APPENDIX F
Snowshed Statistical Reliability Analysis

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Snowshed Statistical Reliability Analysis (SRA)

INTRODUCTION

This appendix documents the updated statistical reliability analysis (SRA) used to recommend potential design values of intact rock unconfined compressive strength (UCS) and global rock mass strength (Global RMS) used for drilled shaft rocket socket analysis and design along Snowshed (SSD) Pier 2. Intact rock UCS was used to develop bearing resistance curves for the shaft rock sockets, following AASHTO procedures; results are presented in Figures 6.17 and 6.18. Global RMS was used as rock mass compressive strength input to DFSAP for analysis of the rock sockets; Global RMS values are presented in Table 6.6.

Prior Documentation and Discussion

This appendix updates the initial SRA documentation contained in Appendix D of the July 2008 *Final Technical Memorandum 3 – Snowshed Replacement* (TM3). The SRA, in the context of rock conditions at the SSD, was further discussed extensively in the April 29, 2009 URS Memorandum to WSDOT "Snowshed Shaft Foundations: Rock Strength Estimates for DFSAP Analysis – I90 Snoqualmie Pass East," and during the August 4, 2009 WSDOT-URS meeting at Hyak, and the follow-up Go-To meeting on August 17, 2009.

Statistical Reliability Analysis (SRA) Purpose

The purpose of the SRA was to extrapolate, in a technically defensible manner, the available rock strength data from the seven 2009 borings drilled along Pier 2 to the 44 proposed Pier 2 shaft locations. Based on interpretation of the available rock data at the Snowshed and generally along the project alignment, the spatial variability of rock strength was considered too high and the available data too limited to reasonably assume direct (linear or nonlinear) interpolation between borings for all 44 locations. From a practical standpoint, it is therefore recommended that the design be based on the same selected rock strength at all shaft locations; this appendix documents how the recommended design rock strength was selected.

The SRA provides a technically defensible basis of extrapolation that is consistent with the observed spatial variability of rock strengths. Besides helping to fill the spatial variability data gap, the SRA helped qualify and quantify design-performance risk given the available data, and it helped to qualify and quantify the effect of obtaining additional boring data.

Conceptual details of the SRA are discussed in the following paragraphs followed by a series of annotated tables and figures that present the SRA results.

STATISTICAL RELIABILITY ANALYSIS METHODOLOGY

Borehole Rock Strength Data: Table F1 and Figure F1

The borehole average UCS for each boring along Pier 2 are listed in Table F1, along with supporting testing results as identified in the table. Figure F1 graphs the various borehole averages, including UCS, along the Pier 2 alignment from the west portal (left) to east portal (right).

The SRA used the borehole average intact rock UCS as an estimator of the average intact rock UCS for a shaft.

Rock Mass Strength: Figure F2

Rock mass strengths were estimated from intact rock UCS using 2002 Hoek-Brown failure criteria[1]. Figure F2 shows that the ratios of rock mass UCS and global RMS (rock mass strength) to intact rock UCS are both a function of rock mass rating RMR, or geologic strength index; GSI = RMR – 5.[2] Rock along Pier 2 was estimated to have an RMR of 45 and be composed of lapilli tuff with a "m_i" parameter of m_i=13.[3]

Using the Hoek-Brown failure criteria, Global RMS was estimated as 0.16 of the average intact rock UCS assuming tuff with RMR=45. This ratio is independent of the intact UCS. Therefore, Global RMS is simply scaled down from intact UCS by a constant factor of 0.16. Assuming constant rock type and RMR, Global RMS variability is the same as the average intact UCS spatial variability, not including model uncertainty in the Hoek-Brown failure criteria or parameter uncertainty associated with RMR and tuff as the rock type.[4]

Average Intact UCS at Shaft Locations Assumed Lognormally Distributed: Figure F3

The borehole UCS averages were found to be reasonable consistent with a lognormal (LN) probability distribution, suggesting that shaft strengths along Pier 2 would be approximately LN distributed. A linear-regression-analysis best-fit LN distribution to the borehole averages, as graphed in Figure F3, was used to characterize the spatial distribution of average intact UCS at the 44 Pier 2 shaft locations.

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[1] Hoek-Brown 2002: Evert Hoek, Carlos Carranza-Torres, and Brent Corkum, "Hoek-Brown Failure Criterion—2002 Edition."

[2] NCHRP 2006: Rock Socketed Shafts for Highway Structure Foundations, TRB. GSI=RMR-5 is Eq 7, p.24.

[3] Evert Hoek. "Practical Rock Engineering" Table 3. Downloaded April 15, 2009 from Rocscience web site http://www.rocscience.com.

[4] It should also be noted (and cautioned) that because it assumes certain principal stress conditions at failure, the Hoek-Brown Global RMS exceeds the rock mass shear strength (τ) for relatively high ratios of intact UCS to the vertical effective overburden stress (σ_V) that is calculated directly from Hoek-Brown failure criteria by setting the minor principal stress to σ_V. Very simply, the Global RMS can greatly exceed the calculated rock mass shear strength for rock with relatively high intact UCS and relatively low overburden stresses.

Global RMS would also follow a LN probability distribution because Global RMS is linearly related to intact UCS, as discussed in the preceding section. The only difference between the Global RMS and intact UCS distributions is that the former is reduced from the later by a constant multiplicative factor of 0.16 at all probability values.

Simulated Drilling at 44 Potential Shaft Locations: Rank Order U of Intact Rock UCS: Figure F4

The best-ft LN distribution of Figure F3 was virtually sampled 1,000 times using Monte Carlo techniques to develop simulated probability distributions of intact rock average UCS at 44 random shaft locations along the Pier 2 alignment. This procedure simulated drilling 44 shafts along Pier 2 where the rock strengths followed the Figure F3 LN distribution.

The 44 locations are random along Pier 2 and cannot be associated with specific locations. If there was a boring at each of the 44 shaft locations there would be no need to do the simulation because conditions would be known at each shaft location to the accuracy of the co-located boring. From a practical standpoint, borings located within say one or two B (B=shaft diameter) of a given shaft could be considered co-located, and data for that boring could be used as site-specific to that shaft. The SRA did not make refinements for arguably co-located borings except at the two portals.

The simulated shaft average intact UCSs were rank ordered from lowest (weakest), U=1, to highest (strongest), U=44; where U is the rank order, 1 to 44. Figure F4 shows selected results:

- U=1 (weakest shaft)
- U=2 (2nd weakest)
- U=10 (10th weakest)
- U=22 (the median strength shaft)
- U=33 (12th strongest)
- U=40 (5th strongest)
- U=44 (strongest shaft).

Results for Global RMS look the same as for intact UCS except all values are multiplied by a factor of 0.16, consistent with an RMR of 45 and rock type tuff.

Selected simulation results are also tabulated in Table F2 for both intact UCS and Global RMS.

A design based on U=1 would provide the most conservative intact rock UCS in terms of rock socket capacity because 43 of the shafts (44 – 1) could be expected to be stronger. For a design based on U=10, 9 shafts could be expected to be weaker and 34 (44 – 10) shafts stronger. And so on.

Weaker rock strengths (lower U) are more critical in terms of rock socket axial and lateral resistance capacity, whereas stronger rock strengths (higher U) are more critical in term of inducing higher shear stresses within the shaft at the soil-rock interface. Higher rock strengths (higher U) are also more problematic in terms of shaft drilling effort. To summarize and reiterate:

- Lower U is more conservative where weaker rock strengths are more critical, as is the case for rock socket axial and lateral resistance capacity.

- Higher U is more conservative where stronger rock strengths are more critical, as is the case for induced shaft shear stresses at the soil rock interface and shaft drilling effort.

Reliability R of Uncertain Rock Strength for a Given Rank Order U

There is uncertainty associated with the actual or true average intact rock UCS for a given shaft rank order U. The uncertainty is measured by a probability distribution of potential strengths, as graphed in Figure F4 for selected U.

For a given U, the lower a rock strength estimate, the higher the probability, or reliability "R," that the estimated rock strength is exceeded by the actual or *true* average strength, which is uncertain at any given location. Equivalently, R represents the probability that the (uncertain) true strength exceeds the given estimate, and is thus the rock strength exceedance probability.

The complement of R, 1 – R, represents the probability that the given estimate exceeds the true strength. R is the probability of an under-estimate of the true strength; 1 – R is the probability of an over-estimate of the true strength. R is the rock strength exceedance probability; 1 – R is the rock strength non-exceedance probability.

While lower U and higher R are appropriate for rock socket axial and lateral resistance capacity, for stronger rock strengths higher U and lower R are more critical in term of higher shaft shear stresses at the soil rock interface and shaft drilling difficulty. In terms of R:

- Higher R is more conservative where weaker rock strengths are more critical, as is the case for rock socket axial and lateral resistance capacity.

- Lower R, and correspondingly higher 1 – R, is more conservative where stronger rock strengths are more critical, as is the case for induced shaft shear stresses at the soil-rock interface and shaft drilling effort.

The rank ordering is completely consistent for a given R. There is, however, overlap between U and R. For example, the U=1 R=50% estimate is about the same as the U=2 R=90% estimate. Where weak rock is the critical conditions, for a given U a "high reliability" estimate would be associated with a high value of R (e.g., R=90%); an intermediate reliability estimate could use the median estimate, R = 50%.

ANALYSIS RESULTS FOR AVERAGE ROCK STRENGTH: FIGURE F5 (INTACT UCS) AND F6 (GLOBAL RMS)

Figure F5 and F6 provide graphical analysis results for shaft rock strength ranks U=1, U=10, and U=22 (median). Figure F5 displays intact rock UCS. Figure F6 displays Global RMS. The strength at the median reliability, R=50%, is prominently displayed for each U rank.

If the weakest shaft (U=1) is used for rock socket axial and lateral capacity, R=50% was considered adequate and was thus used for the intact rock UCS (6.8 ksi) for rock socket capacity analysis and the recommended Global RMS (1.1 ksi) for DFSAP input as the rock mass compressive strength.

CONCLUSIONS

The SRA, as summarized here, was used in a technically defensible way to extrapolate the available rock strength data from the seven 2009 borings drilled along Pier 2 to the 44 proposed shaft locations. The summary results are presented here in Table F2 and Figures F5 and F6. Besides helping to fill the spatial variability data gap, the SRA helped qualify and quantify design-performance risk given the available data, and it helped to qualify and quantify the effect of obtaining additional boring data.

The SRA focused on rock strength spatial variability. There are many sources of uncertainty that the SRA did not explicitly address. These uncertainty sources include statistical uncertainty associated with the estimators of average boring intact UCS and model uncertainty associated with the Hoek-Brown failure criteria for estimating rock mass UCS and Global RMS from intact UCS. Model uncertainty associated with the procedures used to estimate rock socket capacity from intact rock UCS were outside the scope of the SRA, as was model uncertainty associated with using Global RMS in DFSAP analyses.

Table F1.—Summary of Borehole Intact Rock Strength Average Along SSD Pier 2

SSD Pier 2 Borehole Avgs [ksi] -- in BH order								Delta BH Avg w and w/o UCS
SSD Boring	PLT Avg w/UCS	All PLT Avg	PLT w/o UCS	UCS Avg	UCS SD	Number of UCS Specimens	PMT	
SSD-006-09	28.4	28.2	28.2	40.0	6.6	2	4.0	0.2
SSD-007-09	6.2	7.4	7.8	17.0	8.2	2	2.9	-1.6
SSD-008-09	8.0	5.7	3.9	20.4	3.7	3	3.6	4.0
SSD-009-09	9.1	10.2	10.7	20.3	5.6	3		-1.6
SSD-010-09	9.8	9.8	9.8	15.0	0.9	4		-0.1
SSD-011-09	8.3	8.9	9.4	15.0	3.6	4		-1.1
SSD-012-09	11	5.5	4.6	11.7	2.5	2		5.9
Avg:	11.5	10.8	10.6	19.9	4.4	2.9	3.5	0.83
SD:	7.6	7.9	8.2	9.4	2.5	0.8	0.55	3.0
CV:	0.66	0.73	0.77	0.47	0.56	na	0.16	3.6
Geomean:	10.1	9.2	8.8	18.5	3.7	na	3.5	na
Std Error:	2.9	3.0	3.1	3.5	na	na	0.21	1.1

PLT = Point Load Test; UCS = Unconfined Compressive Strength; PMT = Pressure Meter Test.
Avg = Arithmetic Mean; SD = Population Standard Deviation; CV = Coefficient of Variation =SD/Avg;
Geomean = Geometric Mean; Std Error = Standard Error of the Mean = SD/N^{½}; N=Number of Specimens.

Figure F1.—Graph of Borehole Averages of Intact Rock Strength Along Pier 2, Measured East from West Portal. Borehole Averages are Estimators for Shaft Averages.

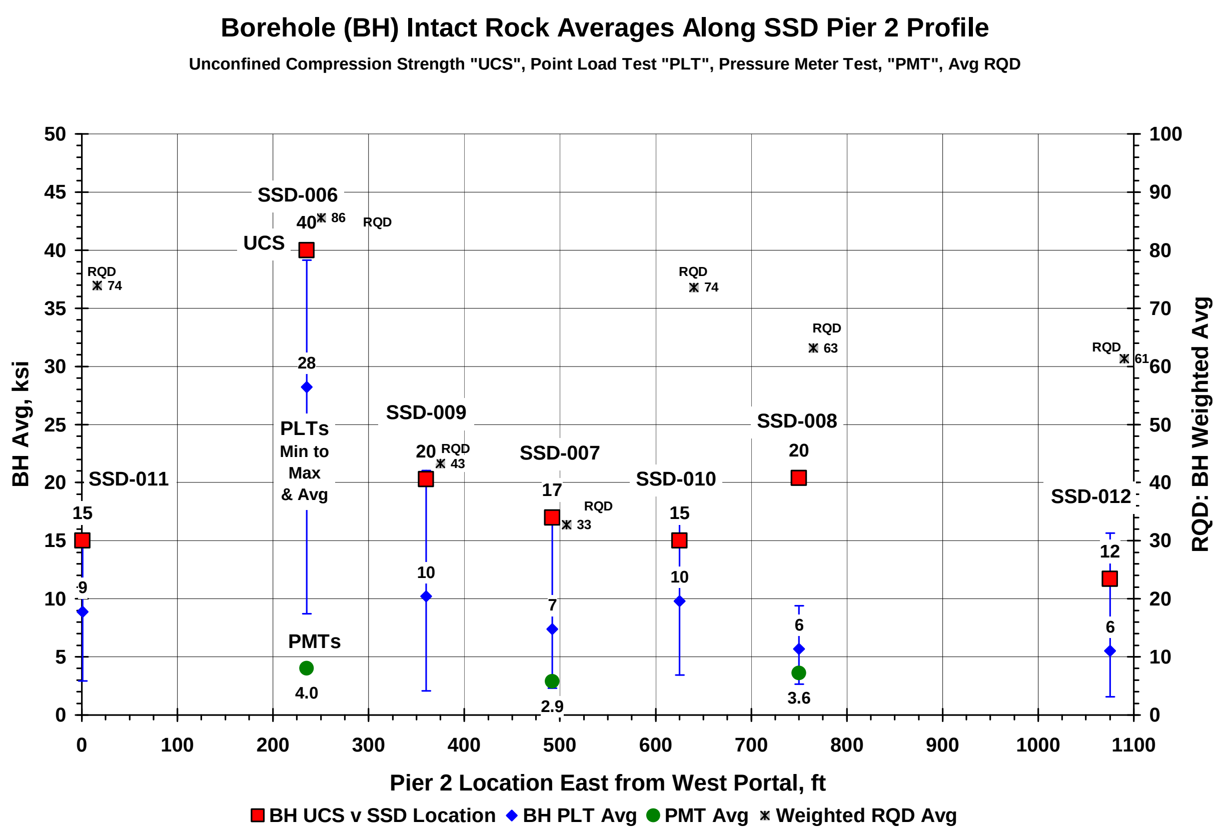
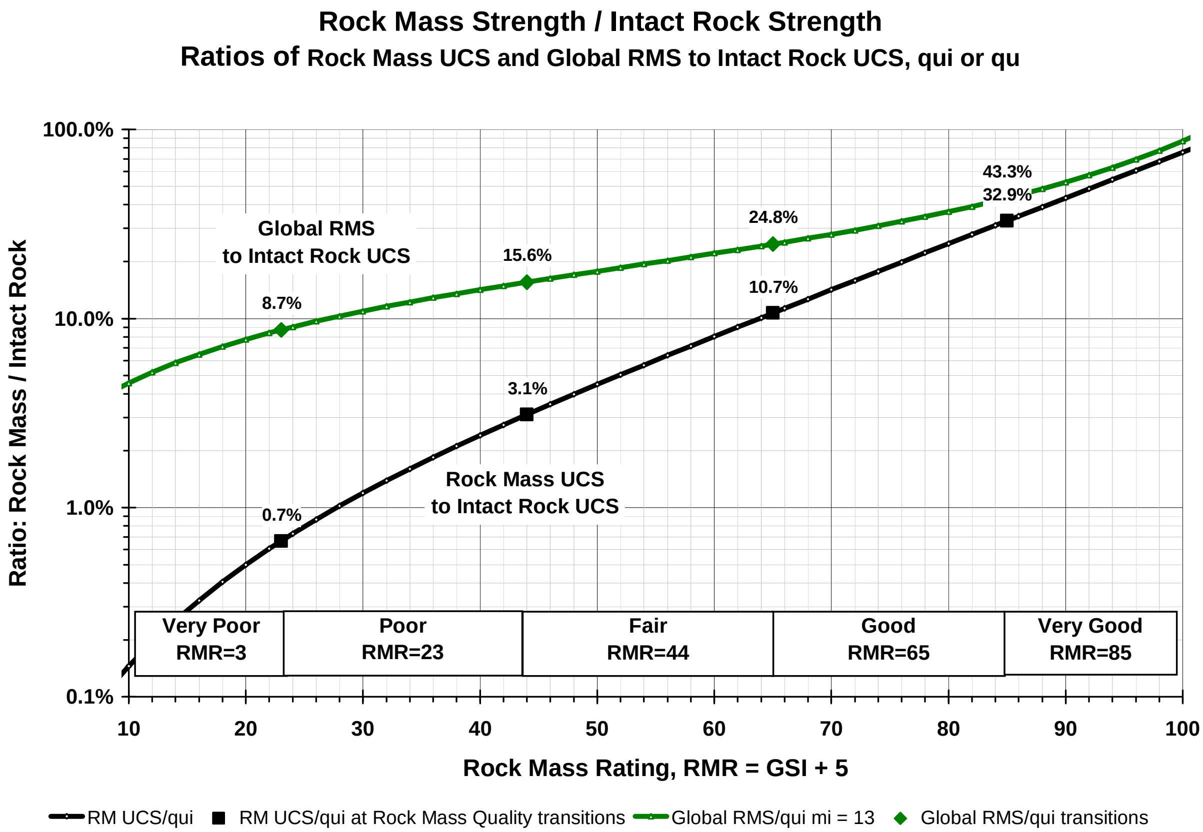


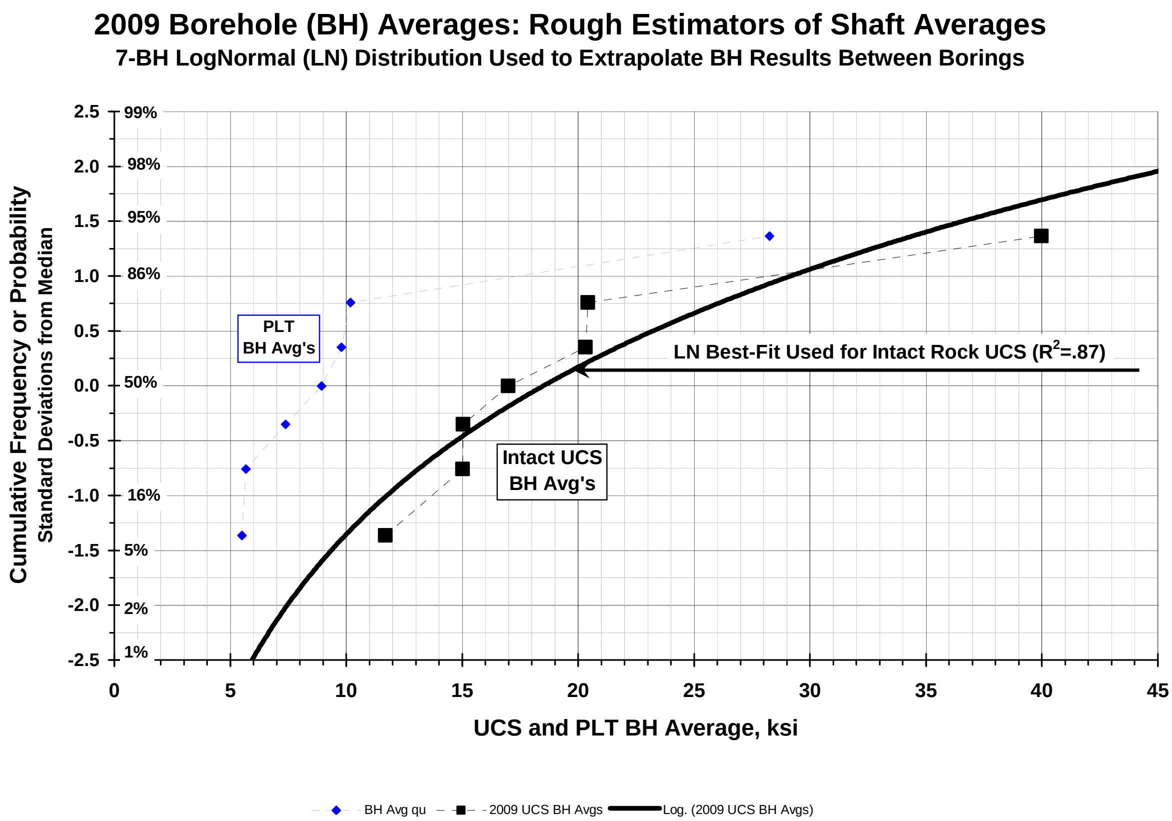
Figure F2.—Ratio of Rock Mass UCS and Global Rock Mass Strength (Global RMS) to Intact Rock UCS "qui" Based on Hoek-Brown 2002 Failure Criteria. Ratio for Global RMS Uses Parameter mi = 13 for Tuff (mi affects Global RMS only, not UCS). Ratios are a function of Rock Mass Rating, RMR, and Geologic Strength Index, GSI; RMR = GSI + 5.



RMR = 45 used for Global RMS Estimates (DFSAP Input) and Shaft Rock-Socket Capacity Design. For RMR = 45 in Tuff (mi=13), Global RMS is 16% of Intact Rock UCS. Note, as stated under Figure 4, shaft rock-socket capacities were calculated using *intact* rock UCS (qu) under 2008 AASHTO LRFD Bridge Design Specification 10.8.3.5.4, based on FHWA O'Neill and Reese (1999), which uses 1988 Hoek-Brown parameters, as specified in AASHTO Table 10.4.6.4-4.

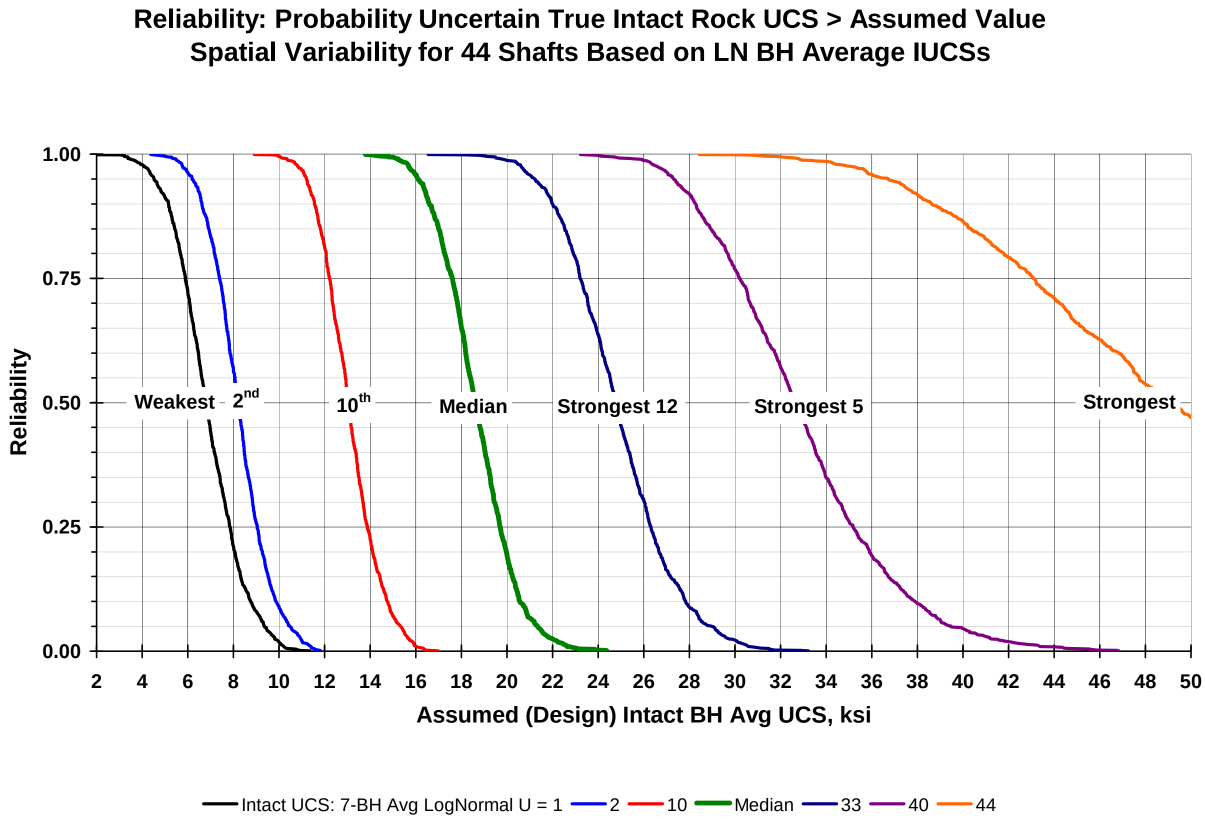
Note on rock UCS symbolism: The symbol "qui" is often used for *intact* rock UCS, while AASHTO uses "qu" as the "average unconfined compressive strength of rock core" (10.4.6.4-1) or the "uniaxial compressive strength of rock" (10.8.3.5.4b-1) and the "unconfined compressive strength of rock (10.8.3.5.4c-1 and c-2). For AASHTO, therefore, qu = qui. Hoek-Brown uses "σ_{ci}" as "the uniaxial compressive strength of intact rock pieces." Each of these symbols (qui, qu, or σ_{ci}) can represent the *intact* rock UCS. However, the symbol "qu" is often used to represent the *rock mass* UCS, which can be confused for the intact rock UCS. Hoek-Brown uses "σ_c" for the rock mass UCS, and "σ_{cm}" as the global rock mass strength.

Figure F3.—Probability Plot of Borehole Average Intact UCS, and PLT Averages for Comparison. Graph Indicates UCS Borehole Averages are Approximately Lognormally (LN) Distributed. Note Best-fit LN Regression Model Used to Extrapolate Intact UCS Averages to 44 Random Shaft Locations.



LN best fit curve by linear regression: y = 2.20 ln(x) – 6.42; R2 = 0.87; y = standard normal deviate; x = UCS. Resulting average UCS = 20.5 ksi; standard deviation = 9.83 ksi; coefficient of variation = 0.48; median (same as geomean) = 18.5 ksi.

Figure F4.—Results from Simulated Drilling at 44 Random Locations Along Pier 2, Representing Potential Range of Intact Rock UCS Averages At 44 Individual Shaft Locations. Reliability R (vertical axis) is the Probability that the Actual, or True, but Uncertain Average Intact Rock UCS Exceeds an Assumed Strength (horizontal axis), at the Weakest to the Strongest Potential Shaft Location. Exact Locations Cannot Be Specified With Available Information.



U is the rank order of shaft intact rock UCS, lowest to highest (U=1 to U=44). U=1 is the weakest of 44 shafts; U=2 the second weakest; U=10 the 10th weakest; U=22 the median shaft strength; U=33 is the 12th strongest shaft; U=40 is the 5th strongest; U=44 is the strongest shaft. Conversely, (44+1 – U) is the rank order of shaft intact rock UCS from highest (U=44) to lowest (U=1).

Each curve represents 1,000 independent simulations.

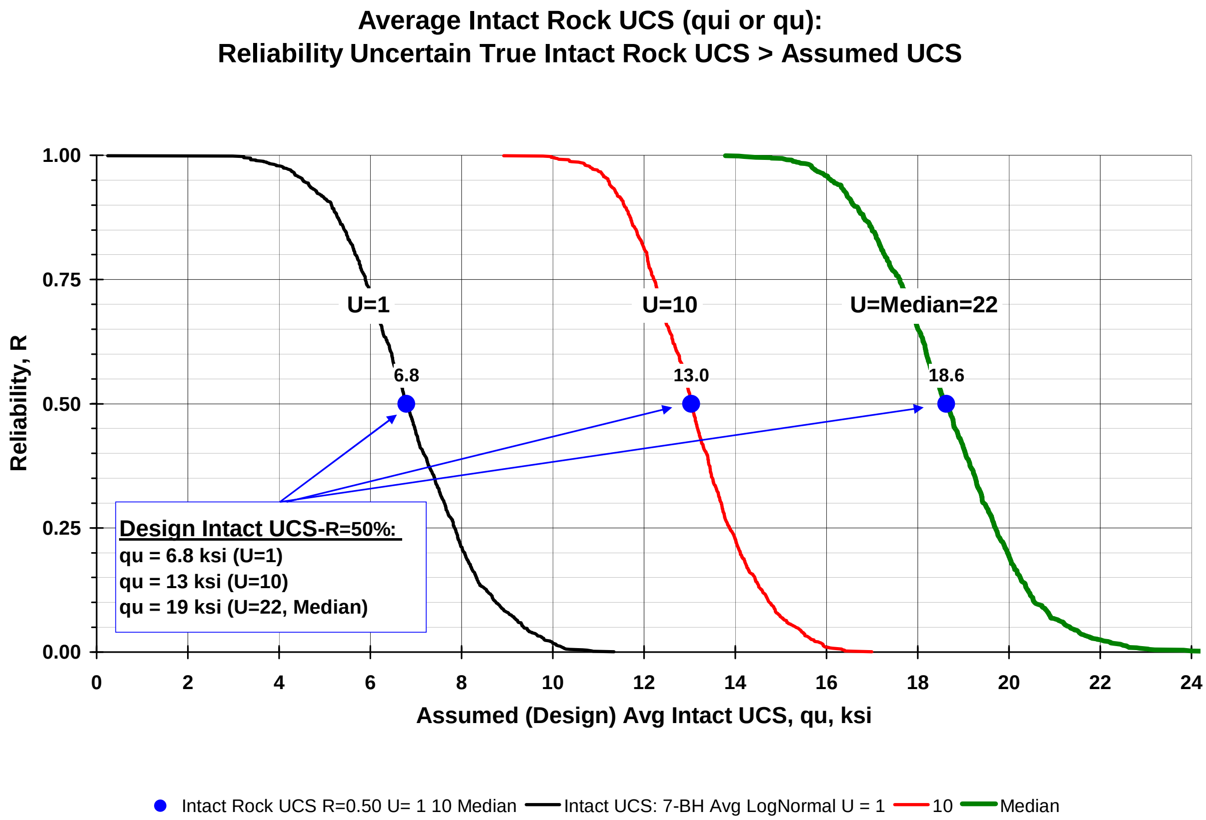
Table F2.—Summary Statistics on 44 Simulated Borehole Averages. Full Distributions (1,000 points) are Graphed in Figures F3, F4, and F5.

Estimated Shaft Rock Strengths for Selected Strength Rank U										
	Shaft Strength Rank Low (Weaker) to High (Stronger), U:									
	1	2	3	4	5	10	Median	33	40	44
	Weakest →									→ Strongest
	Shaft Strength Reverse Rank High (Stronger) to Low (Weaker), 45-U:									
	44	43	42	41	40	35	Median	12	5	1
	Borehole Average Intact UCS, ksi									
Avg:	6.9	8.2	9.1	9.9	10.5	13.1	18.7	24.9	32.9	50.9
SD:	1.4	1.3	1.3	1.2	1.2	1.2	1.6	2.3	3.8	11.4
CV:	0.21	0.15	0.14	0.13	0.12	0.09	0.09	0.09	0.12	0.22
Reliability	Strength Estimates at Selected Reliabilities, R = Probability Estimate Exceeds True Strength									
R=10%	8.8	9.9	10.8	11.5	12.1	14.8	20.6	27.8	37.9	65.0
R=25%	7.8	9.0	9.9	10.8	11.3	13.9	19.7	26.3	35.2	56.6
R=50%:	6.8	8.2	9.0	9.8	10.4	13.0	18.6	24.8	32.7	49.1
R=75%	5.9	7.4	8.3	9.0	9.7	12.2	17.6	23.2	30.2	43.1
R=90%	5.2	6.6	7.6	8.5	9.0	11.6	16.6	22.0	28.3	38.6
	Borehole Global Rock Mass Strength, ksi									
Avg:	1.1	1.3	1.5	1.6	1.7	2.1	3.0	4.0	5.3	8.1
SD:	0.23	0.20	0.20	0.20	0.19	0.20	0.25	0.37	0.61	1.82
CV:	0.21	0.15	0.14	0.13	0.12	0.09	0.09	0.09	0.12	0.22
Reliability	Strength Estimates at Selected Reliabilities, R = Probability Estimate Exceeds True Strength									
R=10%	1.4	1.6	1.7	1.8	1.9	2.4	3.3	4.5	6.1	10.4
R=25%	1.3	1.4	1.6	1.7	1.8	2.2	3.2	4.2	5.6	9.1
R=50%:	1.1	1.3	1.4	1.6	1.7	2.1	3.0	4.0	5.2	7.9
R=75%	0.9	1.2	1.3	1.4	1.6	2.0	2.8	3.7	4.8	6.9
R=90%	0.8	1.1	1.2	1.4	1.4	1.9	2.7	3.5	4.5	6.2

Avg = arithmetic average; SD = standard deviation; CV = coefficient of variation; R = reliability.

Global RMS = 0.16 Intact UCS

Figure F5.—Average Intact Rock UCS (qui or qu)*. Same as Figure F4 for U=1 (weakest), U=10 and U=22 (median) Intact Rock UCS.

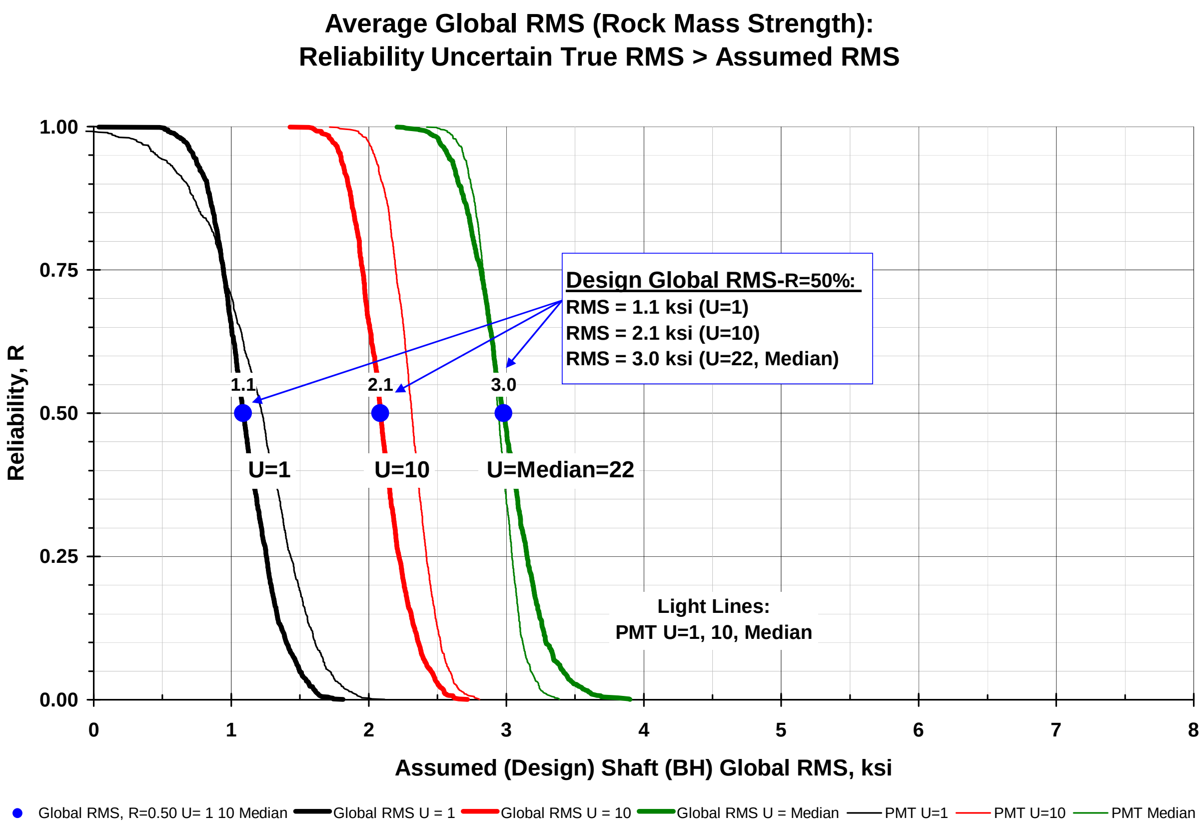


At the weakest shaft location (U=1) a design qu of 6.8 ksi has a 50 percent reliability of exceeding the true average qu at that location, which in general may be any of the 44 shafts – however, shafts within about 2B of a boring could use specific qu data for that boring. B=Shaft Diameter.

Shaft rock-socket capacities were calculated from intact rock UCS (qu) following 2008 AASHTO LRFD Bridge Design Specification 10.8.3.5.4, based on FHWA O'Neill and Reese (1999), which uses 1988 Hoek-Brown parameters, as specified in AASHTO Table 10.4.6.4-4. Rock socket capacities were developed for U=1, U=10 and U=22.

*** Note on rock UCS symbolism: The symbol "qui" is often used for *intact* rock UCS, while AASHTO uses "qu" as the "average unconfined compressive strength of rock core" (10.4.6.4-1) or the "uniaxial compressive strength of rock" (10.8.3.5.4b-1) and the "unconfined compressive strength of rock (10.8.3.5.4c-1 and c-2). For AASHTO, therefore, qu = qui. Hoek-Brown uses "σ_{ci}" as "the uniaxial compressive strength of intact rock pieces." Each of these symbols (qui, qu, or σ_{ci}) can represent the *intact* rock UCS. However, the symbol "qu" is often used to represent the *rock mass* UCS, which can be confused for the intact rock UCS. Hoek-Brown uses "σ_c" for the rock mass UCS, and "σ_{cm}" as the global rock mass strength.**

Figure F6.—Average Global RMS. Global RMS Calculated as 0.16 of Intact UCS based on Results Shown in Figure F2 and Figure F5. Global RMS is used for DFSAP input as "Compressive Strength of Rock Mass."*



*** The April 2006 DFSAP final report calls for entering the "Compressive Strength of Rock Mass" as *the value of the unconfined compressive strength (qu) of the rock mass*, which is the Rock Mass UCS. While significantly exceeding the Rock Mass UCS, the Global RMS is considered the more appropriate parameter value for DFSAP analysis at the Snowshed. For an RMR = 45 Rock Mass UCS is calculated at 3% of the intact rock UCS whereas the Global RMS is calculated at 16% of the intact rock UCS (tuff mi = 13).**