Circular Curves

A circular curve is a segment of a circle — an arc. The sharpness of the curve is determined by the radius of the circle (R) and can be described in terms of "degree of curvature" (D). Prior to the 1960's most highway curves in Washington were described by the degree of curvature. Since then, describing a curve in terms of its radius has become the general practice. Degree of curvature is not used when working in metric units.

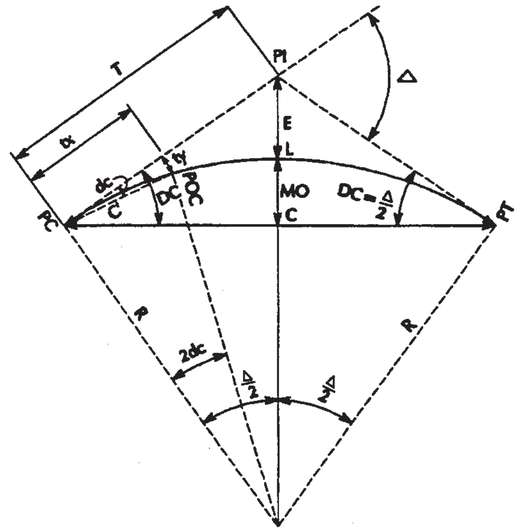
Nomenclature For Circular Curves

P.O.T.	Point on tangent outside the effect of any curve
P.O.C.	Point on a circular curve
P.O.S.T.	Point on a semi-tangent (within the limits of a curve)
P.I.	Point of intersection of a back tangent and forward tangent
P.C.	Point of curvature - Point of change from back tangent to circular curve
P.T.	Point of tangency - Point of change from circular curve to forward tangent
P.C.C.	Point of compound curvature - Point common to two curves in the same direction with different radii
P.R.C.	Point of reverse curve - Point common to two curves in opposite directions and with the same or different radii
L	Total length of any circular curve measured along its arc
L_c	Length between any two points on a circular curve
R	Radius of a circular curve
Δ	Total intersection (or central) angle between back and forward tangents
DC	Deflection angle for full circular curve measured from tangent at PC or PT
dc	Deflection angle required from tangent to a circular curve to any other point on a circular curve
C	Total chord length, or long chord, for a circular Curve
C'	Chord length between any two points on a circular Curve
T	Distance along semi-tangent from the point of intersection of the back and forward tangents to the origin of curvature (From the PI to the PC or PT).
E	External distance (radial distance) from PI to midpoint on a simple circular curve

M.O.	The (radial) distance from the middle point of a chord of a circular curve to the middle point of the corresponding arc.
tx	Distance along semi-tangent from the PC (or PT) to the perpendicular offset to any point on a circular curve. (Abscissa of any point on a circular curve referred to the beginning of curvature as origin and semi-tangent as axis)
ty	The perpendicular offset, or ordinate, from the semi-tangent to a point on a circular curve

Circular Curve Equations

Equations	Units
$R = \frac{180°}{\pi} \cdot \frac{L}{\Delta}$	m or ft.
$\Delta = \frac{180°}{\pi} \cdot \frac{L}{R}$	degree
$L = \frac{\pi}{180} \cdot R\Delta$	m or ft.
$T = R \tan \frac{\Delta}{2}$	m or ft.
$E = \frac{R}{\cos \frac{\Delta}{2}} - R$	m or ft.
$C = 2R \sin \frac{\Delta}{2}, \text{or} = 2R \sin DC$	m or ft.
$MO = R\left(1 - \cos \frac{\Delta}{2}\right)$	m or ft.
$DC = \frac{\Delta}{2}$	degree
$dc = \frac{L_c}{L}\left(\frac{\Delta}{2}\right)$	degree
$C' = 2R \sin(dc)$	m or ft.
$C = 2R \sin(DC)$	m or ft.
$tx = R \sin(2dc)$	m or ft.
$ty = R[1 - \cos(2dc)]$	m or ft.



Constant for π = 3.14159265

Simple Circular Curve

Figure 11-1

After the length of the curve (L) and the semi-tangent length (T) have been computed, the curve can be stationed.

When the station of the PI is known, the PC station is computed by subtracting the semi-tangent distance from the PI station. (Do not add the semi-tangent length to the PI station to obtain the PT station. This would give you the wrong value) Once the PC station is determined, then the PT station may be obtained by adding L to the PC station.

All stationing for control is stated to one hundredth of a foot. Points should be set for full stations and at half station intervals. Full stations are at 100 ft intervals and half station intervals are at 50 ft. (10+00.00).

Example Calculations for Curve Stationing

Given: (See Figure 11-1)

PI = 12 + 78.230

R = 500’

Δ = 86° 28’

Find the PC and PT stations

Calculate T

T = R tan (Δ/2)

= 500 tan 43° 14’

= 470.08’

Calculate L (Δ must be converted to decimal degrees)

$\Delta = 86° \ 28'$

$= 86° + (2?°/ 60)$

$L = (\Delta/360°)\ 2\pi R$ or $R\Delta\ (0.017453293)$

$= 754.56'$

Calculate the PC station

$PI - T = 1278.23' - 470.08'$

$T = 808.15'$

PC station is 8 + 08.15'

Calculate the PT station

$PC + L = 808.15' + 754.56'$

$= 1562.71'$

PT station is 15 + 62.71

Deflections

To lay out a curve it is necessary to compute deflection angles (dc) to each station required along the curve. The deflection angle is measured from the tangent at the PC or the PT to any other desired point on the curve. The total deflection (DC) between the tangent (T) and long chord (C) is Δ/2.

The deflection per foot of curve (dc) is found from the equation: $dc = (L_c / L)(\Delta/2)$. dc and Δ are in degrees.

Since $L_c = 1'$, the deflection per foot becomes:

$$dc / ft = (\Delta/2) / L$$

If only the radius is known, dc / ft can still be found:

$$dc / ft = (360°/4\pi) \text{ or } 28.6479/R \text{ [Expressed in degrees]}$$

or 1718.87338/R [Expressed in minutes]

The value obtained can then be multiplied by the distance between stations to obtain the deflection.

Example Calculations for Curve Data

Given:

PI = 100 + 00.00

$R = 1100'$

$\Delta = 16° \ 30'$

Find the deflection angles through the curve

Calculate T

$T = R \tan(\Delta/2)$

$= 1100 \tan 8° 15'$

$= 159.49'$

Calculate L

$\Delta = 16° 30'$

$= 16.5°$

$L = (\Delta/360°)\ 2\pi R$ or $R\Delta(0.017453293)$

$= 1100' (16.5°) (0.017452293)$

$= 316.78'$

Calculate the PC station

PI - T = 10,000 - 159.49’

= 9840.51’

PC station is 98 + 40.51

Calculate the PT station

PC + L = 9840.51 + 316.78’

= 10157.29

PT station is 101 + 57.29

Calculate the deflection per foot

$dc / ft = (\Delta/2) / L$

$= 8° 15' / 316.78'$

$= 0.0260433° / ft$

The first even station after the PC is 98 + 50.

Calculate the first deflection angle

$L_c = 9850' - 9840.51'$

$= 9.49'$

$dc = L_c(dc / ft)$

$= 9.49(0.0260433°/ft)$

$= 0.2471509°$

$= 0° 14' 50''$

The last even station before the PT is 101 + 50

Calculate the last deflection angle from the PC

L_c = 10150 - 9840.51

= 309.49’

dc = L_c (dc / ft)

= 309.49’ (0.0260433° / ft)

= 8.0601409°

= 8° 03’ 37”

Chord distances would now be calculated using C_I = 2 R sin (dc)

Curve Data

Station	Point	dc	Curve Data
101 + 57.29	PT	8º15’00”	PI 100 + 00.00
		Δ = 16°30’	
		R = 1100’	
		L = 316.78’	
		T = 159.49’	
101 + 50		8º03’37”	
101+ 00		6º45’29”	
100 + 50		5º27’21”	
100 + 00		4º09’13”	
99 + 50		2º51’05”	
99 + 00		1º32’58”	
98 + 50		0º14’50”	
98 + 40.5	PC	0º00’00”	

The deflection at the PT must equal Δ/2.

Figure 11-2

Running the Curve

After completing the computations, it is necessary to establish the curve on the ground. When running the curve ahead on line (from PC to PT) the instrument is set on the PC, the plate set at zero and the telescope inverted for a sight on the back tangent. An alternative method would be to sight the PI without the telescope inverted if the PI has already been set and is visible.

Turn the deflection for the first even half station and accurately measure the proper distance to the desired station. Be sure to measure the chord distance and not the curve distance. The chord distance must be calculated. The backsight should be checked periodically to be certain that the instrument has not drifted.

The curve may be backed in from the PT by entering the total deflection and backing off to the PC. When the PC is sighted, zero should be read.

Radial Layout Method

With the advent of electronic surveying, the need to occupy control points such as PCs, POCs, and PTs no longer exists. Control points that are set off the roadway in the vicinity of the curve are used for layout.

There are computer and data collector programs that calculate angles and distances from control points off the curve for setting points on the curve.

Coordinates of the curve alignment (such as 25 ft stationing) must be input into the Data Collector or computer with the off-the-curve control point coordinates.

The program then calculates the angles and distances from control points to layout the curve.

Specific information about this procedure may be found in the Data Collector reference manual under the “Roading” chapter.

Also, the design engineering software has commands and procedures to generate radial layout data.

Instrument Set At POC

Assuming that the first part of a curve has been located by deflections from the PC, if the next part of the curve is not visible from the PC it must be located by deflections from some point on the curve, usually at a full station.

The instrument can be set on a point from which the PC can be backsighted (Methods A and B) or on any point from which some intermediate POC can be backsighted (Method C).

Method A uses deflection angles turned from the auxiliary tangent at the POC being occupied. Methods B (preferred) and C (for remote locations) use the original calculated (book) deflections turned from the extension of the chord from the PC to the occupied POC.

Method A

- Set the scale at the deflection angle for the point being occupied, but to the “wrong side”.
- Backsight on the PC with the scope inverted
- 0° is now an auxiliary tangent.
- Turn deflection angles for the forward points based on their distance from the occupied POC and therefore turned from the auxiliary tangent.

Method B

- Set the scale to 0°
- Backsight on the PC with the scope inverted.
- 0° is now the extension of the chord from the PC to the occupied POC.
- Turn deflection angles to the forward points using the original calculated (book) deflection angles.

Method C

- Set the scale at the deflection angle for an intermediate point being sighted (POC_1): d_{f1}
- Backsight on the intermediate POC_1 with the scope inverted.
- 0° is now the extension of the chord from the PC to the occupied POC.
- Turn deflection angles to the forward points using the original calculated (book) deflection angles.

Offset Curves

Frequently it is necessary to locate outer and inner concentric curves, such as property lines, curb lines and offset curves for reference during construction. The full lengths of these offset curves are also desirable. Curve data can be calculated using the adjusted radius or by proportioning the center line data.

The subscripts "o" for outside and "i" for inside are commonly used to identify elements on offset curves.

For example, the length of an inside curve would be $L_i = (\pi/180)\ R_i\,\Delta$; R_i being a shorter radius than the center line radius R. Or, by proportion, $L_i = (R_i\ /R)\ L$.

It is convenient to note that the difference in arc length L between the offset curve and the center line curve, for the same internal angle D, where w is the offset distance, is $(2\pi\Delta\ / 360°)\ w$.

Degree of Curvature

The two common definitions of degree of curvature (D) are the arc definition used in highway work and the chord definition used by some counties and in railroad work.

By the arc definition, a D degree curve has an arc length of 100 feet resulting in an internal angle of D degrees. (So, the stationing and angles are known and the chords remain to be calculated.)

By the chord definition, a D degree curve has a chord of 100 feet resulting in an internal angle of D degrees. (So, the chords and angles are known and the arc stations would remain to be calculated.)

In terms of radius, a 1° curve by the arc definition would have a radius of 5729.578 feet. And by the chord definition, its radius is 5729.65 feet. In the days of slide rules, a radius of 5,730 feet might have been used as the formula R = 5730 / D in the Field Tables for Engineers, Spirals, 1957.

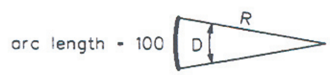
Degree of Curvature for Various Lengths of Radii

Exact for Arc Definition

$$D = \frac{100\left(\frac{180}{\pi}\right)}{R} = \frac{18000}{\pi R}$$

Where D is Degree of Curvature

Length of Radii for Various Degrees of Curvature



$$R = \frac{100\left(\frac{180}{\pi}\right)}{D} = \frac{18000}{\pi D}$$

Where R is Radius Length

Degree of Curvature (Highway)
Figure 11–3

Field Record

When not using a data collector it is necessary to handwrite field notes in the traditional manner

The field notes for curves are kept on transit note sheets. These are available on regular or “rite in rain” paper.

The left page is for station, curve data, deflections, and other such data. The right page is for ties to curve points, descriptions of points set and any pertinent drawings that may make clear to others just what was done in the field. This is very important, as the work may have to be reproduced years later.

It is useful to show the location of points relative to permanent objects. Note what the point is (spike, hub & tack, etc.), and how it is referenced.

Make notes that are neat and accurate. Title and index the first page of each operation. On the first page of each day’s work, show the date, crew, weather, and instrument by Serial number. Number and date every page. Notes written with the book turned are written with the right edge of the book toward the writer.

Do not use “scratch” notes intending to put them in the book later.

Field records are an important aspect in any survey. A well executed surveying job is worthless unless it is well documented. Take the time necessary, in the field, to create good records on what you have done.

Spiral Curves

WSDOT does not use spiral curves on new highway design but a knowledge of spiral curves is necessary when an existing highway alignment contains spiral curves.

A spiral curve is for the transition of a vehicle traveling at a sustained speed from a straight tangent to a circular curve. It is an attempt to approximate the path followed by a vehicle's wheels from when the operator begins to turn his steering wheel until he has reached the maximum degree of curvature at the circular portion of the curve.

Spiral curves are divided into an entering spiral transition, a circular curve, and an exiting spiral transition. In most cases the entering and exiting spiral will be equal. The major difference between a spiral and a circular curve is that the change of direction varies as the square of the length for a spiral rather than as the first power of the length for a circular curve. The degree of curvature on a spiral increases directly as the distance increases along the spiral curve from the tangent. The degree of curvature at any point in the spiral is the same as the degree of curvature of a circular curve having the same radius. A spiral curve will be tangent to a circular curve at the point where they share the same radius.

Spiral Curve Elements

For circular curve elements see circular curve segment in this chapter.

a — Rate of change in the degree of curve of a spiral per 100 feet of length which equals the degree of curve on a spiral at a point 100' (one station) from the TS (or ST).

$a = D/L_S$ or $a = D_S/L_S'$

CΔ — Central angle of circular curve between connecting spiral curves.

C Δ = ?Δ 2DE for equal spirals

C?Δ?= Δ - (DE_1+ DE_2) for unequal spirals

CS — Point of change from circular curve to spiral.

CS = ST - (L_S x 100)

C_S — Total chord length for a spiral curve from its beginning (TS or ST) to its end (SC or CS).

$C_S = 100' L_S - 0.000338\ a^2(L_S)^5$;

also = $\sqrt{(X^2 + Y^2)}$

C_S' — Chord length to any point on a spiral from TS or ST.

$C_S' = 100' L_S' - 0.000338\ a^2(L_S')^5$;

also = $\sqrt{[(x')^2 + (y')^2]}$

Δ — (delta) Total intersection or "central" angle between back and forward tangents.

D — When used in reference to a spiral, D indicates the maximum degree of curvature of the spiral, which is at SC or CS.

$D = aL_S$

DE — Deviation angle of spiral measured from back tangent (or forward tangent) to tangent through the spiral at its maximum degree.

$DE = [a(L_S)^2]/2$; also = $DL_S/2$; also = DF + DH

de — Deviation angle of spiral measured from back tangent (or forward tangent) to tangent through any point on spiral.

$de = [a(L_S')^2]/2$; also = $D_S L_S'/2$; also = df + dh df — Deflection angle to any point on spiral measured from tangent at TS (or ST).

$df = a(L_s')^2/6 - dfk$ in degrees

DF — Deflection angle for full spiral measured from tangent at TS (or ST) to SC (or CS) respectively.

$DF = a(L_s)^2/6 - DFk$; also $D_sL_s/6 - DFk$;

also = DE/3 – DFk

dfk — A correction, in minutes, to be applied to the equation for df when the angle de is 15° and over.

$dfk = 0.000053(DE)^3$

DFk — A correction, in minutes, to be applied to the equation for DF when the angle DE is 15° and over.

$DFk = 0.000053(DE)^3$

dh — Deflection angle required to establish tangent to any point on the spiral measured between tangent to the spiral at that point and the beginning of the spiral (TS or ST).

dh = de – df

DH — Deflection angle required to establish tangent to the spiral at its maximum degree when instrument is sighted on the beginning of the spiral (TS or ST).

DH = DE – DF

dr — Deflection angles required to be computed for each intermediate setup on a spiral to locate other points on the spiral. When sighting on TS or ST to establish tangent to the spiral, dr = dh.

$dr = df + D_s L_s'/6$

D_s — Degree of curve at any point on spiral.

$D_s = aL_s'$

L_s — Total length of a spiral curve measured along its arc in stations of 100'.

$L_s = D/a$

L_s' — Length of spiral curve from T.S. (or S.T.) to any point on spiral measured along its arc in stations of 100'

$L_s' = D_s/a$

P.C.O. — Point where circular curve, if extended around its center, has a tangent that is parallel to the spiral semi-tangent.

POS — Point on spiral.

Q — Offset distance perpendicular to the forward and back tangents that the circular curve is moved to accommodate the spiral transitions.

$Q = (0.0727a)(L_s)^3 - (0.0000002a^3)(L_s)^7$

(See Note 1)

R — Radius of circular curve in feet (minimum length of R_s for spiraled curve).

R_S Radius in feet at any point on spiral curve.

$R_S = 5730/D_S$

SC Point of change from spiral to circular curve.

$SC = TS + (L_S \times 100)$

ST Point of change from spiral to forward tangent.

t Distance along semi-tangent of a spiral curve from TS (or ST) to perpendicular offset through PCO in feet.

$t = 50L_S - (0.000127a^2)(L_S)^5$

T Distance along semi-tangent from the point of intersection of the back and forward tangents to the origin of curvature from that tangent.

$T = t + (R + Q) \tan \triangle/2$

TS Point of change from back tangent to spiral.

$TS = PI - T$

x Distance along semi-tangent of a spiral curve from TS (or ST) to perpendicular offset to the end of spiral in feet.

$x = 100L_S - [(0.000762a^2)(L_S)^5 + (0.0000000027a^4)(L_S)^9]$ (See Note 1)

x' Distance along semi-tangent of a spiral curve from TS (or ST) to perpendicular offset to any point on spiral in feet.

$x' = 100L_S' - [(0.000762a^2)(L_S')^5 + (0.0000000027a^4)(L_S')^9]$

(See Note 1)

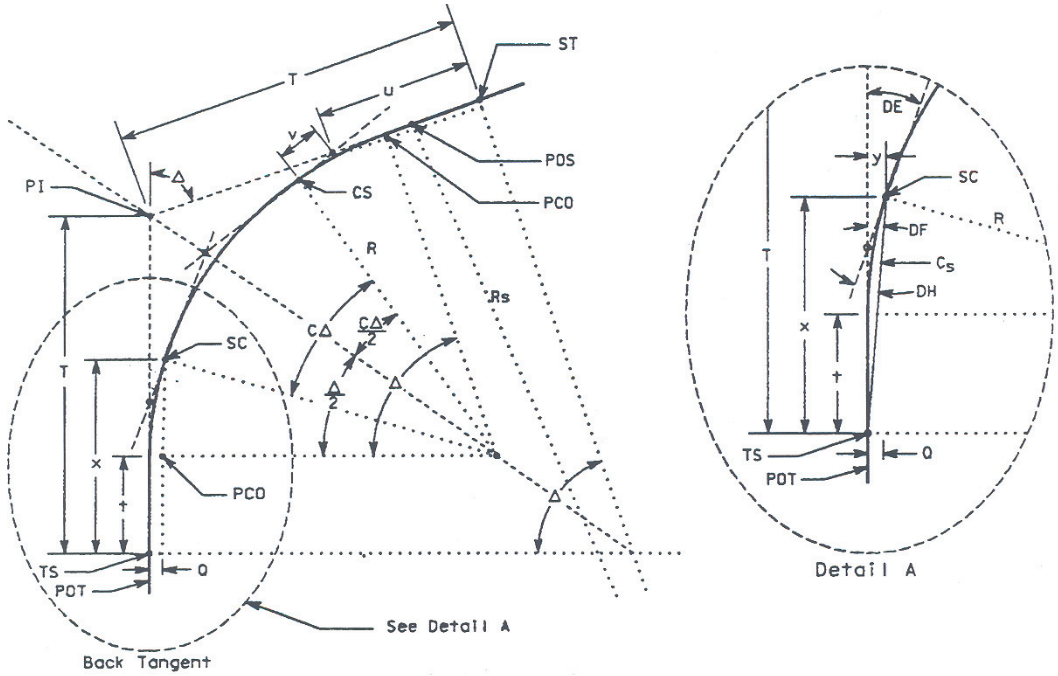
y Offset distance from the semi-tangent to the SC (or CS) measured perpendicular to the semi-tangent in feet.

$y = (0.291a)(L_S)^3 - (0.00000158a^3)(L_S)^7$ (See Note 1)

y' Offset distance from the semi-tangent to any point on the spiral measured perpendicular to the semi-tangent.

$y' = (0.291a)(L_S')^3 - (0.00000158a^3)(L_S')^7$

(See Note 1)



Spiral Curve Elements

Figure 11–4

SPIRAL TABLES a = 1⅔—Continued 1° in 60 ft.

Ls'	Ds	de	df	Q	R + Q	t	x	y	Cs'
Stations	Degrees and Minutes	Deg. & Min.	Deg. & Min.	Feet	Feet	Feet	Feet	Feet	Feet
3.1	5°-10'	8°-00.50'	2°-40.1394'	3.607	1112.639	154.900	309.393	14.428	309.731
3.2	5°-20'	8°-32.00'	2°-50.6337'	3.967	1078.342	159.882	319.289	15.867	319.685
3.3	5°-30'	9°-04.50'	3°-01.4604'	4.350	1046.168	164.862	329.170	17.398	329.633
3.4	5°-40'	9°-38.00'	3°-12.6193'	4.757	1015.933	169.840	339.037	19.024	339.573
3.5	5°-50'	10°-12.50'	3°-24.1102'	5.189	987.474	174.815	348.886	20.747	349.507
3.6	6°-00'	10°-48.00'	3°-35.9332'	5.645	960.645	179.787	358.718	22.570	359.432
3.7	6°-10'	11°-24.50'	3°-48.0879'	6.128	935.317	184.756	368.529	24.497	369.349
3.8	6°-20'	12°-02.00'	4°-00.5744'	6.638	911.374	189.721	378.319	26.529	379.256
3.9	6°-30'	12°-40.50'	4°-13.3921'	7.174	888.712	194.682	388.085	28.669	389.153
4.0	6°-40'	13°-20.00'	4°-26.5410'	7.739	867.239	199.639	397.827	30.920	399.039
4.1	6°-50'	14°-00.50'	4°-40.0209'	8.332	846.868	204.592	407.540	33.284	408.912
4.2	7°-00'	14°-42.00'	4°-53.8316'	8.955	827.526	209.539	417.225	35.764	418.773
4.3	7°-10'	15°-24.50'	5°-07.9728'	9.608	809.142	214.482	426.877	38.362	428.620
4.4	7°-20'	16°-08.00'	5°-22.4440'	10.291	791.654	219.419	436.496	41.080	438.452
4.5	7°-30'	16°-52.50'	5°-37.2453'	11.006	775.006	224.350	446.078	43.922	448.267
4.6	7°-40'	17°-38.00'	5°-52.3760'	11.753	759.144	229.274	455.621	46.889	458.066
4.7	7°-50'	18°-24.50'	6°-07.8361'	12.532	744.021	234.191	465.122	49.983	467.847
4.8	8°-00'	19°-12.00'	6°-23.6249'	13.345	729.595	239.102	474.578	53.207	477.608
4.9	8°-10'	20°-00.50'	6°-39.7421'	14.192	715.824	244.004	483.987	56.563	487.348
5.0	8°-20'	20°-50.00'	6°-56.1874'	15.073	702.673	248.898	493.344	60.053	497.066
5.1	8°-30'	21°-40.50'	7°-12.9603'	15.989	690.106	253.783	502.648	63.679	506.761
5.2	8°-40'	22°-32.00'	7°-30.0603'	16.941	678.094	258.659	511.894	67.442	516.430
5.3	8°-50'	23°-24.50'	7°-47.4868'	17.930	666.609	263.525	521.079	71.346	526.074
5.4	9°-00'	24°-18.00'	8°-05.2395'	18.955	655.621	268.381	530.199	75.391	535.689
5.5	9°-10'	25°-12.50'	8°-23.3176'	20.018	645.108	273.225	539.251	79.578	545.275
5.6	9°-20'	26°-08.00'	8°-41.7208'	21.118	635.046	278.058	548.230	83.911	554.829
5.7	9°-30'	27°-04.50'	9°-00.4481'	22.258	625.415	282.878	557.131	88.389	564.351
5.8	9°-40'	28°-02.00'	9°-19.4990'	23.426	616.194	287.685	565.952	93.015	573.838
5.9	9°-50'	29°-00.50'	9°-38.8729'	24.654	607.365	292.478	574.686	97.789	583.288
6.0	10°-00'	30°-00.00'	9°-58.5690'	25.912	598.912	297.257	583.330	102.713	592.699

Sample Spiral Table

Figure 11-5

Note 1

The last term in the equation may be omitted when the value of DE is 15° or less.

An example of a circular curve with equal spirals is calculated.

Given:

$\Delta = 100° 00'$

PI = Station 120 + 10.54

D =6° 00'

L_S =3.6 Stations

Determine the information necessary to establish the control points for the curve.

First, determine a, the rate of change in the degree of curvature of the spiral per 100 ft of curve.

$a = D/L_S$

$=6°/3.6$

=1 2/3 degrees/station

Now the spiral tables can be used to find information used to establish the control points for the curve.

From the *Field Tables for Engineers, Spirals 1984,* page 57, (a=1 2/3) the following information is found.

de = 10° 48' t = 179.787'

df = 3° 36' x = 358.718'

Q = 5.645' y = 22.570'

R + Q = 960.645' C_s = 359.432'

Or, if spiral tables are not available, the calculations go as follows.

First the angles DE, DF, DH and CΔ are calculated.

DE = $DL_s/2$

= (6)(3.6)/2

= 10.8°

=10° 48'

DF = DE/3 - DFk

(DFk = 0 because D < 15°)

= (10° 48')/3 - 0

=3° 36'

DH = DE - DF

=10° 48' - 3° 36'

=7° 12'

CΔ = Δ - 2DE

= 100° - (2)(10° 48')

=78° 24'

Once CΔ is known, the length of the circular curve can be found.

L =CΔ /D

= (78° 24')/6

= 13.0667 stations

The radius for the circular curve is calculated using the formula in Field Tables for Engineers Spirals 1984, page 8.

R = 5730/D

= 5730/6

= 955.0'

To find the stations, first calculate T. Since T = t + (R + Q) tan D/2, it is necessary to first find Q and t.

$Q = (0.0727a)(L_S)^3$

$= (0.0727)(5/3)(3.6)^3$

= 5.6532'

$t = 50L_S - (0.000127a^2)(L_S)^5$

$= 50(3.6) - (0.000127)(1\ 2/3)^2(3.6)^5$

= 180 - 0.2133

= 179.7867'

Now find T.

$T = t + (R + Q)\tan \Delta/2$

= 179.7867 + (955 + 5.6532)tan100/2

= 1324.65'

Stationing can now be determined for the control points.

PI = station 120 + 10.54 is given

TS = PI - T

= 12010.54 - 1324.65

= station 106 + 85.89

$SC = TS + (L_S \times 100)$

= 106 + 85.89 + 360

= station 110 + 45.89

CS = SC + (L x 100)

= 110+45.89 + 1306.67

= station 123+52.56

$ST = S + (L_S x100)$

= 360

= station 127+12.56

Spiral Deflections

For the deflections the spirals are divided into equal arcs. In the example above, where L_S = 360', the spirals may be divided into nine equal arcs of 40 feet each. (A railroad would have used chords.) A curve with equal spirals uses the same arcs and deflections at either end.

Since the deflection varies as the square of the distance (L_S'), the deflection angle df = $a(L_S')^2/6$ in degrees or $10a(L_S')^2$ in minutes at any point on the spiral. The rate of change of curvature was calculated first and is 1 2/3 or, to calculate more simply, a =5/3.

The deflections for the previous example are, therefore:

Station	Deflection
106+85.89 TS	0°
107+25.89	$(5/3)(0.4)^2/6$ = 0.044° = 0° 02' 40"
107+65.89	$(5/3)(0.8)^2/6$ = 0.177° = 0° 10' 40"
108+05.89	$(5/3)(1.2)^2/6$ = 0.399° = 0° 24' 00"

This process is continued until the S.C. is reached at station 110+45.89 where L_S' is 3.6. The deflections are the same for the spiral at the other end of the curve. The circular curve is run in the same manner as a circular curve without spirals on either end.

For further information concerning spiral curves consult the *Field Tables for Engineers, Spirals 1984.*

Field Procedures

One method for running a spiral is by occupying the TS (or ST). Set zero in the instrument and backsight a point on tangent or foresight a point on semi-tangent. Turn the calculated deflection and chain from station to station along the spiral.

Another method is to set out coordinate values for selected stations calculated with coordinate geometry software.

Offset Curves

It may be necessary to locate outer or inner concentric curves such as lane edges and offset curves for reference during construction. Spiral curve data can be calculated using the offset width (w) and the deflection angles for the segment: one measured from the forward tangent to the POS_1 at the beginning of the segment (df_1), and the other measured to the POS_2 at the end of the segment (df_2).

To sight along a line tangent to the spiral at any point (POS_2) on a spiral:

1. Occupy the TS.
2. Sight an intermediate point (POS1) for the deflection angle df_1.
3. Sight POS_2 for angle df_2.
4. Occupy POS_2 and sight POS_1.
5. Turn an angle = $2(df_2 - df_1)$ away from the TS to sight the tangent.

To sight radially at POS_2 follow steps 1 through 4 above and then turn 90° in either direction.

The length of a spiral curve segment of an offset curve is the center line segment plus (outside) or minus (inside curve) the amount $3[\sin(df_2 - df1)]w$.

See Circular Curves.

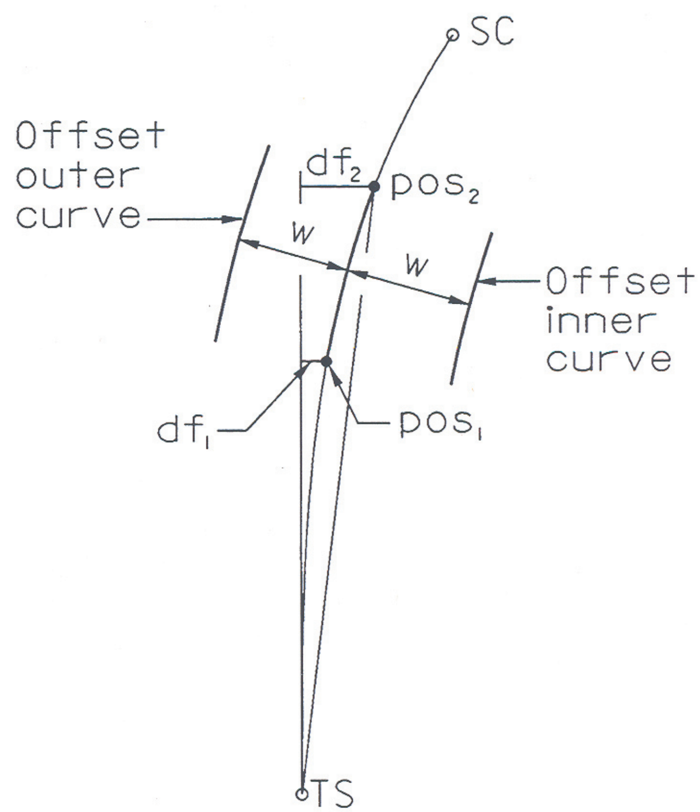


Figure 11–6

Vertical Alignment and Superelevation

Profile Grade

Grade is the rate of change in vertical elevation per unit of horizontal length. This rate is expressed in percent. For example, a 1 percent grade means a rise or fall of 1 foot in 100 feet of horizontal distance (rise over run).

To determine the grade between two points on a line, divide the difference in elevation in feet by the distance between the two points in stations (1 station = 100 feet) and multiply by 100 so the result will be in percent.

Example (metric)

Station	Elevation
193+60	16.00
211+75	80.61

distance 18.15 m difference in elevation 64.61 m

The grade is then +64.61/18.15 = +3.56%

Since the elevation ahead on line is higher than the elevation of the beginning station it is a plus grade. If the ahead elevation were lower, it would be a minus grade.

This **profile grade** describes the vertical alignment of the roadway and is shown on the profile sheets of the contract plans.

A finished roadway is not a flat plane. Instead, the roadway is sloped slightly to the sides to allow water to run off.

This requires that a definite point on the cross section be chosen to "carry" the grade. The lateral location of profile grade is shown in the roadway sections portion of the contract plans.

On construction, it is important to study the profile sheets and roadway sections and know where every change in the profile grade occurs. Serious staking errors can be made in the field by overlooking a lateral shift in the location of the profile grade.

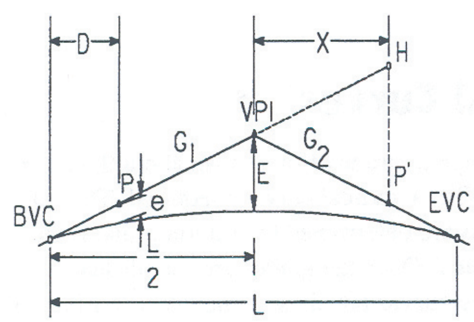
Vertical Curves

When a change in grade of more than about 0.5 percent occurs at a VPI, a vertical curve is required. The vertical curve lengths are determined by criteria found in the *Design Manua*l. Once the grades are established and the lengths of the curves are chosen, the vertical offsets of the curves can be computed. A vertical curve is a parabolic curve. When the grades form a peak or hill at the VPI, the curve is known as a **crest vertical** or **summit vertical** (Figure 11-7). When the grades form a valley or dip at the VPI the curve is known as a **sag vertical** (Figure 11-8).

When l_1 does not equal l_2 as shown in Figure 11-9 the curve is nonsymmetrical.

The vertical curve is computed by figuring offsets from the tangent grades. Subtract the offsets from the tangent grade elevations for crest verticals and add the offsets to the tangent grade elevations for sag verticals.

The following nomenclature and formulas are from the *Highway Engineering Field Formulas,* M 22-24.

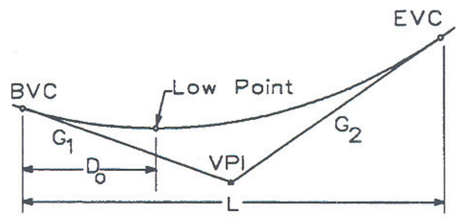


Crest Vertical Curve
Figure 11-7

Equations for Crest Vertical Curve

$$e = \frac{AD^2}{200L}$$

$$L_1 = \frac{2(AX + 200e + 20\sqrt{AXe + 100e^2})}{A}$$



Sag Vertical Curve
Figure 11-8

Equations for Sag Vertical Curve

$$E = \frac{AL}{800}$$

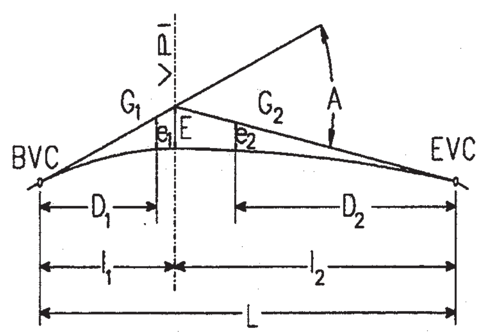
$$E = \frac{1}{2}\left(\frac{Elev.BVC + Elev.EVC}{2} - Elev.VPI\right)$$

$$e = \frac{4ED^2}{L^2}$$

Notes: All equations use units of length (not stations or increments)

The variable **A** is expressed as an absolute in (%) percent

Example: If G_1 = +4% and G_2 = -2%
Then **A**=6



Nonsymmetrical Vertical Curve
Figure 11-9

Equations for Nonsymmetrical Vertical Curve

$$A = |(G_2) - (G_1)|$$

$$L = l_1 + l_2$$

$$E = \frac{l_1 l_2}{200(l_1 + l_2)} A$$

$$e_1 = m\left\{\frac{D_1}{l_1}\right\}^2$$

$$e_2 = m\left\{\frac{D_2}{l_2}\right\}^2$$

Superelevation

On horizontal curves, the roadway is tilted so that the edge of the pavement at the outside of the curve is higher than the edge of the pavement at the inside of the curve. This is called **superelevation** and is done to counteract the centrifugal force, which tends to push the vehicle off the roadway at the outside of the curve.

The rate of superelevation is a function of the radius of the curve and the design speed. As the radius shortens for a given design speed or as the design speed increases for a given radius, the **super rate** must increase to keep the vehicle on the road. The maximum super rate is 0.10 ft/ft. For further information see Chapter 642 of the *Design Manua*l.

Going from a tangent section, which has a **normal crown** to a curve section, which has **full supe**r, there is a gradual change in the rate of superelevation called a **transitio**n.

Approximately three fourths of the transition is in the tangent section of the roadway and one fourth is in the curve. See the *Design Manual* for superelevation transition designs. The edge of the pavement that is on the outside of the curve begins to rise in relation to the center line (or reference point). The edge rises until the roadway is on one (sloping) plane from edge to edge. This is called **crown slope** and is usually 0.02 ft/ft. The entire roadway then rotates about the **pivot point** until it reaches its maximum or **full supe**r. Exiting a curve, the process is simply reversed, going from full super to crown slope to normal crown. See Figure 11-10.

The survey crew should not have to design supers but may have to compute grades for a station or offset on a section in transition. The contract plans will contain superelevation diagrams on the same sheet as the roadway profiles.

Figure 11-11 shows a super diagram for the roadway shown in Figure 11-10.

The contract plans and/or the roadway elevation listing (grade sheets) show the stations of the beginning of the transition, crown slope, and full super.

To find the super rate for any station in a transition:

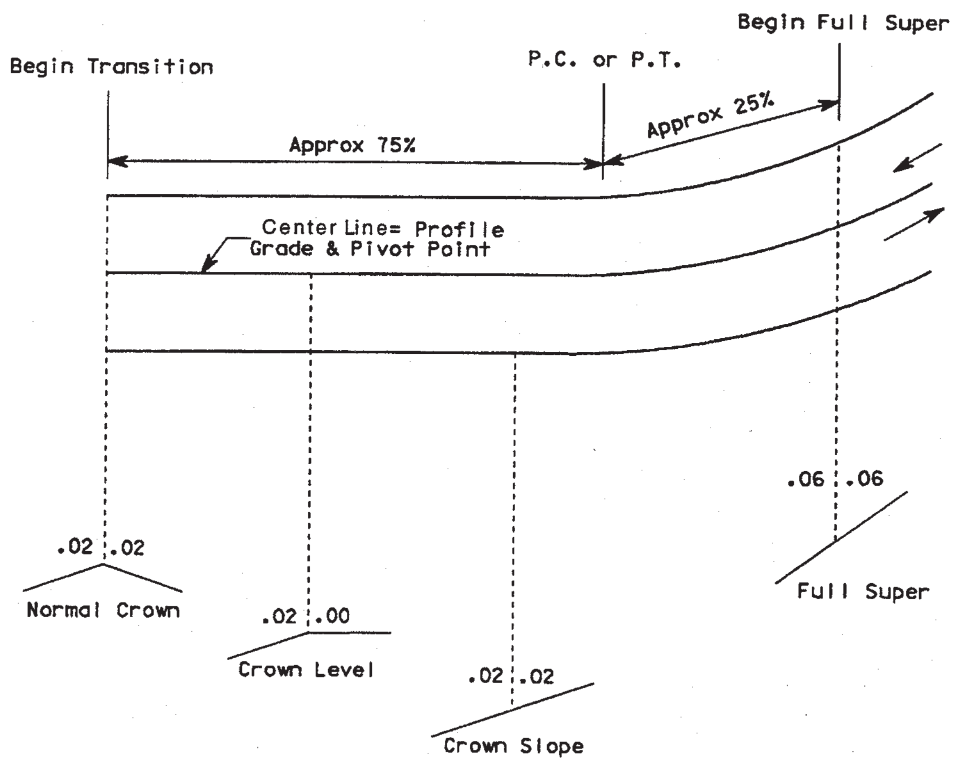
1. Subtract the begin transition station from the end transition station.
2. Subtract the desired station from the end transition station.
3. Subtract, algebraically, the crown slope from the full super rate.

4. Divide the super difference (#3) by the station difference (#1).
5. Multiply the desired station difference (#2) by the rate of change (#4).
6. Subtract #5 from the full super rate which gives the super rate for the station in question.

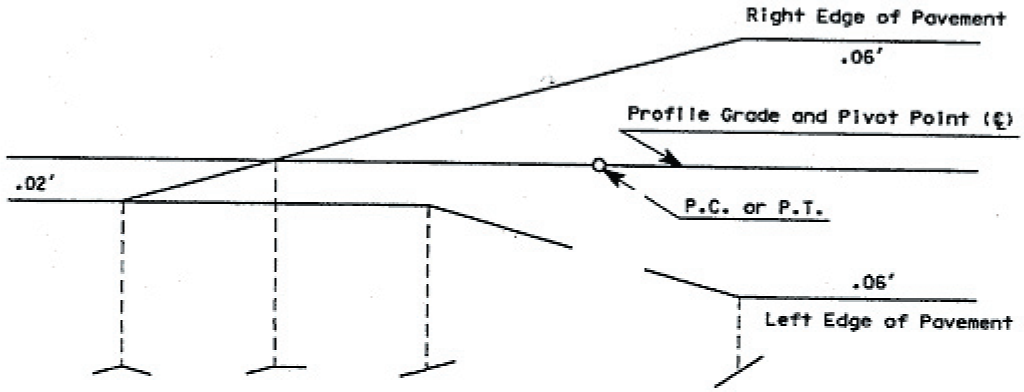
Example

Find the super rate at station 17+50 on the right edge of the pavement using Figure 11-11.

1. 18+20.68 – 16+04.68 = 216 ft
2. 18+20.68 – 17+50 = 70.68 ft
3. 0.06 - (-0.02) = 0.08 ft/ft
4. 0.08/216’ = 0.000370370 ft/ft/ft = rate of change
5. 0.000370370 x 70.68’ = 0.02618 ft/ft
6. 0.06 – 0.02618 = 0.03382 ft/ft



Superelevation for Two-Lane Highway
Figure 11-10



Normal Crown Begin Transition 16+04.68	Crown Level 16+57.70	Crown Slope 17+11.70	Begin Full Super 18+20.68

Superelevation Diagram for Two-Lane Highway
Figure 11-11