{4\Delta_{pc}}{L}\left(x - \frac{x^2}{L}\right) - \delta_h + e_h & bL \le x \le L(1-b) \\ \frac{4\Delta_{pc}}{L}\left(x - \frac{x^2}{L}\right) + \frac{e'}{bL}(L-x) + e_e & L(1-b) \le x \le L \end{cases}$$

$$\theta(x) = \begin{cases} \int -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(x - \frac{x^2}{L}\right) + \frac{e'}{bL}x + e_e\right]dx & 0 \le x \le bL \\ \int -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(x - \frac{x^2}{L}\right) - \delta_h + e_h\right]dx & bL \le x \le L(1-b) \\ \int -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(x - \frac{x^2}{L}\right) + \frac{e'}{bL}(L-x) + e_e\right]dx & L(1-b) \le x \le L \end{cases}$$

$$\theta(x) = \begin{cases} -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^2}{2} - \frac{x^3}{3L}\right) + \frac{e'}{bL}\frac{x^2}{2} + e_e x + K_1\right] & 0 \le x \le bL \\ -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^2}{2} - \frac{x^3}{3L}\right) + (e_h - \delta_h)x + K_2\right] & bL \le x \le L(1-b) \\ -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^2}{2} - \frac{x^3}{3L}\right) + \frac{e'}{bL}\left(Lx - \frac{x^2}{2}\right) + e_e x + K_3\right] & L(1-b) \le x \le L \end{cases}$$

$$\theta\left(\frac{L}{2}\right) = 0$$

$$-\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{\left(\frac{L}{2}\right)^2}{2} - \frac{\left(\frac{L}{2}\right)^3}{3L}\right) + (e_h - \delta_h)\frac{L}{2} + K_2\right] = 0$$

$$K_2 = -\left[\frac{\Delta_{pc}L}{3} + (e_h - \delta_h)\frac{L}{2}\right]$$

$$\theta_1(bL) = \theta_2(bL)$$

$$-\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{(bL)^2}{2} - \frac{(bL)^3}{3L}\right) + \frac{e'}{bL}\frac{(bL)^2}{2} + e_e(bL) + K_1\right] = -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{(bL)^2}{2} - \frac{(bL)^3}{3L}\right) + (e_h - \delta_h)bL + K_2\right]$$

$$K_1 = (e_h - \delta_h)(bL) + K_2 - \frac{e'}{bL}\frac{(bL)^2}{2} - e_e(bL)$$

$$K_1 = (e_h - \delta_h)(bL) - \frac{\Delta_{pc}L}{3} - (e_h - \delta_h)\frac{L}{2} - \frac{e'}{bL}\frac{(bL)^2}{2} - e_e(bL)$$

$$K_1 = (e_h - e_e - \delta_h)(bL) - \frac{e'}{2}(bL) - (e_h - \delta_h)\frac{L}{2} - \frac{\Delta_{pc}L}{3}$$

$$K_1 = e'(bL) - \frac{e'}{2}(bL) - (e_h - \delta_h)\frac{L}{2} - \frac{\Delta_{pc}L}{3}$$

$$K_1 = \frac{e'}{2}(bL) - (e_h - \delta_h)\frac{L}{2} - \frac{\Delta_{pc}L}{3}$$

$$y(x) = \begin{cases} \int -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^2}{2} - \frac{x^3}{3L}\right) + \frac{e'}{bL}\frac{x^2}{2} + e_e x + K_1\right]dx & 0 \le x \le bL \\ \int -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^2}{2} - \frac{x^3}{3L}\right) + (e_h - \delta_h)x + K_2\right]dx & bL \le x \le L(1-b) \\ \int -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^2}{2} - \frac{x^3}{3L}\right) + \frac{e'}{bL}\left(Lx - \frac{x^2}{2}\right) + e_e x + K_3\right]dx & L(1-b) \le x \le L \end{cases}$$

$$y(x) = \begin{cases} -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^3}{6} - \frac{x^4}{12L}\right) + \frac{e'}{bL}\frac{x^3}{6} + e_e\frac{x^2}{2} + K_1 x + K_4\right] & 0 \le x \le bL \\ -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^3}{6} - \frac{x^4}{12L}\right) + (e_h - \delta_h)\frac{x^2}{2} + K_2 x + K_5\right] & bL \le x \le L(1-b) \\ -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^3}{6} - \frac{x^4}{12L}\right) + \frac{e'}{bL}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right) + e_e\frac{x^2}{2} + K_3 x + K_6\right] & L(1-b) \le x \le L \end{cases}$$

$$y(0) = 0$$

$$K_4 = 0$$

$$y_1(bL) = y_2(bL)$$

$$-\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{(bL)^3}{6} - \frac{(bL)^4}{12L}\right) + \frac{e'}{bL}\frac{(bL)^3}{6} + e_e\frac{(bL)^2}{2} + K_1(bL) + K_4\right]$$
$$= -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{(bL)^3}{6} - \frac{(bL)^4}{12L}\right) + (e_h - \delta_h)\frac{(bL)^2}{2} + K_2(bL) + K_5\right]$$

$$\frac{e'}{6}\frac{(bL)^3}{bL} + \frac{e_e}{2}(bL)^2 + K_1(bL) = \frac{(e_h - \delta_h)}{2}(bL)^2 + K_2(bL) + K_5$$

$$K_5 = \frac{e'}{6}(bL)^2 + \frac{e_e}{2}(bL)^2 + K_1(bL) - K_2(bL) - \frac{(e_h - \delta_h)}{2}(bL)^2$$

$$K_1 - K_2 = \frac{e'}{2}(bL)$$

$$K_5 = \frac{e'}{6}(bL)^2 + \frac{e_e}{2}(bL)^2 - \frac{(e_h - \delta_h)}{2}(bL)^2 + \frac{e'}{2}(bL)^2$$

$$K_5 = \frac{e'}{6}(bL)^2 + \frac{(bL)^2}{2}(e_e - e_h + \delta_h + e')$$

$$e' = e_h - e_e - \delta_h$$

$$K_5 = \frac{e'}{6}(bL)^2 + \frac{(bL)^2}{2}(e_e - e_h + \delta_h + e_h - e_e - \delta_h)$$

$$K_5 = \frac{e'}{6}(bL)^2$$

$$y(x) = -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{x^3}{6} - \frac{x^4}{12L}\right) + (e_h - \delta_h)\frac{x^2}{2} - \left[\frac{\Delta_{pc}L}{3} + (e_h - \delta_h)\frac{L}{2}\right]x + \frac{e'}{6}(bL)^2\right], bL \le x \le L(1-b)$$

$$\Delta_{hs} = y\left(\frac{L}{2}\right) = -\frac{P}{EI}\left[\frac{4\Delta_{pc}}{L}\left(\frac{\left(\frac{L}{2}\right)^3}{6} - \frac{\left(\frac{L}{2}\right)^4}{12L}\right) + (e_h - \delta_h)\frac{\left(\frac{L}{2}\right)^2}{2} - \left[\frac{\Delta_{pc}L}{3} + (e_h - \delta_h)\frac{L}{2}\right]\left(\frac{L}{2}\right) + \frac{e'}{6}(bL)^2\right]$$

$$\Delta_{hs} = -\frac{P}{EI}\left[4\Delta_{pc}\left(\frac{L^2}{48} - \frac{L^2}{96}\right) + (e_h - \delta_h)\frac{L^2}{8} - \frac{\Delta_{pc}L^2}{6} - (e_h - \delta_h)\frac{L^2}{4} + \frac{e'}{6}(bL)^2\right]$$

$$\Delta_{hs} = -\frac{P}{EI}\left[-\frac{5}{48}\Delta_{pc}L^2 - (e_h - \delta_h)\frac{L^2}{8} + \frac{e'}{6}(bL)^2\right]$$

$$e' = e_h - e_e - \delta_h$$

$$e_h - \delta_h = e' + e_e$$

$$\Delta_{hs} = -\frac{P}{EI}\left[-\frac{5}{48}\Delta_{pc}L^2 - (e' + e_e)\frac{L^2}{8} + \frac{e'}{6}(bL)^2\right]$$

$$\Delta_{hs} = -\frac{P}{EI}\left[-\frac{5}{48}\Delta_{pc}L^2 + \frac{e'}{6}(bL)^2 - e'\frac{L^2}{8} - e_e\frac{L^2}{8}\right]$$

$$\Delta_{hs} = -\frac{P}{EI}\left[-\frac{5}{48}\Delta_{pc}L^2 + \frac{8e'(bL)^2 - 6e'L^2}{48} - e_e\frac{L^2}{8}\right]$$

$$\Delta_{hs} = -\frac{P}{EI}\left[-\frac{5}{48}\Delta_{pc}L^2 + \frac{e'L^2(4b^2 - 3)}{24} - e_e\frac{L^2}{8}\right]$$

$$\Delta_{hs} = \frac{5P}{48EI}\Delta_{pc}L^2 + \frac{Pe'L^2(3 - 4b^2)}{24EI} + \frac{Pe_eL^2}{8EI}$$

$$\Delta_{hs} = \frac{5P}{48EI}\Delta_{pc}L^2 + \frac{Pe'L^2(3 - 4b^2)}{24EI}\frac{bL}{bL} + \frac{Pe_eL^2}{8EI}$$

$$\Delta_{hs} = \frac{5P}{48EI}\Delta_{pc}L^2 + \frac{Pe'}{bL}\frac{bL^3(3 - 4b^2)}{24EI} + \frac{Pe_eL^2}{8EI}$$

$$N = \frac{Pe'}{bL}$$

$$\Delta_{hs} = \frac{5P}{48EI}\Delta_{pc}L^2 + \frac{NbL^3(3 - 4b^2)}{24EI} + \frac{Pe_eL^2}{8EI}$$

Temporary strands

Temporary top strands are post-tensioned in ducts that parallel the top surface of the girder. Since the strand is not bonded to the concrete, the deflection is caused by an end moment and a uniformly distributed force from the strand bearing againsted the curved duct. The deflection is

$$\Delta_{ts} = \frac{5P}{48EI}\Delta_{pc}L^2 + \frac{Pe_{ts}L^2}{8EI}$$